ELECTRIC TRACTION

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BY

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PREFACE
TO THE THIRD EDITION

THE task of bringing the previous (1929) edition up to date has presented many difficulties due to the extraordinary high costs of book production at the present day, which has necessitated much pruning in order to obtain a volume suitable for students and engineers interested in Electric Traction.

The task has been doubly difficult because of the very recent developments in single-phase traction at 50 cycles per second, the application of mercury-arc rectifiers to railway vehicles, and the increasing use of self-propelled thermo-electric locomotives, for which space had to be provided. The author is aware of many omissions necessitated by exigencies of space.

The present volume follows the lines of the preceding editions, i.e. all the essential principles of the subject are covered together with representative examples of railway and road vehicles and their equipment.

In the task of revision the author has had the generous co-operation of a large number of engineers, in this country and abroad, to whom he is under considerable obligation and to whom he tenders his grateful thanks.

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To the Senate of the University of London, the Examinations Board of the City and Guilds of London Institute, and the Council of the Institution of Electrical Engineers, the thanks of the author are due for permission to use questions from their examination papers, and, in the case of the Institution of Electrical Engineers, the loan of blocks and the use of illustrations from the *Journal* and *Proceedings of the I.E.E.*

A. T. DOVER

LONDON, 1953

CONTENTS

LIST OF ABBREVIATIONS

Ampere	A
Centimetre	cm
Square centimetre	cm^2
Cycles per second	c/s
Electromotive force	e.m.f.
Foot, feet	ft
Horse-power	h.p.
Inch	in.
Square inch	$in.^2$
Kilogramme	kg
Kilometre	km
Kilovolt	kV
Kilovolt ampere	kVA
Kilowatt	kW
Kilowatt-hour	kWh
Miles per hour	m.p.h.
Miles per hour per second	m.p.h.p.s.
Millimetre	mm
Pound	lb
Revolutions per minute	r.p.m.
Revolutions per second	r.p.s.
Root-mean-square	r.m.s.
Second	sec
Volt	V
Watt	W
Watt hour	Wh

LIST OF TABLES

LIST OF SYMBOLS

A, a = Cross-sectional area
a = Number of circuits in armature winding
B = Flux density (B_m = maximum flux density)
D = Distance (in miles) between stops
D = Distance (in miles) from start to cut off
D_a, D_c, D_w = Diameter of armature, commutator, or driving wheel
E = e m f generated in armature or stator winding
E_a = Phase e m f of rectifier
E_d = Output voltage of rectifier
E_{av} = Average voltage between commutator segments
E_s = Static (induced) e m f
E_t = Transformer e m f induced in armature coils short circuited by brushes
F = Force or tractive effort
f = Frequency (cycles per second)
G = Gradient per cent
h = Number of hours per annum
I = Current (I_1, I_2, upper and lower limits, respectively)
I_d = Current output from cathode of rectifier
K, k = Constants
k = Radius of gyration
= Air resistance coefficient for shape of end of train
L, l = Length
L = Inductance
m = Number of armature coils short circuited by a brush
= Number of phases in supply to rectifier
N = Number of turns per circuit in armature or stator winding
n = Revolutions per minute
n_s = Synchronous speed in revolutions per minute
n_c = Cascade synchronous speed
P = Power
= Pull on poles in span wire construction
p = Number of poles
= Price (in pence) of 1 kilowatt hour
Q = Specific electric loading
R = Resistance in ohms
= Total train resistance in lb
r = Specific train resistance in lb per ton
R, r = Radius
S = Longitudinal exposed surface of coach
s = "Slip" of rotor of induction motor
= Half length of span of span wire
T, t = Time in seconds

T = Tension in trolley wire (or catenary wire) at lowest point.
U = Hypothetical speed of train.
V = Speed of train or car in m.p.h. (V_m = maximum speed, V_a = average speed).
= Terminal voltage.
v_c = Peripheral speed of commutator.
W = Dead weight of train in tons.
= Weight per span of trolley wire (or span wire).
W_e = Accelerating weight of train in tons.
w = Weight per foot of trolley wire (or span wire).
X = Reactance.
y = Deflection at dropper in catenary suspension.
z = Distance between trolley wire and catenary wire at mid-span.
Z = Impedance.
α = Acceleration in m.p.h.p.s.
β = Braking retardation in m.p.h.p.s.
β_c = Coasting retardation in m.p.h.p.s.
β_r = Retardation when braking regeneratively.
Δ = "Delta" connexion of three-phase circuits.
δ = Sag of trolley wire (or catenary wire).
η = Efficiency.
Φ = Flux (Φ_m = maximum flux).
ϕ = Phase angle between e.m.f. and current.
γ = Gear ratio.
= Grading coefficient in rheostat calculations ($= I_2/I_1$).
λ = Ratio exposed transverse surface of coach/cross section of coach body.
= Spacing of droppers in catenary suspension.
= Grading coefficient in rheostat calculations ($= (\Phi_1/I_1)/(\Phi_2/I_2)$).
τ_c = Pitch of commutator segments.
θ = Temperature.
ω = $2\pi \times$ frequency.
$\mathfrak{T}$ = Torque.

NOTES

LOGARITHMS, where used, are to the base 10.

VECTOR DIAGRAMS.—All diagrams have been drawn for *counter-clockwise* rotation.
E.M.F. vectors are represented by an ordinary arrow-head.
Flux vectors are represented by a double arrow-head.
Ampere-turn vectors are represented by a solid arrow-head.
Current vectors are represented by a closed arrow-head.

WEIGHTS.—The British ton (2,240 lb) is used throughout this book.

CHAPTER I

INTRODUCTORY

THIS book is concerned mainly with the technical considerations involved in the distribution and application of electric power to the working of railway trains, tramcars and trolleybuses. The equipment of commercial battery road vehicles is also considered.

Systems of Operation for Tramways and Trolleybus Routes. In Great Britain the system of operation and the operating voltage are prescribed by the Regulations of the Ministry of Transport. Thus only direct current is permitted and the voltage is limited to a maximum of 650 volts at the power station. In practice the working voltage at the vehicles is about 525 volts.

The d.c. system is also employed in other countries (because of its advantages over a.c. systems) but the working voltage on some trolleybus routes is higher than that in this country (e.g. 1,000 to 1,500 V).

The power is supplied to the vehicles by overhead conductors fed at suitable points from either a central power station or sub-stations. A single overhead conductor (of positive polarity) is employed for tramways, the rails forming the return conductor. Two overhead conductors are necessary for trolleybuses. The current collector on the vehicle takes the form of either a wheel or slider at the end of a pole (called a "trolley collector") or a sliding contact strip carried at the end of a light pantograph framework fixed to the roof of the vehicle.

Comparison of Tramway, Trolleybus and Other Forms of Road Transport. In any system of electric traction a heavy initial expenditure is necessary for the power supply system, and, in the case of tramways, for the track rails and electrical return system, in addition to the cost of the vehicles, depots, etc., before any revenue can be earned. In this country Parliamentary powers have to be obtained for tramways and trolleybus systems and details of the vehicles have to be submitted to the Ministry of Transport for approval. Moreover, a number of somewhat severe statutory regulations, concerning leakage currents from overhead conductors and the voltage drop in the track rails, have to be complied with, while in the case of a tramway system operating through public thoroughfares the authority is responsible not only for the maintenance of the track rails but also for the paving (used by other vehicles) between these rails and a distance of 15 in. on either side.

On the other hand a transport system operating oil-engined vehicles is involved initially only with the cost of the vehicles and garages. But the propulsive equipment of these vehicles has a much shorter life than the electrical equipment of a tramcar or a trolleybus, and this equipment together with the brakes requires considerably more maintenance than the corresponding equipment of electrically propelled vehicles. The oil-engined vehicle is also dependent upon imported fuel, the cost of which is liable to considerable fluctuation. Another disadvantage of this vehicle is its limited maximum power due to the speed/torque characteristics of the engine, whereas the propulsive equipment of electric vehicles can withstand large temporary overloads and can draw relatively large power from the distribution system.

The trolleybus can accommodate slightly more passengers than an oil-engined bus, and because of its higher acceleration and braking retardation the schedule speed over a given route will be higher than that of the latter vehicle, so that fewer trolleybuses will be required for operating a given service. A disadvantage of the trolleybus is that it is tied to a definite route, but for the conditions favourable to the operation of trolleybuses—i.e. frequent service with moderately heavy traffic density—this limitation cannot be considered as a real disadvantage, as the routes are selected with discrimination and must in this country be approved. Moreover, immobilization of vehicles, due to diversions necessitated by road repairs or other contingencies, can be prevented by increasing the size of the lighting battery so that it can be used as a source of power for propelling the vehicle at low speed over such diversions.

In general the *modern* tramcar, with equipment for high acceleration and retardation, is best suited for high traffic density in large cities, as such cars can accommodate many more passengers than buses. In European cities, for example, where trailers are permitted, a single-deck metre-gauge car with two trailers can, under peak traffic conditions, transport 300 passengers. Under such conditions the revenue earned is sufficient to provide for adequate maintenance of the track and distribution system, and the traffic congestion is less than that which would occur if a larger number of buses were employed to handle the same traffic.

Tramcars, trolleybuses and oil-engined buses, therefore, have their particular spheres of operation, and, when free from political influence or harsh statutory regulations, each can be applied to give satisfactory service and provide adequate revenue. For example, the city of Zürich has a co-ordinated transport system of tramcars, trolleybuses and oil-engined buses. Tramcars are employed in the central districts where the traffic is very dense and the streets are congested with private motorcars, trolleybuses are employed for the inner suburbs with medium traffic density and oil-engined buses for the outer suburbs and country districts.

Methods of Supplying Power to Railway Trains. These comprise (i) the overhead system, (ii) the conductor rail system.

The *overhead system* must, obviously, be adopted when the trains are supplied at high voltage. Under these conditions heavy trains may be supplied through conductors of relatively small cross-section, and the collection of the current required by a heavy train can be performed satisfactorily by a collector with a sliding contact. Overhead construction is universal for all a.c. railways, and it is also adopted for d.c. railways operating at voltages of 1,500 volts and above. In all these cases the track rails are utilized as the return conductor, so that with d.c. and single-phase systems only one overhead wire is required for each track.

The *conductor rail system* is adopted for heavy electric traction systems operating at voltages up to about 800 volts*, since, in these cases, large currents may be required by the trains. The power is supplied to the trains through high conductivity steel rails, which are supported on insulators parallel with the track rails and fed at suitable points from sub-stations. The current is conveyed from the conductor rails to the train equipment by means of collector shoes. In the majority of cases the track rails are used as the return conductor, so that only one conductor rail is required.

* The Manchester-Bury section of the London Midland Region operates at 1,200 volts with conductor rails.

Technical Aspects of Railway Electrification. In this country electrification has been confined to the urban and suburban lines in the vicinity of our large cities (e.g. London, Liverpool, Newcastle, Manchester).* On the Southern Region, however, the suburban electrification has been extended to main lines serving the South Coast (e.g. Brighton, Hastings, Portsmouth), the longest route being 87 miles. These lines operate with conductor rails at the same voltage (660 V) as the suburban lines. In the United States of America and on the Continent electrification has been carried to trunk lines and also to freight lines operating in mountainous districts. The tendency in the U.S.A., however, is towards the adoption of Diesel-electric traction on main lines as fuel oil is readily available at low cost.

The chief difficulties in the way of electrification of our trunk lines are: first, the existence of the modern steam locomotive; and, second, the large cost of converting the lines from steam to electric operation. The modern steam locomotive in service on our trunk lines is capable of fulfilling all the requirements of the traffic department for fast passenger traffic.

The case is entirely different with suburban railways. On these railways large numbers of passengers, have to be transported daily over relatively short distances in competition, in many cases, with other methods of transportation. For the railway to retain its traffic and create additional traffic, the passengers must be transported over a given distance in a much shorter time than that required by its competitors.

Now, the frequent starting and stopping of steam trains at stations spaced a short distance apart does not lead to economical operation and, moreover, it is impossible to obtain high schedule speeds unless exceptionally heavy locomotives are adopted. On the other hand, an electric train is capable of handling such a service economically at a schedule speed which will attract traffic. This schedule speed may be from 50 to 100 per cent higher than that corresponding to steam operation, the increase being due to the higher acceleration of the electric train. Since the electric train is capable of running the service at a higher schedule speed, it follows that the train miles which can be run with a given equipment in a given time are greater with electric service than with steam service. A given number of electric trains is therefore capable of dealing with a greater volume of traffic than the same number of steam trains with equal seating accommodation. The electric train has the additional advantage that it may be divided and run in sections during the periods of light traffic, thereby enabling a frequent service of trains to be maintained, leading to increased traffic during these periods.

Electric traction also forms a solution to the problem of relieving congestion at terminal stations. Thus the number of trains which can be got into and out of a terminus in a given time depends on the number of signal and train movements required. With electric trains consisting of motor-coaches, the number of signal and train movements required for a train entering and leaving a terminus is only one-fourth of the number required for a steam train. Now the number of trains which can be run over the tracks in a given time is limited by the terminal facilities. It is obviously more desirable to increase these facilities by adopting electric traction than by the alternative of carrying out the widening of the tracks and the extensions and additions to the station platforms, since the electric train service will, in most cases, lead to additional

* Brief details of these electrifications are given in a paper, by C. M. Cook, on "Railway Electrification in Great Britain." *Proc. I.E.E.* Vol. 97, Pt. IA, p. 6.

traffic, the revenue from which will go towards meeting the cost of electrification.

When main-line passenger traffic is operated by electric traction increased comfort and amenities are available for the passengers due to the absence of grit, cinders, and smoke in the corridors; moreover, with electric operation, the rolling stock requires less cleaning and maintenance than with steam operation and the fire hazard in forest regions is eliminated.

Electric traction also possesses advantages over steam traction for the handling of freight traffic, particularly on railways having heavy gradients and long tunnels. In deciding upon the system of electrification to be adopted for a railway with heavy gradients, consideration would naturally be given to those systems in which electric regenerative braking could be used. The trains descending the gradients would be braked electrically, so that, instead of the kinetic energy of the train being dissipated in the brake shoes and wheel tyres, it would be converted into electrical energy and returned to the supply system. Thus, in addition to the saving in the power consumption, the maintenance of the brake shoes, wheel tyres, and track rails would be reduced. The reduction in the latter items alone may be sufficient to cover a fair percentage of the costs of electrification.

Systems of Railway Electrification. These comprise (i) the direct-current system, (ii) the single-phase alternating-current system, (iii) the three-phase alternating-current system, and (iv) the composite single-phase three-phase, and single-phase direct-current systems.

In the *d.c. system* the energy required by the trains must be obtained from sub-stations (except in the case of a very short railway where a direct supply from a direct-current generating station may be possible) which receive energy from a three-phase high-voltage transmission system. The sub-stations must, therefore, contain converting plant, e.g. transformers and mercury-arc rectifiers. For heavy and dense traffic on suburban railways the sub-stations will have to be spaced about 2 to 3 miles apart, but for main-lines operating at high voltage (3,000 V) the distance apart of the sub-stations will be between 10 and 30 miles. All of these sub-stations may be of the unattended type.

The train equipments must be built for operation at the line voltage, but with voltages above 1,500 volts two or more motors may be operated permanently in series, so that the terminal voltage per motor is limited to one half, or a smaller fraction, of the line voltage. The insulation of the motors, however, must be designed to withstand the full line voltage.

In the *single-phase system* the energy required by the trains may be obtained directly, at high voltage, from a generating station when the extent of the electrification is within a radius of about 20 miles from the generating station. For longer distances the economic voltage for the power transmission system is higher than that which is desirable for the traction system, and therefore transformer sub-stations become necessary.

As single-phase traction motors are inherently low-voltage machines, a transformer must be carried on each train to supply the motors at suitable voltage. The motors are of the series type.

In the *three-phase system* the energy required by the trains may be obtained either directly from the generating station, or from transformer sub-stations which receive energy from a high-voltage transmission system. Since two trolley wires per track are necessary, the line voltage of the traction system

has to be limited to values below the highest voltages employed for single-phase railways, and as the traction motors (which are of the induction, or constant-speed, type) can be built economically for operation at moderately-high voltages, the line voltage is usually chosen so that the motors may be supplied directly from the trolley wires. The induction type of motor is employed, as, at present, no variable-speed polyphase traction motor has been developed. The induction motor, however, possesses the merits of a high efficiency and of operating as a generator when it is driven at speeds above the synchronous speed and is connected to the supply system. The motor equipment of a train can, therefore, be used for regenerative braking during the descent of gradients, and the constant-speed characteristic of the motors limits the speed of the train to a definite value.

In the *composite systems* the energy is distributed in the form of single-phase current at high voltage, and is converted on the locomotives into either three-phase or direct current at low voltage for utilization in the traction motors; a transformer and phase-converter being necessary in one case, and a motor generator in the other case, for the conversion. These systems, therefore, combine the advantages of high-voltage single-trolley-wire distribution with efficient traction motors suitable for regenerative braking. Their development has been due to the inferiority of 25-c/s, single-phase motors, compared with three-phase and direct-current motors for regenerative braking on a large scale, and their present application is limited to mountain ranges having heavy mineral traffic. Recent developments in mercury-arc rectifiers for locomotive service have enabled such equipment to be employed, instead of a.c./d.c. motor generators, on locomotives for operation over routes where regenerative braking is not required.

Operating Voltages. *Direct current* urban and suburban electrifications in the vicinity of large cities have operating voltages between 600 and 750 volts. For the longer distance suburban services, however, a higher voltage, e.g. 1,500 volts, is desirable. For main-line electrification, voltages of 1,500 and 3,000 volts are employed.

Single-phase railways have operating voltages between 15,000 and 25,000 volts, and the frequency is $16\frac{2}{3}$, 25 or 50 c/s.

Three-phase main-line railways operate at a voltage between 3,300 and 3,600, and a frequency of $16\frac{2}{3}$ c/s.

Comparison and Applications of Systems of Railway Electrification. The *d.c. system* is the oldest of the systems of railway electrification. Its greatest application has been to urban and suburban services at operating voltages between 500 and 800 volts, the first installations being made in about 1890 in this country and a few years earlier in America.* Its application to long-distance main-line service required the development of high-voltage d.c. machines and control apparatus. This development started in America, and the first installation (at an operating voltage of 2,400 V) was made in 1913.

The 1,500-volt system is in use in this country on suburban lines in the London, Manchester and Birkenhead districts, and is the recommended system

* The pioneer developments in the d.c. system of electric traction started about five years earlier, when electric tramways commenced operation, the overhead-trolley system and an operating voltage of 500 volts being employed.

for future electrifications.* It is extensively used for suburban and main lines in many countries (e.g. France, Holland, Denmark, Spain, Portugal, India, Australia, New Zealand, Japan). The 3,000-volt system is in use in Italy, Belgium, South Africa, North and South America, Poland.

The early applications of the *three-phase system* were to Swiss mountain railways, and the first application to main-line railways (operating at 3,000 volts) was in 1902. Subsequent developments of the three-phase system have been entirely for main-line electrification, and the principal applications are in Northern Italy. These lines, however, will be ultimately converted to 3,000 volts, d.c., which is now the standard system for main-line electrification in Italy.

The *single-phase system* required the development of a suitable single-phase traction motor. The system has received its greatest development on the Continent (although much pioneer work was done in America), and extensive applications are to be found in Switzerland, Germany, Austria and Scandinavia, on suburban as well as main-line railways. These electrifications are all at low frequency (16⅔ c/s).

In America about 500 route miles (on the Pennsylvania and New Haven lines) are electrified on the single-phase system, but the frequency is 25 c/s and the operating voltage is 11 kV.

In France a line in the Savoy Alps (Aix les Bains—Annecy—La Roche sur Foron) is operating at the industrial frequency of 50 c/s (20 kV) to try out locomotives and motor coaches equipped with single-phase commutator motors, which have recently been developed for this frequency. Other trial locomotives and motor-coaches are equipped with rectifiers and d.c. motors, and a locomotive equipped for dual operation at full power on both the single-phase and d.c. (1,500 V) systems has been built. This (50-c/s) system is employed for extensive electrifications in the industrial districts of Northern France.

The single-phase 50-c/s system has also been adopted for the electrification of the Katanga lines in the Belgian Congo, which serve rapidly developing industrial districts with increasing traffic. The first stage of the electrification (completed in 1952) comprised 105 km (65 miles) of route, and extensions will bring the total mileage to about 200 miles. Locomotives equipped with 50-c/s series motors are employed.

The *composite systems* were developed later than the 25-c/s system to overcome the disadvantages of the straight single-phase system for heavy mineral traffic on mountain grades. They must, therefore, be considered as entirely special systems of electrification, applicable only to exceptional operating conditions.

The single-phase/three-phase system is in operation in Hungary (50-c/s supply) and America (Virginian Railway, 25-c/s supply). The single-phase/d.c. system is in operation in America (Great Northern Railway (Cascade Mountains section) and Virginian Railway.)

Hence the *systems available for any new electrification* are (i) the d.c. system and (ii) the single-phase system.

For *heavy suburban service* the low-voltage d.c. system is undoubtedly superior to the single-phase system, as the following comparison will show—

The dynamical characteristics of the d.c. traction motor are better suited for the frequent and rapid acceleration of heavy trains than those of the single-phase traction motor.

* British Transport Railway Commission Report, 1951.

The d.c. train equipments are lighter, more efficient, and less costly (both initially and in maintenance) than the corresponding single-phase equipments. Moreover, the energy consumption of a d.c. train will be lower than that of a single-phase train operating under similar service conditions.

The conductor-rail distribution system will be less costly, both initially and in maintenance, than the high-voltage overhead distribution system.

The d.c. system causes no interference with neighbouring overhead communication (telephone) circuits. This interference may be serious with the single-phase system, and may necessitate placing the communication circuits either along another route or underground. Moreover, due to the high impedance of the track rails, when used as conductors for alternating current, special arrangements (involving transformers and insulated cables) are necessary with the single-phase system to relieve the track rails of the return current, and so avoid troubles due to earth currents (such currents affecting telegraph and other weak-current circuits using the earth as a return).

These advantages of the d.c. system will generally more than compensate for the chief disadvantage of this system, namely the necessity for converting sub-stations at relatively short distances apart.

For *main-line long distance services*, however, both high-voltage d.c. and single-phase systems are capable of giving satisfactory results. The single-phase system appears to possess advantages over the d.c. system for long-distance electrification, on account of the higher line voltage and the fewer sub-stations. The chief disadvantages are—interference with neighbouring tele-communication circuits, unbalancing of the primary supply system (with 50-c/s traction), high voltage drop in the rails. Although the d.c. system requires sub-stations equipped with mercury-arc rectifiers, such sub-stations are now always unattended and remote controlled.

Further points of comparison between the high-voltage d.c. and single-phase systems for long-distance electrification are*—

Single-phase locomotives are more expensive initially than d.c. locomotives.

Regenerative braking can be obtained more efficiently, and with less complication, and less additional equipment, with the d.c. system than with the single-phase system.

* The technical problems (including energy consumption, capital outlay and annual expenditure) concerned with d.c. and a.c. systems of railway electrification are discussed by A. E. Müller in an article, "Considerations on single-phase 50-c/s traction from public supply systems," *Brown Boveri Review*, Vol. 39, p. 263. [This publication and other "house" journals are available for reference in the library of the I.E.E.]

CHAPTER II

THE MECHANICS OF TRAIN MOVEMENT

I. PRELIMINARY STUDY OF SPEED-TIME CURVES

THE motion of a train or any vehicle is made up of periods of acceleration, of retardation, and, in some cases, of constant speed. Now, acceleration and retardation represent the rate of change of speed with respect to time; therefore, a curve which shows the speed of the train with respect to time will also supply information concerning the acceleration and retardation. For example, the acceleration or retardation at any instant can be obtained by determining the tangent of the angle of inclination of this curve (at the given instant) to the time axis—an upward slope (tangent positive) indicating acceleration, and a downward slope (tangent negative) indicating retardation. The acceleration, or retardation, obtained by this method will be given in terms of the units adopted for the axes of speed and time. If the former is represented in miles per hour and the latter in seconds, the acceleration or retardation will be expressed in miles per hour per second (abbreviated, m.p.h.p.s.).

Further, the distance travelled by the train during a given interval of time can be obtained by determining the area between the curve and the time axis corresponding to this interval.

Such speed-time curves are of *fundamental importance* in electric traction, as when the speed-time curve is given and the resistances to motion of the train or vehicle are known the energy required for propulsion can be calculated. Moreover, when the characteristic curves of the motors are known the energy input to the train or vehicle can also be calculated.

Analysis of Speed-Time Curve for Electric Train. A speed-time curve, for a run between two station stops, may consist of periods of (i) acceleration; (ii) constant speed, or "free running" (which may be zero for short distance runs); (iii) coasting, i.e. running with power shut off, the retardation being due to the resistances to motion; and (iv) retardation due to braking.

With electric trains, equipped with series motors, the period of acceleration is made up of (*a*) an initial period, during which the acceleration is practically constant, followed by (*b*) a period in which the acceleration gradually decreases until the maximum speed is reached.

The period of constant acceleration corresponds to the "notching" or starting period, during which the current input to the motors can be maintained at a definite mean value.

The period of decreasing acceleration commences when full voltage is applied to the motors, as then the current and torque decrease as the speed increases. Therefore the acceleration will gradually decrease until the torque is just sufficient to balance that due to resistances to motion. The shape of this portion of the speed-time curve will depend entirely on the shape of the speed-torque curve of the motor, and will be affected to some extent by variations in the line voltage and in the resistances to motion.

These two portions of the accelerating period are called respectively "*rheostatic acceleration*" or "acceleration while notching," and "acceleration on the speed curve" or "*speed-curve running.*"

The duration of the free-running and coasting periods will depend on the nature of the service (that is, the distance between the stops and the average speed between the stations), and will be affected by the acceleration and retardation, as discussed below.

Typical speed-time curves for electric trains operating on passenger services are given in Figs. 1, 2, 3. Each curve corresponds to a particular class of traffic, (1) urban or city service, where the distance between stops is of the order of 0·5 mile; (2) suburban service, where the distance between the stops may average from 1·5 to 2 miles over a distance of from 15 to 20 miles from the city terminus; (3) main-line service, where the stops are infrequent.

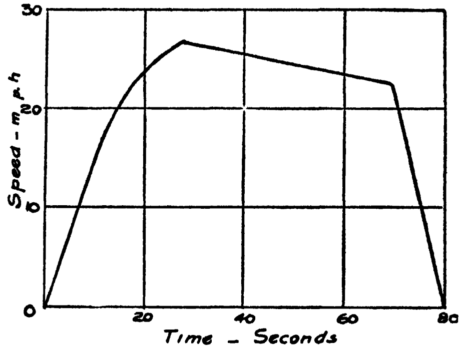


Fig. 1. Speed-Time Curve for City Service

In Fig. 1, which is representative of city service, relatively high values must be adopted for the acceleration and retardation in order to obtain a moderately high average speed between the stations. The short distance between the stations does not permit of a free-running period since it is desirable to include a short coasting period in order to obtain a reasonable energy consumption. This class of traffic requires a frequent service of trains.

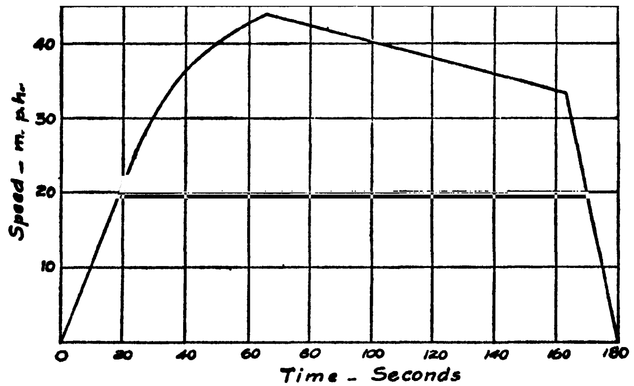


Fig. 2. Speed-Time Curve for Suburban Service

In suburban services (Fig. 2) the longer distance between the stations permits of a longer coasting period than is possible with city service. In this case, also, relatively high values for the acceleration and braking retardation are required in order to render the service as attractive as possible.

Main-line service (Fig. 3) is characterized by the long periods of free-running at high speeds, the accelerating period being relatively unimportant.

The *free-running* (i.e. *constant* speed) period occurs, on level track, when the power output from the driving axles balances the rate at which energy is expended against the resistances to motion.

The *coasting* period, during which the retardation (due to the resistances to motion) is practically constant, is desirable for two reasons—(i) it provides a leeway in operation which can be used for making-up time lost by temporary speed checks, (ii) it utilizes some of the kinetic energy of the train, which would otherwise be wasted during braking, and therefore tends to reduce the energy consumption of the train.

Approximate Values of Acceleration and Retardation. The *initial acceleration* of electric trains on suburban and urban services is usually between 1·0 and 2·5 m.p.h.p.s., the higher value being desirable for urban services with dense

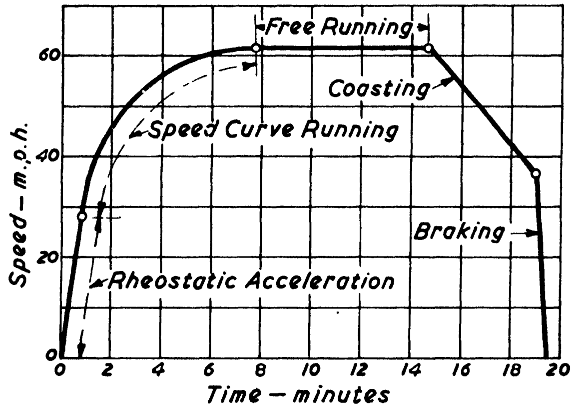


FIG. 3. SPEED-TIME CURVE FOR MAIN-LINE SERVICE

traffic. In contrast, when suburban railways were operated by steam locomotives the initial acceleration was of the order of 0·5 m.p.h.p.s. With trolley-buses the initial acceleration is between 2·5 and 4 m.p.h.p.s., the higher value being employed in America.

For a given service the initial acceleration depends on a number of limiting factors, such as (i) the size and weight of the equipment; (ii) the peak load on the sub-stations; (iii) the discomfort of the passengers; (iv) the cost of the equipment and the maintenance charges thereon; (v) the cost of that portion of the rolling stock which is attributable to the motor equipment and acceleration; (vi) the cost of energy.

The *coasting retardation* is directly related to the resistances to motion during coasting. For suburban trains running on level straight track it is of the order of 0·1 m.p.h.p.s.

The *braking retardation* for urban and suburban services is between 2 and 3 m.p.h.p.s., a high retardation being necessary in these cases to obtain the full advantages of high acceleration and relatively long coasting periods.

Simplified Speed-Time Curve. When comparative performances for a given service, at various schedule speeds,* are required (for example, in preliminary calculations for time-tables, etc.), the actual speed-time curves of Figs. 1, 2, 3, are replaced by simplified speed-time curves, which do not involve

* Schedule, or "booked," speed is the average speed obtained when the duration of the stop is included.

a knowledge of the motor characteristic. These simplified speed-time curves have simple geometric shapes, and, in consequence, the relationship between the acceleration, retardation, average speed, and distance can be deduced by simple mathematics.

For example, the speed-time curve of Fig. 1 can be replaced by either of the equivalent quadrilateral diagrams shown in Fig. 4, in both of which the

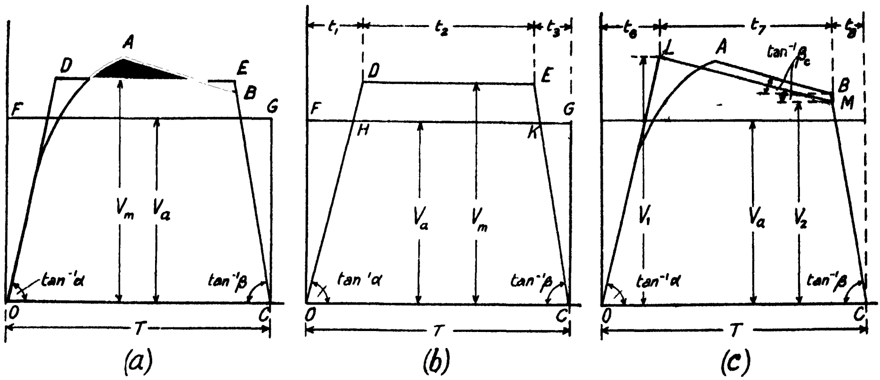


FIG. 4. METHODS OF SIMPLIFYING SPEED-TIME CURVES FOR PRELIMINARY CALCULATIONS

acceleration and braking retardation have the same values as those in Fig. 1, and all diagrams enclose the same area. The speed-curve-running and coasting periods of the actual speed-time curve, Fig. 1, are replaced in one case, Fig. 4 (*b*), by a constant-speed period, and in the other case, Fig. 4 (*c*), by extensions of the initial accelerating and coasting periods.

The trapezoidal diagram, Fig. 4 (*b*), gives the simpler relationship between the principal quantities concerned with speed-time diagrams, and also gives a close approximation of the actual energy required for propulsion on long-distance runs on level track. On the other hand, the quadrilateral diagram, Fig. 4 (*c*), approximates more closely to the actual conditions on short-distance runs in which coasting is an important item.

Relationship between Principal Quantities in Speed-Time Curves. Considering the *trapezoidal speed-time curve*, Fig. 4 (*b*), let

D = distance between stops, in miles.

T = running time, in seconds.

α = acceleration, in miles per hour per second.

β = retardation, in the same units.

V_a = average speed, in miles per hour (= $3{,}600D/T$).

V_m = free-running, or maximum, speed in miles per hour.

t_1 = time of acceleration, in seconds (= V_m/α).

t_3 = time of braking, in seconds (= V_m/β).

t_2 = time of free running, in seconds [= $T - (t_1 + t_3)$].

Then the area of the trapezium—which represents the distance between stops—is given by*

$$\begin{aligned} D &= V_m(\tfrac{1}{2}t_1 + t_2 + \tfrac{1}{2}t_3)/3{,}600 \\ &= V_m[\tfrac{1}{2}(t_1 + t_3) + T - (t_1 + t_3)]/3{,}600 \\ &= V_m[T - \tfrac{1}{2}(t_1 + t_3)]/3{,}600 \end{aligned}$$

Substituting for t_1 and t_3 in terms of V_m, α, β, we have

$$D = \frac{1}{3{,}600}\left\{V_mT - \tfrac{1}{2}V_m^2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right\} \quad . \quad . \quad . \quad (1)$$

When α or β is to be determined we re-arrange this equation in the form

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7{,}200}{V_m^2}\left(\frac{V_mT}{3{,}600} - D\right)$$

or, since $T/3{,}600 = D/V_a$, we have

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7{,}200D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right) \quad . \quad . \quad . \quad . \quad (2)$$

When V_m is to be determined, equation (1) is re-arranged as an ordinary quadratic equation, thus

$$\tfrac{1}{2}V_m^2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) - V_mT + 3{,}600D = 0$$

and the solution is obtained by general algebraic rules. For example, in the case of an equation of the form $ax^2 + bx + c = 0$, the solution is given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \text{ or } x = -\frac{b}{2a} \pm \sqrt{\left[\left(\frac{b}{2a}\right)^2 - \left(\frac{c}{a}\right)\right]}$$

Hence, in the present case, where

$$x = V_m,\ a = \tfrac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right),\ b = -T,\ c = 3{,}600D$$

$$\text{we have } V_m = \left(\frac{\alpha\beta}{\alpha + \beta}\right)T \pm \sqrt{\left[\left(\frac{\alpha\beta}{\alpha + \beta}\right)^2T^2 - 7{,}200D\left(\frac{\alpha\beta}{\alpha + \beta}\right)\right]} \quad . \quad (3)$$

The numerical evaluation of this equation is effected expeditiously by the use of the slide rule, but it becomes tedious when ordinary arithmetic is employed. An *alternative equation* for V_m, which avoids the extraction of a square root, is readily derived from the geometry of Fig. 4 (*b*). Thus, since the trapezium *ODEC* and the equivalent rectangle *OFGC* are of equal area, the area of the smaller trapezium *HDEK* must be equal to the combined area of the two triangles *OFH*, *KGC*. If the periods *FH*, *KG* are denoted by t_4, t_5, respectively, the equality of these areas is given by

$$\tfrac{1}{2}(V_m - V_a)[t_2 + T - (t_4 + t_5)] = \tfrac{1}{2}V_at_4 + \tfrac{1}{2}V_at_5$$

Now $\qquad t_1 = V_m/\alpha,\ t_3 = V_m/\beta,\ t_4 = V_a/\alpha,\ t_5 = V_a/\beta.$

Hence, substituting for these quantities and re-arranging terms we have

$$(V_m - V_a)\left\{T - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\left[\tfrac{1}{2}(V_m + V_a)\right]\right\} = \tfrac{1}{2}V_a^2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

and, finally,

$$\begin{aligned} V_m &= V_a + \frac{\tfrac{1}{2}V_a^2(\alpha + \beta)/\alpha\beta}{T - \tfrac{1}{2}(V_m + V_a)(\alpha + \beta)/\alpha\beta} \\ &= V_a + \frac{\tfrac{1}{2}V_a^2(\alpha + \beta)/\alpha\beta}{T - \tfrac{1}{2}V_a(1 + V_m/V_a)(\alpha + \beta)/\alpha\beta} \quad . \quad . \quad . \quad (3a) \end{aligned}$$

* For the area to represent distance in miles, ordinates (i.e. speed) must be expressed in miles per hour and abscissae (i.e. time) in hours.

The evaluation of this equation requires a knowledge of the value of the ratio V_m/V_a (which depends upon the shape of the trapezium $ODEC$). But for speed-time curves of the shapes of Figs. 1 and 2 sufficient accuracy will usually be obtained by assuming V_m/V_a as 1·25. The correctness of the assumption can always be checked from the separate values of V_m and V_a derived by calculation, and, if necessary, a closer degree of accuracy may be obtained by a further calculation of V_m [from equation (3a)] using a revised value for V_m/V_a.

Considering the *quadrilateral speed-time curve*, Fig. 4 (c), let V_1, V_2, denote the speeds—in miles per hour—at the beginning and end, respectively, of the coasting period, t_7, the duration, in seconds, of this period; t_6, t_8, the duration, in seconds, of the accelerating and braking periods, respectively; β_c the coasting retardation in m.p.h.p.s.; and D, T, α, β, V_a refer to the same quantities as employed in the trapezoidal speed-time curve. Then the area of the quadrilateral $OLMC$ is given by

$$
\begin{aligned}
D &= [\tfrac{1}{2}V_1(t_6 + t_7) + \tfrac{1}{2}V_2(t_7 + t_8)]/3{,}600 \\
&= [\tfrac{1}{2}V_1(T - t_8) + \tfrac{1}{2}V_2(T - t_6)]/3{,}600 \\
&= \left[\tfrac{1}{2}T(V_1 + V_2) - \tfrac{1}{2}V_1V_2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\right]\Big/3{,}600
\end{aligned}
$$

Now
$$
\begin{aligned}
V_2 &= V_1 - \beta_c t_7 = V_1 - \beta_c[T - (t_6 + t_8)] \\
&= V_1 - \beta_c T + V_1\beta_c/\alpha + V_2\beta_c/\beta
\end{aligned}
$$

i.e.
$$V_2 = \beta[V_1(1 + \beta_c/\alpha) - \beta_c T]/(\beta - \beta_c) \quad . \quad . \quad . \quad . \quad (4)$$

Substituting this value for V_2 in the equation for D and simplifying, we obtain

$$
D = \frac{1}{7{,}200}\left\{V_1T\left[1 + \frac{\alpha(\beta + \beta_c) + 2\beta\,\beta_c}{\alpha(\beta - \beta_c)}\right] - \frac{\beta}{\beta - \beta_c}\left[V_1^2\frac{(\alpha + \beta_c)(\alpha + \beta)}{\alpha^2\beta} + \beta_c T^2\right]\right\} \quad . \quad (5)
$$

and by further simplification and re-arrangement we have

$$
\alpha^2\left[7{,}200\frac{D}{V_1}(\beta - \beta_c) - T\beta\left(2 - \frac{T\beta_c}{V_1}\right) + V_1\right] + \alpha[V_1(\beta + \beta_c) - 2\beta\beta_c T] + V_1\beta\beta_c = 0 \quad . \quad (6)
$$

In the application of these equations β, β_c, D and the average speed (from which T can be calculated) will usually be known. Solutions can then be obtained for V_1 when α is given, or for α when V_1 is given. V_2 is then obtained from the equation

$$V_2 = [V_1(1 + \beta_c/\alpha) - \beta_c T]\beta/(\beta - \beta_c)$$

If, however, the distance, D, is required, when values of the acceleration, retardations, and the duration of these periods are given, the calculation is more straightforward when carried through step by step (as in Example 3, below) rather than by substituting in equation (5).

Examples. 1. A train runs on a service in which there are two stops per mile and the schedule speed is 17 m.p.h., stops being of 20 seconds' duration. Determine the trapezoidal speed-time curve for the run if the acceleration is 1·2 m.p.h.p.s. and the braking retardation is 2 m.p.h.p.s.

The distance between stops (D) is 0·5 mile. Whence the schedule time is ($3{,}600 \times 0{\cdot}5/17 =$) 105·8 sec. and the running time (T) is ($105{\cdot}8 - 20 =$) 85·8 sec.

Hence, substituting in equation (3) and solving for V_m we obtain

$$V_m = 26{\cdot}4 \text{ m.p.h.}$$

Also $\quad V_a = 3{,}600\, D/T = 3{,}600 \times 0{\cdot}5/85{\cdot}8 = 20{\cdot}97$ m.p.h.

Whence

$t_1 =$ duration of accelerating period	$= V_m/\alpha$	$= 22$ sec.
$t_3 =$ duration of braking period	$= V_m/\beta$	$= 13{\cdot}2$ sec.
$t_2 =$ duration of free-running period	$= T - (t_1 + t_3)$	$= 50{\cdot}6$ sec.
$D_1 =$ distance run during accelerating period	$= \frac{1}{2}V_m t_1/3{,}600$	$= 0{\cdot}0806$ mile
$D_2 =$ distance run during free-running period	$= V_m t_2/3{,}600$	$= 0{\cdot}371$ mile
$D_3 =$ distance run during braking period	$= \frac{1}{2}V_m t_3/3{,}600$	$= 0{\cdot}0484$ mile

2. A train is required to run between stations 1 mile apart at a schedule speed of 25 m.p.h., the duration of the stops being 20 seconds. The braking retardation is 2·25 m.p.h.p.s. Assuming a trapezoidal speed-time curve, calculate the acceleration if the ratio (maximum speed/average speed) is to be 1·25.

The schedule time is ($1 \times 3{,}600/25 =$) 144 sec.

Hence the running time (T) is ($144 - 20 =$) 124 sec, and the average speed (V_a) is ($1 \times 3{,}600/124 =$) 29·05 m.p.h.

Whence the maximum speed (V_m) $= 1{\cdot}25 \times 29{\cdot}05 = 36{\cdot}3$ m.p.h.

The acceleration is calculated from equation (2) thus

$$\frac{1}{\alpha} = \frac{7{,}200D}{V_m^{\,2}}\left(\frac{V_m}{V_a} - 1\right) - \frac{1}{\beta}$$

i.e. $$\frac{1}{\alpha} = \frac{7{,}200 \times 1}{36{\cdot}3^2}(1{\cdot}25 - 1) - \frac{1}{2{\cdot}25} = 0{\cdot}917$$

and $$\alpha = 1{\cdot}09 \text{ m.p.h.p.s.}$$

3. A train is accelerated uniformly from rest until a speed of 25 m.p.h. is reached 20 seconds after starting. Power is then cut off and the train coasts for 40 seconds. The brakes are then applied and the train is brought to rest 70 seconds after starting. The retardation during coasting may be assumed to be uniform at the rate of 0·1 m.p.h.p.s. Determine the distance run from start to stop and the average speed.

The calculation is best effected step by step. First the speed (V_2) at the end of the coasting period is determined. Thus

$$V_2 = V_1 - \beta_c t_7 = 25 - 0{\cdot}1 \times 40 = 21 \text{ m.p.h.}$$

Hence,

Distance run during accelerating period	$= \frac{1}{2} \times 25 \times 20/3{,}600$	$= 0{\cdot}0694$ mile
Distance run during coasting period	$= \frac{1}{2}(25 + 21) \times 40/3{,}600$	$= 0{\cdot}2555$ mile
Distance run during braking period	$= \frac{1}{2} \times 21 \times 10/3{,}600$	$= 0{\cdot}0292$ mile

$$\therefore \text{ total distance run} = 0{\cdot}354 \text{ mile}$$

$$\text{Whence average speed} = 0{\cdot}354 \times 3{,}600/70 = 18{\cdot}2 \text{ m.p.h.}$$

4. A train is required to run between stations 1·2 miles apart at a schedule speed of 25 m.p.h., the duration of the stops being 20 seconds. The run is to be made according to a quadrilateral speed-time curve, Fig. 4 (c), and the coasting and braking retardations may be assumed at 0·1 m.p.h.p.s. and 2 m.p.h.p.s. respectively. Determine the acceleration if the speed at the end of the accelerating period is 38 m.p.h. Determine also the duration of the coasting period.

The schedule time is ($1{\cdot}2 \times 3{,}600/25 =$) 172·7 sec, and the running time (T) is ($172{\cdot}7 - 20 =$) 152·7 sec.

The acceleration is calculated from equation (6) by substituting the appropriate

numerical values for D, V_1, T, β, β_c. Thus, after substitution and simplification, we obtain the equation

$$-18\alpha^2 + 18{\cdot}72\alpha + 7{\cdot}6 = 0$$

from which $\alpha = 1{\cdot}352$ m.p.h.p.s.

Hence, duration of accelerating period $= 38/1{\cdot}352 = 28{\cdot}1$ sec.

The duration of the coasting period can be determined when the speed (V_2) at the end of this period is known. This speed is calculated from equation (4), thus

$$\begin{aligned} V_2 &= \beta[V_1(1 + \beta_c/\alpha) - \beta_c T]/(\beta - \beta_c) \\ &= 2[38(1 + 0{\cdot}1/1{\cdot}352) - 0{\cdot}1 \times 152{\cdot}7]/1{\cdot}9 \\ &= 26{\cdot}88 \text{ m.p.h.} \end{aligned}$$

Whence duration of coasting period

$$= t_7 = (V_1 - V_2)/\beta_c = (38 - 26{\cdot}88)/0{\cdot}1 = 111{\cdot}2 \text{ sec.}$$

A check upon the calculations can be made by calculating (i) the braking retardation from V_2 and the duration of the braking period; (ii) the distances run during acceleration, coasting, and braking.

Thus, the duration of the braking period

$$= 152{\cdot}7 - (28{\cdot}1 + 111{\cdot}2) = 13{\cdot}4 \text{ sec.}$$

and the computed braking retardation is $26{\cdot}88/13{\cdot}4 = 2{\cdot}005$ m.p.h.p.s., which checks closely with the given value of $2{\cdot}0$ m.p.h.p.s.

Distance run during acceleration $= \frac{1}{2} \times 38 \times 28{\cdot}1/3{,}600 = 0{\cdot}148$ mile
Distance run during coasting $= \frac{1}{2}(38 + 26{\cdot}88) \times 111{\cdot}2/3{,}600 = 1{\cdot}002$ miles
Distance run during braking $= \frac{1}{2} \times 26{\cdot}88 \times 13{\cdot}4/3{,}600 = 0{\cdot}05$ mile
Whence the total computed distance $= 0{\cdot}148 + 1{\cdot}002 + 0{\cdot}05 = 1{\cdot}2$ miles, which is correct.

5. A train is required to run between stations 1 mile apart at an average speed of 25 miles per hour. The run is to be made to a quadrilateral speed-time curve, the acceleration being $1{\cdot}25$ m.p.h.p.s. and the coasting and braking retardations being $0{\cdot}1$ and 2 m.p.h.p.s. respectively. Determine the duration of the accelerating, coasting, and braking periods and the distances run during these periods.

In this problem the speeds V_1, V_2, at the beginning and end, respectively, of the coasting period must first be determined, V_1 being calculated from equation (5), and V_2 from equation (4).

The running time (T) $= 1 \times 3{,}600/25 = 144$ sec.

Hence, substituting this value and the known values of D, α, β, β_c in equation (5) and simplifying, we obtain the equation for V_1 as

$$1{\cdot}476V_1^2 - 327{\cdot}3V_1 + 9{,}382 = 0$$

Whence $V_1 = 33{\cdot}96$ m.p.h.

Substituting in equation (4), we obtain

$$V_2 = 23{\cdot}45 \text{ m.p.h.}$$

Hence,

Duration of accelerating period	$= V_1/\alpha$	$= 27{\cdot}17$ sec.
Duration of coasting period	$= (V_1 - V_2)/\beta_c$	$= 105{\cdot}1$ sec.
Duration of braking period	$= V_2/\beta$	$= 11{\cdot}73$ sec.
Distance run during accelerating period	$= \frac{1}{2} \times 33{\cdot}96 \times 27{\cdot}17/3{,}600$ $= 0{\cdot}128$ mile	
Distance run during coasting period	$= \frac{1}{2}(33{\cdot}96 + 23{\cdot}45) \times 108/3{,}600$ $= 0{\cdot}837$ mile	
Distance run during braking period	$= \frac{1}{2} \times 23{\cdot}45 \times 11{\cdot}73/3{,}600$ $= 0{\cdot}0382$ mile	

Total computed distance $= 0{\cdot}128 + 0{\cdot}837 + 0{\cdot}0382 = 1{\cdot}003$ miles.

[NOTE.—The slight discrepancy between the computed distance ($1{\cdot}003$ miles) and the actual distance ($1{\cdot}0$ mile) is due to the slide rule in making the calculations.]

Equivalent Speed-Time Curve. *An important property of speed-time curves of constant shape, but of unequal area, is that they can all be reduced to an* equivalent speed-time curve of the same shape, provided that the co-ordinates of this equivalent speed-time curve are suitably chosen. Thus, consider two trapezoidal speed-time curves A, B, of similar shape but of unequal area,

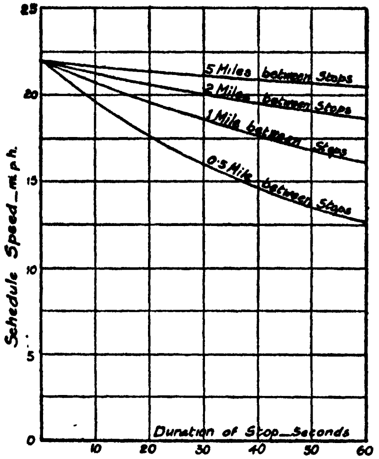


FIG. 5. INFLUENCE OF DURATION OF STOP AND LENGTH OF RUN ON SCHEDULE SPEED

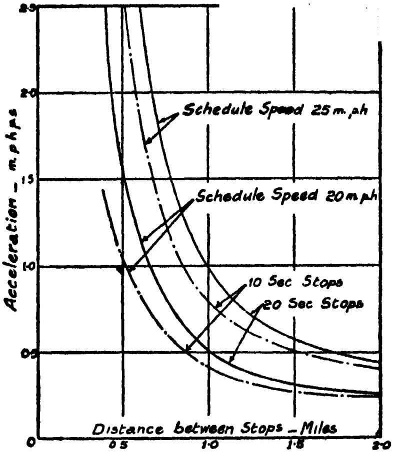


FIG. 6. MINIMUM ACCELERATION TO MAINTAIN GIVEN SCHEDULE SPEEDS WITH VARIOUS DISTANCES BETWEEN STOPS

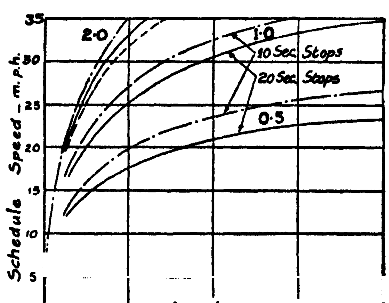


FIG. 7. SCHEDULE SPEEDS CORRESPONDING TO RUNS OF 0·5, 1·0 AND 2·0 MILES, WITH VARIOUS VALUES OF ACCELERATION

plotted to the same co-ordinates. Let the maximum ordinate of B be k times that of A. Then, since the shape is constant in the two cases, the acceleration and the braking retardation must remain constant, and the durations of the accelerating, braking, and free-running periods of speed-time curve B must be k times those of the corresponding periods of speed-time curve A. Consequently, the area of speed-time curve B—which represents the distance run, say D_2 miles—is k^2 times that of speed-time A (which refers to a run of, say, D_1 miles). That is, $D_2 = k^2 D_1$, or $k = \sqrt{(D_2/D_1)} = \sqrt{(s_1/s_2)}$, where s_1, s_2 are the reciprocals

of the distances D_1, D_2 respectively and denote the stops per mile corresponding to services run according to the speed-time curves A and B respectively.

Hence if T_1, T_2, denote the running times, and V_{1m}, V_{2m}, the maximum speeds in the two cases, A and B respectively, then

$$T_2 = kT_1 = T_1\sqrt{(s_1/s_2)}, \text{ or } T_2\sqrt{s_2} = T_1\sqrt{s_1}$$

and

$$V_{2m} = kV_{1m} = V_{1m}\sqrt{(s_1/s_2)}, \text{ or } V_{2m}\sqrt{s_2} = V_{1m}\sqrt{s_1}$$

Therefore, if the co-ordinates of the original speed-time curves are changed to speed × $\sqrt{}$(stops per mile) and time × $\sqrt{}$(stops per mile), a single speed-time curve will be obtained.

The equivalent speed-time curve is of considerable use in estimations of energy consumption, and its application is discussed in the next chapter.

Factors Affecting Schedule Speed. The schedule speed of a given train when running on a given service (i.e. with a given distance between stations) is influenced by the duration of the stop, the acceleration, the braking retardation, and the maximum speed. With different classes of service the distance between stops has also a considerable influence on the schedule speed. In order to show the effect on the schedule speed of these variables, the curves in Figs. 5, 6, 7, have been calculated, using the trapezoidal form of speed-time curve, Fig. 4 (*b*).

TABLE I

SCHEDULE SPEEDS CORRESPONDING TO AN AVERAGE SPEED OF 22 M.P.H. FOR VARIOUS DISTANCES BETWEEN STOPS AND DURATION OF STOP

Distance between stops	0·5 mile	1·0 mile	2 miles	5 miles
Duration of Stop	Schedule Speed			
seconds	m.p.h.	m.p.h.	m.p.h.	m.p.h.
10	19·6	20·8	21 3	21·7
20	17·7	19·6	20 7	21·4
30	16·1	18·6	20·1	21·2
40	14·8	17·7	19·6	20·9
50	13 7	16·9	19·1	20·7
60	12 7	16·1	18·6	20·5

The *effect of the duration of the stop* on the schedule speed, for a given average speed, is shown in Table I, which emphasizes the importance of short stops for urban service. For this class of service a stop of 15 to 20 seconds' duration is generally sufficient. The effect of increasing the stop (in Table I) from 10 to 20 seconds reduces the schedule speed by 10 per cent; and if the stop is increased to 40 seconds, the schedule speed will be 16·4 per cent less than that with a stop of 20 seconds. With longer distances between stations, the duration of the stop can be increased without affecting the schedule speed to any great extent. Thus, comparing the 2 mile and 5 mile runs, the effect of increasing the duration of stop from 20 to 40 seconds reduces the schedule speeds by 5·4 per cent and 2·4 per cent respectively. The results are shown graphically in Fig. 5.

The *minimum acceleration* required to give schedule speeds of 20 and 25 m.p.h., when the distance of the run varies from 0·5 mile to 2 miles and the

stops are 10 and 20 sec, is shown in Fig. 6. The braking retardation is 2 m.p.h.p.s. in all cases, and the ratio of maximum to average speed is 1·4 for the 0·5-mile run, 1·3 for the 1-mile run and 1·2 for the 2-mile run; these particular values being chosen to be representative of practical conditions, since, the longer the distance between stops, the lower will be the ratio of maximum to average speed.

The curves in Fig. 7 have been plotted from the same data as those in Fig. 6, and show the effect of the acceleration on the schedule speed for given distances between stations and given durations of stops. For the 2-mile runs an additional curve, shown in broken line, has been calculated for stops of 30 seconds' duration. A study of these curves (Figs. 6, 7) shows the necessity of a high acceleration if a high schedule speed is desired, in urban and suburban service. It is evident that no steam locomotive could run a service at a schedule speed of 20 m.p.h. with a stop every ½ mile, even when the duration of the stop is only 10 seconds, but it is quite within the range of electric traction to run this service with a stop of 20 seconds' duration.

When we consider longer distances (e.g. above 1 mile) between stops, the importance of a high acceleration is not so marked, and a steam locomotive would be quite capable of running a service at a schedule speed of over 25 m.p.h. with a stop of 30 seconds' duration at stations 2 miles apart.

CHAPTER III

THE MECHANICS OF TRAIN MOVEMENT

II. Preliminary Investigation of Energy Consumption

We have shown how the speed of a train can be represented during any interval of its motion, and it is now necessary to consider the manner in which the energy required can be estimated.

When electricity is applied to the operation of road and railway systems from a central power station, it is necessary to predetermine the amount of power required by the cars and trains, in order that the power-station equipment, sub-stations, feeders, etc., may be designed for economical working. The degree of accuracy to which this pre-determination is possible will depend on the exactness of the available knowledge relating to the conditions of operation, such as schedule speed, distance between stops, frequency of service, weight of cars, etc. Thus, on railways, the schedule speeds are fixed by time-tables, and the operating conditions are known, but with street traffic the schedule speeds and the operating conditions are very variable. In the case of tramway and trolleybus systems an exact estimation of the energy required is not essential, as, apart from the variable conditions met with in street traffic, the maximum power taken by such vehicles does not usually exceed 200 kW. On the other hand, the maximum power required by a train may exceed 2,000 kW, and therefore an accurate estimation of the energy required in this case is essential.

The total energy supplied to an electric train for propulsion may be expended in five ways, i.e. (i) in accelerating the train in a horizontal direction; (ii) in accelerating the revolving parts; (iii) in doing work against gravity, if the train is ascending a gradient; (iv) in doing work against the resistances to motion; and (v) in supplying the losses in the motors and electrical equipment.

For short-distance runs on level track at high schedule speeds, the energy required for acceleration forms a large percentage of the total energy supplied for propulsion. On the other hand, with long-distance runs on level track at high speeds the energy expended against the resistances to motion may be considerably greater than that required for acceleration.

The energy expended in accelerating the train is converted into kinetic energy and therefore represents stored energy which is recoverable for propulsion purposes. A portion of this stored energy is utilized during coasting and the remainder is dissipated as heat in the brake shoes.

Determination of Energy Required for Propulsion. The determination of the energy required for acceleration, and also for the other items (except the losses in the motors) enumerated above, involves only the application of elementary dynamics. The work done during a period of uniform acceleration from rest, the work done against gravity, and the energy expended against the resistances to motion can all be determined directly from simple expressions when no variable factors are involved. But with variable acceleration and resistances to motion the work done must be calculated by a point-to-point method which involves the separate consideration of the quantities (i.e. force, speed, time) concerned. We shall, therefore, show how to calculate the force necessary for acceleration.

Tractive Force for Acceleration. The fundamental dynamical equation for the acceleration of a body in a horizontal direction is

Acceleration = accelerating force/mass of body

When British units are employed the acceleration is expressed in feet per second per second and, if the accelerating force is expressed in pounds, the mass of the body must be expressed in engineer's mass units, i.e. (weight in lb/gravitational acceleration in ft per sec) or w/g where w is the weight in lb and g is the gravitational acceleration in ft per sec per sec. In this country the value of g is taken as 32·2 ft per sec per sec.

If, however, the acceleration is to be expressed in miles per hour per second (α) and the weight in tons (W), then for the accelerating force (F) to be in lb, we must have

$$\alpha(5{,}280/3{,}600) = F/(2{,}240W/32{\cdot}2)$$

whence

$$F = W\alpha[2{,}240 \times 5{,}280/(3{,}600 \times 32{\cdot}2)]$$
$$= 102W\alpha.$$

Therefore to obtain an acceleration of 1 mile per hour per second (or 1·467 feet per second per second) an accelerating force of 102 pounds is necessary for each ton accelerated.

This relationship, however, only holds good provided that the body being accelerated possesses no rotating parts. But with electric trains the wheels, axles, motor armatures, and gearing have to be accelerated in an angular direction at the same time as the whole train is accelerated in a linear direction. The force required for the angular acceleration of these parts is determined from the fundamental relationship.

Angular acceleration = accelerating torque/moment of inertia.

In this equation the angular acceleration is expressed in radians per sec per sec when the accelerating torque is expressed in lb-ft and the moment of inertia is expressed in engineer's mass units and (feet)^2. Thus

$$\alpha_a = \mathfrak{T}/(k^2w/g)$$

where α_a denotes the angular acceleration in radians per sec per sec, $\mathfrak{T}$ the accelerating torque in lb-ft, k the radius of gyration in ft, w the weight of the rotating body in lb, g the value of gravitation acceleration in ft per sec per sec.

[Note.—Moment of inertia = mass × (radius of gyration)^2 = $(w/g)k^2$.]

Hence, if we have a wheel, of weight W_1 tons and radius of tread r_1 ft, rotating about its axis, and a force F_1 lb, in addition to that necessary to balance the rotational frictional resistances, is applied to the periphery, then the angular acceleration of the wheel will be given by

$$\alpha_a = F_1r_1/(k_1^2 \times 2{,}240W_1/32{\cdot}2)$$

But, if the wheel forms part of a train which is being accelerated linearly at α m.p.h.p.s., the angular acceleration in radians per sec per sec = (linear acceleration of tread of wheel in ft per sec per sec/radius of tread in ft), i.e.

$$\alpha_a = (5{,}280/3{,}600)\alpha/r_1$$

whence

$$F_1 = (W_1\alpha k_1^2/r_1^2) \times [2{,}240 \times 5{,}280/(32{\cdot}2 \times 3{,}600)]$$
$$= 102W_1\alpha(k_1/r_1)^2$$

In the case of an armature driven through gearing, of ratio γ, from the axle

of the wheel, the angular acceleration of the armature will be γ times the angular acceleration of the wheel. Hence if W_2 is the weight, in tons, of the armature and k_2 its radius of gyration, in feet, the torque at the *armature shaft* necessary for an angular acceleration $\gamma\alpha_a$ is given by

$$\mathfrak{T}_1 = \gamma\alpha_a k_2^2 W_2 \times 2{,}240/32{\cdot}2$$

But the torque at the axle of the wheel will be $\gamma\mathfrak{T}_1$, since the angular velocity of the wheel is $(1/\gamma)$ of that of the armature. Therefore, the force (F_2 lb) at the tread of the wheel* necessary for the angular acceleration of the armature is

$$\begin{aligned} F_2 = \gamma\mathfrak{T}_1/r_1 &= W_2\alpha_a\gamma^2(k_2^2/r_1) \times 2{,}240/32{\cdot}2 \\ &= W_2\alpha\gamma^2(k_2^2/r_1^2) \times [2{,}240 \times 5{,}280/(32{\cdot}2 \times 3{,}600)] \\ &= 102\, W_2\alpha\gamma^2 k_2^2/r_1^2 \\ &= 102\, W_2\alpha\gamma^2(k_2/r_2)^2(r_2/r_1)^2 \end{aligned}$$

Similarly the force (F_3 lb) at the tread of the wheel necessary for the angular acceleration of the gear wheel (of weight W_3 tons and radius of gyration k_3 ft) is

$$F_3 = 102\, W_3\alpha(k_3/r_1)^2$$

and that (F_4) necessary for the angular acceleration of the axle (of weight W_4 tons and radius of gyration k_4 ft) carrying the wheels is

$$F_4 = 102\, W_4\alpha(k_4/r_1)^2$$

In practice the last item (F_4) will be extremely small in comparison with the other items ($F + F_1 + F_2$) and may, therefore, be neglected.

Hence the force (F_a lb) acting at the treads of the driving wheels (and called, therefore, the *tractive force*) necessary for the acceleration, on level track, of an electric train made up of motor and trailer coaches will be

$$F_a = F + 2n_1F_1 + n_2F_2 + n_2F_3$$

where n_1 is the number of axles, n_2 the number of motors, F the force for the linear acceleration of the dead weight of the train, $2n_1F_1$ the force for the angular acceleration of the wheels (which are all assumed to be identical), and n_2F_2, n_2F_3 the forces for the angular acceleration of the armatures and gear wheels, respectively. Whence

$$\begin{aligned} F_a &= 102\,W\alpha + 102\,W_1\alpha\,.\,2n_1\left(\frac{k_1}{r_1}\right)^2 + 102\,W_2\alpha n_2\gamma^2\left(\frac{k_2}{r_2}\right)^2\left(\frac{r_2}{r_1}\right)^2 + 102\,W_3\alpha n_2\left(\frac{k_3}{r_1}\right)^2 \\ &= 102\alpha\left\{W + 2n_1W_1\left(\frac{k_1}{r_1}\right)^2 + n_2W_2\gamma^2\left(\frac{k_2}{r_2}\right)^2\left(\frac{r_2}{r_1}\right)^2 + n_2W_3\left(\frac{k_3}{r_1}\right)^2\right\} \\ &= 102\alpha W_e \qquad (7) \end{aligned}$$

where W_e is called the *accelerating weight* of the train.

The amount by which W_e exceeds the dead weight of the train varies from 8 per cent to 15 per cent of the latter, the actual value depending on the number of wheels and motors, type of motor, etc.

In order to calculate the accelerating weight of a train, it is necessary to know the radius of gyration of each rotating part. The radius of gyration of a

* It is necessary to consider this force at the tread of the wheel, as the characteristics of the motor are calculated for the output at this point.

cylinder is 0·707 × external radius; an average value for a steel-tyred railway wheel is 0·77 × radius of tread; while, for the armature of a d.c. or a.c. commutator motor, the value of 0·7 × external radius of armature core is approximately correct. The radius of gyration of a gear wheel will depend on the design of the wheel, but, for the class of gear wheels used on motor-coach trains, the value of 0·8 × radius of pitch circle will be sufficiently accurate.

Hence, inserting these values in the above equation, we have

$$W_e = W + 2n_1W_1(0{\cdot}77)^2 + n_2W_2\gamma^2(0{\cdot}7)^2\left(\frac{r_2}{r_1}\right)^2 + n_2W_3(0{\cdot}8)^2\left(\frac{r_3}{r_1}\right)^2$$

$$= W + 1{\cdot}2n_1W_1 + 0{\cdot}49n_2W_2\gamma^2\left(\frac{r_2}{r_1}\right) + 0{\cdot}64n_2W_3\left(\frac{r_3}{r_1}\right)^2 \quad . \quad . \quad (8)$$

Example. Calculation of the accelerating weight of a motor-coach train-unit for suburban service.—The train-unit is made up of three 4-wheel bogie coaches and has a weight of 94 tons without passengers. It is equipped with four direct-current motors and each motor is geared to a driving axle through single-reduction gearing having a gear ratio of 2·81. All wheels are 42½ in. in diameter and each wheel weighs 1,000 lb. Each motor armature weighs 2,350 lb and the diameter of the armature core is 21 in. Each gear wheel weighs 565 lb and has a pitch circle diameter of 30 in.

Hence, Number of axles (n_1) = 12
Number of motors (n_2) = 4

Substituting in equation (8), we have

$$W_e = 94 + [1{\cdot}2 \times 12 \times 1{,}000/2{,}240] + [0{\cdot}49 \times 4 \times (2{,}350/2{,}240) \times 2{\cdot}81^2 \times (21/42{\cdot}5)^2] + [0{\cdot}64 \times 4 \times (565/2{,}240) \times (30/42{\cdot}5)^2]$$

$$= 94 + 6{\cdot}43 + 3{\cdot}96 + 0{\cdot}323$$

$$= 104{\cdot}7 \text{ tons (approximately)},$$

which is 11·5 per cent greater than the dead weight of the train.

Tractive Force for Gravitational Effect and Resistance to Motion. When a train is on a gradient the effect of gravity produces a force which tends to cause motion down the gradient. This force is calculated by resolving the vertical gravitational force due to the weight of the train in a direction parallel to the gradient. Thus, if W is the weight of the train, in tons, and θ is the inclination of the gradient to the horizontal, the force (F_g lb) due to gravity, acting down the gradient is

$$F_g = 2{,}240W \sin\theta$$

But in railway work gradients are expressed either in terms of distance (along the track) corresponding to a rise of 1 ft or in terms of the rise per 100 ft of track. The former is standard in British railway practice while the latter is customary in American railway practice and gives the "percentage gradient."

Hence, since $\sin\theta$ = (rise or elevation/distance along track), the percentage gradient (G) is equal to $100 \sin\theta$. Whence

$$F_g = 2{,}240W(G/100) = 22{\cdot}4WG$$

It should be observed that for motion of a train up a gradient this force must be balanced by a corresponding increase in the tractive force if the acceleration is to be maintained at the same value as on level track. Conversely,

if the train is descending a gradient, the force, F_g, forms parts of the accelerating force.

The determination of the tractive force (F_r) necessary to balance the resistances to motion (which are called collectively "train resistance" or "tractive resistance") involves a knowledge of these resistances.

As train resistance is made up of several variable components, none of which can be calculated from first principles, our present knowledge has had to be obtained experimentally. Train resistance in detail is considered later in Chapter XVIII, where formulae are given for the tractive resistances of locomotive-hauled and motor-coach trains.

Although we have no definite knowledge of train resistance when the speed is changing rapidly, we may assume that these conditions produce additional friction due to oscillation and buffing movements of the rolling stock. Hence, in calculations, the train resistance for the period of initial acceleration is assumed at an arbitrary value, usually between 8 and 10 lb per ton of train weight. Such a procedure is warranted because the energy expended against train resistance during this period is insignificant in comparison with that needed for acceleration.

Tractive Effort for Propulsion of Train. The tractive force (F_t) required to propel a train on level track at a constant acceleration is, therefore,

$$F_t = F_a + F_r = 102W_e\alpha + Wr$$

where W, W_e, are the dead and accelerating weights (in tons), respectively, of the train, α the acceleration in m.p.h.p.s., and r the specific train resistance.

If the train has to ascend a gradient, then

$$F_t = F_a + F_r + F_g = 102W_e\alpha + Wr + 22{\cdot}4WG$$

while, if the train has to descend a gradient, we have

$$F_t = F_a + F_r - F_g = 102W_e\alpha + Wr - 22{\cdot}4WG$$

Examples. 1. A 200-ton train is to be accelerated on level track at 1·2 m.p.h.p.s. What tractive effort must be supplied?

Assuming the accelerating weight as 220 tons and the train resistance as 10 lb per ton of train weight, we have

$$F_t = 102 \times 220 \times 1{\cdot}2 + 200 \times 10 = 28{,}920 \text{ lb}$$

2. If the tractive effort in the preceding example is maintained constant, while the train ascends a gradient of 1 in 250, what will be the resulting acceleration?

The percentage gradient (G) is $(1/250) \times 100 = 0{\cdot}4$.

The acceleration is determined from the equation

$$\begin{aligned}\alpha &= (F_t - Wr - 22{\cdot}4WG)/102W_e \\ &= [28{,}920 - 200(10 + 22{\cdot}4 \times 0{\cdot}4)]/102 \times 220 \\ &= 1{\cdot}12 \text{ m.p.h.p.s.}\end{aligned}$$

Power Output from Driving Axles. The power output from the driving axles at any instant is equal to the product of the tractive effort and speed. If the power (P) is to be expressed in kilowatts, when the tractive effort (F_t) and speed (V) are given in lb and miles per hour respectively, then

$$P = F_tV \times 5{,}280 \times 746/(60 \times 33{,}000 \times 1{,}000) = 0{\cdot}00199F_tV$$

or, to a close approximation,

$$P = 0{\cdot}002F_tV \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Energy Output from Driving Axles. *The energy output during a given* period is obtained by determining the area of the power-time (output) curve for that period or, alternatively, by calculating the mean value of the power output and multiplying this by the time. If the power is expressed in kilowatts and the time is expressed in seconds—as is customary in calculations of speed-time curves—the energy will be given in kilowatt-seconds.

For example, for a run, on level track, made according to the trapezoidal speed-time curve (Fig. 4 (*b*)) the energy output for the accelerating period

$$= \tfrac{1}{2}t_1 \times 0{\cdot}002F_tV_m$$

and the energy output for the free-running period

$$= t_2 \times 0{\cdot}002F_t'V_m$$

where F_t, F_t' denote the tractive efforts during acceleration and free running, respectively, and t_1, t_2 denote the duration of the accelerating and free-running periods, respectively.

Instead of expressing the energy in kilowatt-seconds, it is more convenient for purposes of comparison to introduce the train weight and length of run, and to express the energy in watt-hours per ton-mile, i.e. energy output in watt-hours/(weight of train in tons $\times$ distance of run in miles).

This quantity is called the *specific energy output* and forms a basis of comparison between the dynamical performances of trains operating to different schedules.

The energy output for a given run represents the energy necessary for dynamical purposes, and is entirely independent of the system of propulsion, except in so far as the latter affects W and W_e. The energy input to the motors will depend on the efficiency of the electrical equipment and the method of speed control.

The energy input to the motors is called the "energy consumption" of the train, since it is the energy used for propulsive purposes. The total energy taken from the conductor rails or overhead line will be greater than this by the amount required for lighting, heating, control, and brake apparatus.

The energy consumption can be expressed in "kilowatt-hours per train mile," that is

$$\frac{\text{energy consumption of train in kilowatt-hours}}{\text{length of run in miles}}$$

or in "watt-hours per ton mile," that is,

$$\frac{\text{energy consumption of train in watt-hours}}{\text{length of run in miles} \times \text{weight of train in tons}}$$

the latter being known as the *specific energy consumption.*

Specific Energy Output for Runs made to Simplified Speed-Time Curves. When comparisons are to be made between services operated to simplified speed-time curves (e.g. the trapezium, Fig. 4 (*b*), or the quadrilateral, Fig. 4 (*c*)) the specific energy outputs for acceleration and train resistance may be determined from very simple expressions. Thus, the energy expended in accelerating a train from rest to a speed V_m m.p.h. is given, in kW-sec, by

$$\tfrac{1}{2}(0{\cdot}002V_m \times 102W_e\alpha)t_1$$

Replacing t_1 by V_m/α, *converting into watt-hours, and dividing by the train* weight (W) and distance (D miles), we have

Specific energy expended in acceleration

$$= \tfrac{1}{2}(0\cdot002V_m^2 \times 102W_e) \times 1{,}000/(3{,}600WD)$$

$$= \frac{0\cdot0283V_m^2}{D} \cdot \frac{W_e}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Similarly, if the train resistance is constant at r lb per ton over a distance D' miles, the work done against train resistance is $5{,}280D'Wr$ ft-lb. Converting this into watt-hours per ton mile for a run of D miles, we have

Specific energy expended against train resistance

$$= 5{,}280D'Wr \times 746/(550 \times 3{,}600 \times WD)$$

$$= 1\cdot99rD'/D = 2rD'/D \text{ (approximately)} \quad . \quad . \quad . \quad (11)$$

Whence the energy expended for the run

$$= (0\cdot0283V_m^2/D)W_e/W + 2rD'/D \text{ watt-hours per ton-mile,}$$

A principle which is of importance in comparisons of energy consumption is that, on the assumption of constant train resistance, the specific energy output of a given train is constant for all runs made to speed-time curves of similar shapes. Thus, consider two trapezoidal speed-time curves of similar shape, the ordinates of one (B) being k times those of the other (A). Then the maximum speed of B is k times that of A, and the distance between stops for B is k^2 times that for A (p. 16). Hence the specific energy expended in acceleration is

$$0\cdot0283V_m^2(W_e/W)/D \text{ for speed-time curve } A,$$

and $$0\cdot0283(kV_m)^2(W_e/W/k^2D \text{ for speed-time curve } B,$$

i.e. the two quantities are equal.

A similar equality is obtained for the specific energy expended against train resistance provided that the same value of constant train resistance is assumed in each case.

This principle is of considerable use when estimations of energy consumption are required for operating a train service along a given route at various schedule speeds. It may be extended to actual speed-time curves by assuming a definite shape for the speed-curve-running portion of such curves, and with generalized data of motors the energy consumption for any given conditions may be predicted. Such methods are largely employed by electric railway engineers and manufacturers for preliminary purposes.

Factors Affecting Energy Consumption. The specific energy consumption of trains operating at a given schedule speed is influenced by (i) the distance between stops, (ii) the acceleration, (iii) the retardation, (iv) the maximum speed, (v) the type of train and equipment, (vi) the configuration of the track.

Generally, for a given run at a given schedule speed, the specific energy consumption will be lower the higher the acceleration and retardation, since by these means a longer coasting period can be obtained. However, due consideration must be given to the weight of the equipment and the effect of this on the energy consumption of the train. For runs of short distances, a low

specific energy consumption will generally indicate a low total energy consumption, but for longer distances it does not follow that a similar relation holds, since, in the latter case, the work done against train resistance is a considerable percentage of the total energy output from the axles.

Examples. 1. A train service between two stations 1 mile apart, and between which there is a uniform gradient of 1 in 80, is scheduled at an average speed (excluding stops) of 25 m.p.h. in one direction, i.e. up the gradient, and 27·5 m.p.h. in the opposite direction. The dead weight of the train is 210 tons. When operating on level track the acceleration is 1·275 m.p.h.p.s. and the braking retardation is 2 m.p.h.p.s., the corresponding net tractive efforts being 30,000 lb and 47,000 lb.

Calculate the specific energy output for runs, in both directions, made to trapezoidal speed-time curves, and also for a "down" journey made to a quadrilateral speed-time curve (Fig. 4 (*c*)).

Assume the accelerating weight to be 10 per cent greater than the dead weight, and the train resistance to be 12 lb per ton when power is "on" and 15 lb per ton when coasting.

From the given data we obtain

$W_e = 1{\cdot}1 \times 210 = 231$ tons

G = percentage gradient $= 100/80 = 1{\cdot}25$

F_g = force due to gradient $= 22{\cdot}4WG$

$= 22{\cdot}4 \times 210 \times 1{\cdot}25 = 5{,}880$ lb

Hence, Acceleration up gradient $= (30{,}000 - 5{,}880)/(102 \times 231)$
$= 1{\cdot}022$ m.p.h.p.s.

Acceleration down gradient $= (30{,}000 + 5{,}880)/(102 \times 231)$
$= 1{\cdot}522$ m.p.h.p.s.

Braking retardation up gradient $= (47{,}000 + 5{,}880)/(102 \times 231)$
$= 2{\cdot}245$ m.p.h.p.s.

Braking retardation down gradient $= (47{,}000 - 5{,}880)/(102 \times 231)$
$= 1{\cdot}745$ m.p.h.p.s.

Running time, "up" journey $= 1 \times 3{,}600/25 = 144$ sec

Running time, "down" journey $= 1 \times 3{,}600/27{\cdot}5 = 131$ sec

The maximum speeds, calculated from equation (3), are 29·3 m.p.h. for the "up" journey, and 32·4 m.p.h. for the "down" journey.

Whence, for the "up" journey—

Duration of accelerating period $= 29{\cdot}3/1{\cdot}022 = 28{\cdot}65$ sec

Duration of braking period $= 29{\cdot}3/2{\cdot}245 = 13{\cdot}05$ sec

Duration of free-running period $= 144 - 28{\cdot}65 - 13{\cdot}05 = 102{\cdot}3$ sec

Distance run during acceleration $= \frac{1}{2} \times 29{\cdot}3 \times 28{\cdot}65/3{,}600 = 0{\cdot}1165$ mile

Distance run during free running $= 29{\cdot}3 \times 102{\cdot}3/3{,}600 = 0{\cdot}834$ mile

Distance run during braking $= \frac{1}{2} \times 29{\cdot}3 \times 13{\cdot}05/3{,}600 = 0{\cdot}0531$ mile

Distance run with power $= 0{\cdot}95$ mile

Specific energy output for acceleration $= 0{\cdot}0283 \times 29{\cdot}3^2 \times 1{\cdot}1$
$= 26{\cdot}75$ watt-hours per ton mile

Specific energy output for resistances and gravity $= 2(12 + 22{\cdot}4 \times 1{\cdot}25)0{\cdot}95/1{\cdot}0$
$= 76$ watt-hours per ton mile

Specific energy output for "up" journey $= 26{\cdot}75 + 76$
$= 102{\cdot}75$ watt-hours per ton mile

For the "down" journey—

Duration of accelerating period $= 32{\cdot}4/1{\cdot}522 = 21{\cdot}27$ sec

Duration of braking period $= 32{\cdot}4/1{\cdot}745 = 18{\cdot}56$ sec

Duration of free-running period $= 131 - 21{\cdot}27 - 18{\cdot}56 = 91{\cdot}17$ sec

Distance run during acceleration $= \frac{1}{2} \times 32{\cdot}4 \times 21{\cdot}27/3{,}600 = 0{\cdot}096$ mile

Distance run during free running $= 32{\cdot}4 \times 91{\cdot}17/3{,}600 = 0{\cdot}82$ mile

Distance run during braking $= \frac{1}{2} \times 32{\cdot}4 \times 18{\cdot}56/3{,}600 = 0{\cdot}0834$ mile

Specific energy output during acceleration $= 0{\cdot}0283 \times 32{\cdot}4^2 \times 1{\cdot}1$ mile $+ 2 \times 12 \times 0{\cdot}096/1{\cdot}0 = 35$ watt-hours per ton mile

Specific energy output during free-running $= 0$*

Specific energy output for "down" journey $= 35$ watt-hours per ton mile

When the "down" journey is made to a quadrilateral speed-time curve, we have

Acceleration $= 1{\cdot}522$ m.p.h.p.s.

Braking retardation $= 1{\cdot}745$ m.p.h.p.s.

Coasting retardation $= (15 - 22{\cdot}4 \times 1{\cdot}25)/102 \times 1{\cdot}1$

$= -0{\cdot}116$ m.p.h.p.s.

By substituting in equation (5), we obtain

$$V_1 = 26{\cdot}7 \text{ m.p.h.}$$

and by substituting in equation (4), we have

$$V_2 = 37{\cdot}7 \text{ m.p.h.}$$

Whence,

Duration of accelerating period $= 26{\cdot}7/1{\cdot}522 = 17{\cdot}5$ sec

Duration of braking period $= 37{\cdot}7/1{\cdot}745 = 20{\cdot}6$ sec

Duration of coasting period $= 131 - 17{\cdot}5 - 20{\cdot}6 = 92{\cdot}9$ sec

Distance run during acceleration $= \frac{1}{2} \times 26{\cdot}7 \times 17{\cdot}5/3{,}600 = 0{\cdot}065$ mile

Distance run during coasting $= \frac{1}{2}(26{\cdot}7 + 37{\cdot}7) \times 92{\cdot}9/3{,}600 = 0{\cdot}831$ mile

Distance run during braking $= \frac{1}{2} \times 37{\cdot}7 \times 20{\cdot}6/3{,}600 = 0{\cdot}108$ mile

Specific energy output for acceleration $= 0{\cdot}0283 \times 26{\cdot}7^2 \times 1{\cdot}1$

$= 22{\cdot}2$ watt-hours per ton mile

Specific energy output for train resistance $= 2 \times 12 \times 0{\cdot}065/1{\cdot}0$

$= 1{\cdot}56$ watt-hours per ton mile

Specific energy output for "down" journey

$= 23{\cdot}76$ watt-hours per ton mile

2. Referring to example (4), p. 14, if the weight of the train is 200 tons, the accelerating weight 220 tons, and the train resistance during the accelerating period is 10 lb per ton, calculate (*a*) the specific energy output for the run, (*b*) the energy dissipated in the brakes, (*c*) the energy utilized during coasting, (*d*) the mean train resistance during coasting.

From p. 14, we have

Speed (V_1) at end of accelerating period $= 38$ m.p.h.

Speed (V_2) at end of coasting period $= 26{\cdot}88$ m.p.h.

Distance run during accelerating period $= 0{\cdot}148$ mile.

Hence the specific energy expended against train resistance during the accelerating period is

$$2 \times 10 \times (0{\cdot}148/1{\cdot}2) = 2{\cdot}47 \text{ watt-hours per ton-mile,}$$

* As the resultant train resistance is negative ($= 12 - 22{\cdot}4 \times 1{\cdot}25 = -16$) the brakes must be applied during the free-running period to maintain the speed constant.

and the specific energy expended in acceleration is

$$0{\cdot}0283 \times 38^2 \times (220/200)/1{\cdot}2 = 37{\cdot}4 \text{ watt-hours per ton-mile.}$$

Whence—

(*a*) Specific energy output for the run

$$= 37{\cdot}4 + 2{\cdot}47 = 39{\cdot}87 \text{ watt-hours per ton-mile}$$

(*b*) The energy dissipated in the brakes is equal to the energy stored in the train at the end of the coasting period (i.e. at the speed V_2) and is equal to

$$0{\cdot}0283 \times 26{\cdot}88^2 \times (220/200)/1{\cdot}2 = 18{\cdot}72 \text{ watt-hours per ton-mile.}$$

(*c*) The energy utilized during coasting is, therefore

$$37{\cdot}4 - 18{\cdot}72 = 18{\cdot}68 \text{ watt-hours per ton-mile.}$$

(*d*) The mean train resistance during coasting may be calculated either from the energy utilized during the coasting period or from the given value of the coasting retardation. In the latter case, if r_c is the mean train resistance (in lb per ton of train weight) during coasting, β_c the coasting retardation in m.p.h.p.s., then

$$r_c = 102\beta_c(W_e/W)$$

and, since $\beta_c = 0{\cdot}1$, therefore

$$r_c = 102 \times 0{\cdot}1 \times (220/200) = 11{\cdot}22 \text{ lb per ton.}$$

If we wish to calculate this quantity by the alternative method—utilizing equation (11)—we require a knowledge of the distance run during coasting. From p. 15, this distance is given as 1·002 miles. Hence, substituting in equation (11), we have

$$1{\cdot}99r_c \times (1{\cdot}002/1{\cdot}2) = 18{\cdot}68$$

or

$$r_c = 11{\cdot}22 \text{ lb per ton.}$$

CHAPTER IV

DIRECT-CURRENT TRACTION MOTORS

General Considerations

With a knowledge of the dynamical requirements involved in the movement of trains and vehicles we can now investigate the dynamical performances of the different types of motors to ascertain their suitability for traction purposes.

Since, with all forms of electric traction, the tractive force at the treads of the driving wheels is always proportional to the torque at the armature shaft of the motor, the relationship between the speed, torque, and current input to the motor are of fundamental importance.

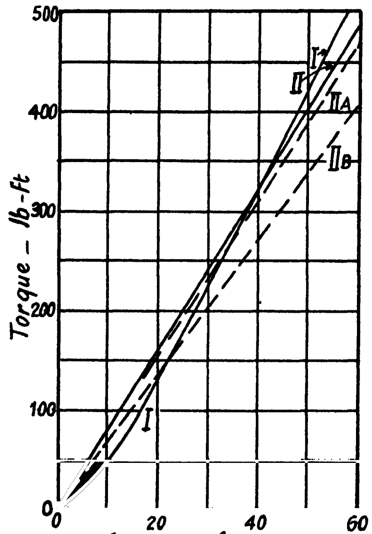


Fig. 8. Torque Curves for d.c. Series (I) and Shunt (II) Motors

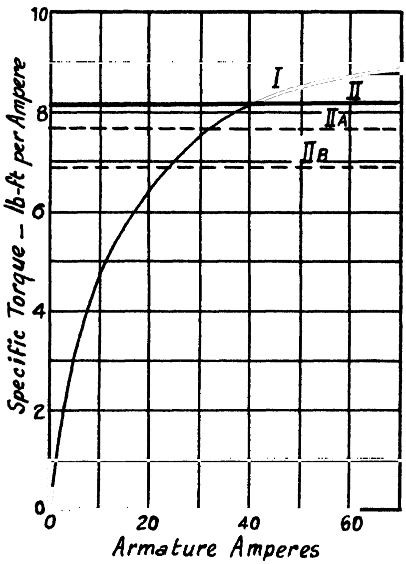


Fig. 9. Specific Torque Curves for d.c. Series (I) and Shunt (II) Motors

Torque-Current Characteristics. The relationship between the torque and armature current of any d.c. motor is given by the equation

$$\mathfrak{T} = (p/a)\Phi Iz/852 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

or

$$\mathfrak{T} = p\Phi \,.\, IN/426 \quad . \quad . \quad . \quad . \quad (12a)$$

where $\mathfrak{T}$ is the gross torque (in lb-ft), p the number of poles, a the number of circuits in the armature winding, Φ the flux per pole (in megalines), I the current input to the armature, z the number of active armature conductors, N $(= \frac{1}{2}z/a)$ the number of turns per armature circuit.

Hence for a given number of poles the torque is proportional to the product of flux and armature ampere-turns, or for any given motor the torque is proportional to the product of flux and armature current.

Therefore with a motor in which the flux is constant (e.g. a shunt motor supplied at constant voltage) the torque is directly proportional to the *armature* current. But if the voltage at the terminals of the shunt winding decreases, due to voltage drop in the line wires supplying the motor, the proportionality between torque and armature current will, for the higher values of the latter, depart from the straight-line relationship.

With a motor in which the flux varies with the armature current (e.g. a series motor) the variation of torque will always be greater than the corresponding variation of current. Moreover, with such a (series) motor the torque corresponding to a given current will be unaffected by variations in the line voltage.

In Fig. 8 are given calculated torque/armature-current curves for shunt and series motors in which the armature windings and magnetic circuits are assumed to be identical and the effects of armature reaction are ignored. These curves are calculated from equation (12), together with data of the armature winding and the magnetization curve.

The effect of voltage variation at the terminals of the shunt winding upon the torque is indicated by the curves IIA, IIB, which correspond to voltages of 80 per cent and 60 per cent of normal, respectively, curve II corresponding to normal voltage.

The variation of torque with current is shown better in Fig. 9, in which the specific torque (i.e. torque per ampere of armature current) is plotted against armature current. Incidentally, these curves show also the variation of flux with armature current, since—from equation (12*a*)—

$$\mathfrak{T}/I = (pN/426)\Phi \quad . \quad . \quad . \quad . \qquad (12b)$$

The curves of Figs. 8 and 9 show also the starting performances of shunt and series motors.

From a study of these curves we conclude that when a large torque is required the current input to a series motor will be lower than that to a corresponding shunt motor, even when the latter is operated under the most favourable conditions, i.e., constant voltage. In traction systems, however, the line voltage is subject to large fluctuations, and the conditions at starting may result in the voltage at the car being appreciably below normal. Hence under these conditions the difference between the armature currents for a given torque will become greater as the line voltage is reduced below normal. Of course, by highly saturating the magnetic circuit of the shunt motor, the effect of voltage variation upon the torque will be smaller, but this procedure would be impracticable with traction motors owing to the restricted space available for the field winding.

Speed-Current and Speed-Torque Characteristics. The equation for the speed of a d.c. motor is obtained from the fundamental equation for the e.m.f. generated in the armature, i.e.

$$V - Ir = \frac{n}{60} \cdot \frac{p}{a} \cdot z\Phi \times 10^{-2}.$$

whence
$$n = \frac{6{,}000(V - Ir)}{z\Phi p/a} = \frac{3{,}000(V - Ir)}{pN\Phi} \quad . \quad . \quad . \quad (13)$$

where n is the speed in revolutions per minute, V the voltage at the terminals

of the motor, r the resistance of the main circuit (including the series field winding, if any), and I, p, a, z, N, Φ have the same significance as previously. For a given motor the speed is, therefore, proportional to the ratio $(V - Ir)/\Phi$, i.e. counter-e.m.f./flux.

Hence if the line voltage is constant and the effects of armature reaction are ignored, the speed-current and speed-torque characteristic for a shunt motor can be calculated quite easily from equations (12), (13). But with a series motor a knowledge of the magnetic characteristic or saturation curve is necessary, together with data of the armature and field windings, before these characteristics can be pre-determined. If, however, the torque-current curve is available the speed-current curve can be readily obtained, as, by combining equations (12*b*) and (13), we have

$$n = \frac{3{,}000(V - Ir)}{426\mathfrak{T}/I} = 7{\cdot}05\,\frac{(V - Ir)}{\mathfrak{T}/I} \qquad (14)$$

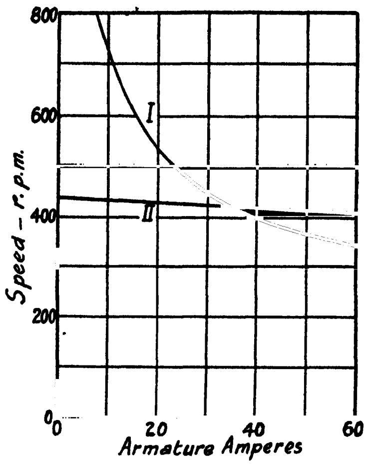


FIG. 10. SPEED-CURRENT CURVES FOR SERIES (I) AND SHUNT (II) MOTORS

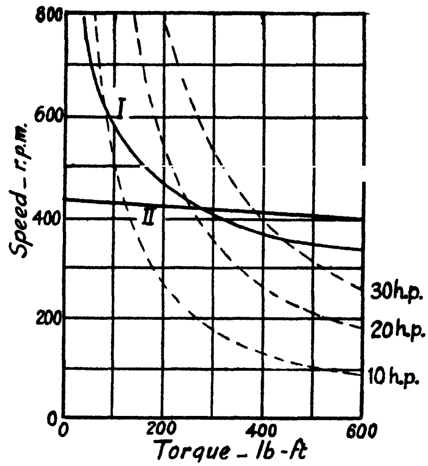


FIG. 11. SPEED-TORQUE CURVES FOR SERIES (I) AND SHUNT (II) MOTORS

With the speed-current and torque-current curves available, the speed-torque characteristic can be determined quite easily.

The calculated speed-current and speed-torque characteristics, at normal voltage, for the shunt and series motors previously considered are shown in Figs. 10 and 11 respectively, and represent the dynamical performances of the motors. The dotted curves in Fig. 11 correspond to constant outputs: their points of intersection with the appropriate motor characteristic give the speeds and torques for the given outputs.

EXAMPLES. 1. The relationship between the current and torque of a series motor—determined by a static test—is as follows—

Current (amp)	10	20	30	40	50	60	70	80
Torque (lb-ft)	33	95	170	258	346	450	565	670

Deduce the speed curve of the motor when supplied at a constant voltage of 500 volts, having given: resistance of main circuit of motor = 0·5 ohm.

The calculation is best effected in tabular form, employing equation (13*a*), i.e. $n = 7{\cdot}05(V - Ir)/(\mathfrak{T}/I)$.

I . .	10	20	30	40	50	60	70	80
$\mathfrak{T}/I$.	3·3	4·75	5·66	6·45	6·92	7·5	8·07	8·37
Ir . .	5	10	15	20	25	30	35	40
$V{-}Ir$.	495	490	485	480	475	470	465	460
$(V{-}Ir)/(\mathfrak{T}/I)$	150	103·2	85·7	74·4	68·6	62·7	57·6	55
$n = 7{\cdot}05\,(V{-}Ir)/(\mathfrak{T}/I)$	1,057	728	604	524	483	422	406	388

2. The magnetization curve of a 4-pole series traction motor—determined by separately exciting the field winding, and connecting a voltmeter across the brushes and driving the armature at a constant speed of 500 r.p.m.—is as follows—

Field amperes	50	100	150	200	250	300	350
Armature volts . . .	240	380	446	488	522	550	576

Determine the speed-torque curve for this motor when operating at a constant voltage of 600, having given that the armature has a two-circuit winding with 97 turns per circuit, the resistance of the armature winding and brushes is 0·06 ohm, and the resistance of the field windings is 0·05 ohm.

The relationship between the flux per pole and exciting current is obtained from the no-load magnetization curve by the application of the fundamental e.m.f. equation: $E = (p/a)z\Phi(n/60) \times 10^{-2}$, or $E = 2pN\Phi(n/60) \times 10^{-2}$. In the present case $p = 4$, $N = 97$, $n/60 = 500/60 = 8{\cdot}33$, so that

$$\Phi = E \times 10^2/(8 \times 97 \times 8{\cdot}33) = 0{\cdot}01546E$$

Whence—

Field amperes	50	100	150	200	250	300	350
Armature volts . . .	240	380	446	488	522	550	576
Flux per pole (megalines) . .	3·71	5·88	6·9	7·55	8·07	8·5	8·9

The speed can now be calculated by the application of equation (13), i.e.

$$n = 3{,}000(E - IR)/(pN\Phi) = 7{\cdot}74(600 - 0{\cdot}11I)/\Phi$$

and the torque can be determined by the application of equation (12*a*), i.e. $\mathfrak{T} = p\Phi IN/426 = 0{\cdot}91\Phi I$, Φ being expressed in megalines in both cases. The calculations are given in tabular form—

Amperes input (I) . . .	50	100	150	200	250	300	350
Internal e.m.f. ($= 600{-}0{\cdot}11I$) . .	594·5	589	583·5	578	572·5	567	561·5
Speed—r.p.m. [$= 7{\cdot}74(600{-}0{\cdot}11I)/\Phi$] .	1,240	775	654	592	549	516	488
Torque—lb-ft ($= 0{\cdot}91\Phi I$) . . .	169	536	942	1,375	1,837	2,320	2,835

Comparison of Dynamical Performances of Shunt and Series Motors. Comparing the dynamical performances of each type of motor with the dynamical requirements of tramway and suburban railway services, we observe that the characteristics of the series motor are better suited to these conditions than those of the shunt motor. Thus (i) the series motor is capable of exerting a large torque at starting, which, for a given current, is independent of fluctuations of the line voltage; (ii) this motor possesses a high free-running speed; (iii) the speed automatically decreases when the torque increases, thereby protecting the motor against excessive overloading. This self-protective property is of immense value in practice, and is shown by the torque-output curves of Fig. 12, which have been deduced from the speed-torque curves of Fig. 11.

Speed-output curves are also given in Fig. 12. These curves show that if large outputs at high speeds—such as would be necessary for express passenger service on main-line railways—are required a number of speed/output characteristics, instead of a single characteristic, must be available from the motor equipment of the locomotive; the lower speeds being obtained by operating the motors at voltages below normal (in combination, if necessary, with field

weakening), and the higher speeds being obtained by field weakening. Both methods are discussed later in this chapter and in chapters VIII and IX.

Electrical Operation of Series and Shunt Motors under Traction Service Conditions. We will now compare the electrical operation of series and shunt motors under conditions likely to occur in actual service. Confining our attention to tramway and suburban railway services—which require the use of passenger cars equipped with two or four motors per car—the first item of importance is the manner in which the *division of load between the several motors* is affected by (*a*) differences in the diameters of the driving wheels, such as would be caused in practice by unequal wear of the tyres; (*b*) differences in the speed curves of the motors, such as would occur with slight inequalities in the air gaps of the motors, or slight inequalities in the magnetic characteristics of the materials employed.

In general, these conditions have an adverse effect upon the operation of shunt motors but have very little effect upon the operation of series motors, the inequality of the division of load being greater for motors possessing "flat" speed-torque characteristics than for machines having "steep" speed-torque characteristics.

The comparison is best shown quantitatively.

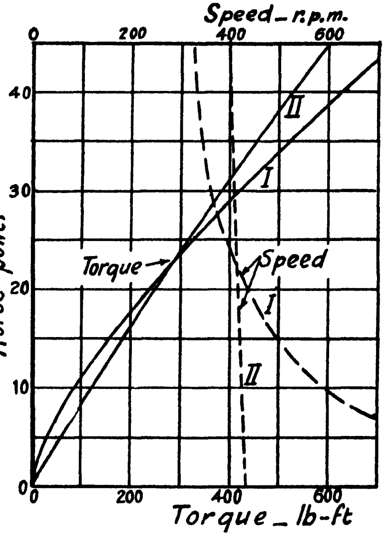


FIG. 12. OUTPUT CHARACTERISTICS OF SERIES (I) AND SHUNT (II) MOTORS

I. **Series Motors, Parallel Operation, Unequal Wheel Diameters.** Consider a car to be equipped with two motors each having a speed curve identical with curve I, Fig. 13. Let one motor (A) drive wheels 30 in. in diameter, and let the other motor (B) drive wheels $29\frac{1}{2}$ in. in diameter. Then for a given speed of the car the speed of armature B will be ($30/29{\cdot}5 =$) 1·017 times that of armature A. Hence when the motors are connected in parallel the current input to each will be obtained from points on the speed curve corresponding to these speeds. For example, if for a given speed of the car the speed of A is 390 r.p.m., the speed of B will be ($1{\cdot}017 \times 390 =$) 397 r.p.m. Whence, from Fig. 13, the current inputs are 41·5 A and 39·5 A respectively. The division of the load between the motors is, therefore, affected only slightly by the inequalities in the wheel diameters.

IA. **Series Motors, Series Operation, Unequal Wheel Diameters.** The current in each motor is now the same, but the terminal voltages are unequal. In these circumstances the results are best obtained analytically. Thus, if V is the line voltage, V_A, V_B the terminal voltages of the motors A, B respectively, n_A, n_B the speeds of these motors, r the resistance of each, and I the current input, then, from equation (13), we have

$$n_A/n_B = (V_A - Ir)/(V_B - Ir),$$

and

$$V_A + V_B = V$$

Solving these equations, we obtain

$$V_A = \frac{V - Ir}{1 + n_B/n_A} + \frac{Ir}{1 + n_A/n_B}$$

$$V_B = \frac{V - Ir}{1 + n_A/n_B} + \frac{Ir}{1 + n_B/n_A}$$

If n_A' is the speed corresponding to a current I and normal voltage V, then

$$n_A = n_A'(V_A - Ir)/(V - Ir).$$

Example. If $n_A/n_B = 1/1{\cdot}017$ as above, $V = 500$, $Ir = 45$, $n_A' = 390$,

$$V_A = \frac{500 - 45}{1 + 1{\cdot}017} + \frac{45}{1 + 1/1{\cdot}017} = 248{\cdot}2 \text{ volts}$$

$$V_B = \frac{500 - 45}{1 + 1/1{\cdot}017} + \frac{45}{1 + 1{\cdot}017} = 251{\cdot}8 \text{ volts}$$

$$n_A = 390(248{\cdot}2 - 45)/(500 - 45) = 174{\cdot}2 \text{ r.p.m.}$$

$$n_B = 1{\cdot}017 n_A = 177{\cdot}2 \text{ r.p.m.}$$

II. **Shunt Motors, Parallel Operation, Unequal Wheel Diameters.** If the car were equipped with two motors having speed curves identical with curve II, Fig. 13, the current inputs corresponding to 390 r.p.m. and 397 r.p.m. are 41·5 A and 24·2 A, respectively. In this case the motors are loaded very unequally.

IIA. **Shunt Motors, Series Operation, Unequal Wheel Diameters.** When the armatures are connected in series and each motor is normally excited, the load is almost equally divided. Thus, applying the equations obtained for the series-connected series motors, but taking $Ir = 30$ volts instead of 45 volts, we have

$$V_A = \frac{500 - 30}{1 + 1{\cdot}017} + \frac{30}{1 + 1/1{\cdot}017} = 248{\cdot}12 \text{ volts}$$

$$V_B = \frac{500 - 30}{1 + 1/1{\cdot}017} + \frac{30}{1 + 1{\cdot}017} = 251{\cdot}88 \text{ volts}$$

$$\text{Input to armature } A = 41{\cdot}5 \times 248{\cdot}12 = 10{,}290 \text{ watts}$$

$$\text{Input to armature } B = 41{\cdot}5 \times 251{\cdot}88 = 10{,}450 \text{ watts}$$

$$n_A = 390(248{\cdot}12 - 30)/(500 - 30) = 181 \text{ r.p.m.}$$

$$n_B = 1{\cdot}017 \times 181 = 184{\cdot}2 \text{ r.p.m.}$$

III. **Compound Motors, Parallel Operation, Unequal Wheel Diameters.** With motors having speed curves identical with curve III, Fig. 13, the current inputs corresponding to armature speeds of 390 r.p.m. and 397 r.p.m. are 41·5 A and 35·8 A respectively.

IV. **Parallel Operation, Equal Wheel Diameters, Dissimilar Speed Curves.** Assuming a 2½ per cent difference at normal speed (390 r.p.m.), the current inputs to the motors are obtained from the points on the appropriate speed curves which correspond to the actual armature speeds. For example, with

series motors having speed curves represented by curves I$_A$, I$_B$, *Fig. 13, the* currents corresponding to equal armature speeds of 390 r.p.m. are 41·5 A and 44·5 A respectively. With shunt motors under similar conditions (curves II$_A$, II$_B$, Fig. 13) the armature currents are 41·5 A and 72 A respectively.

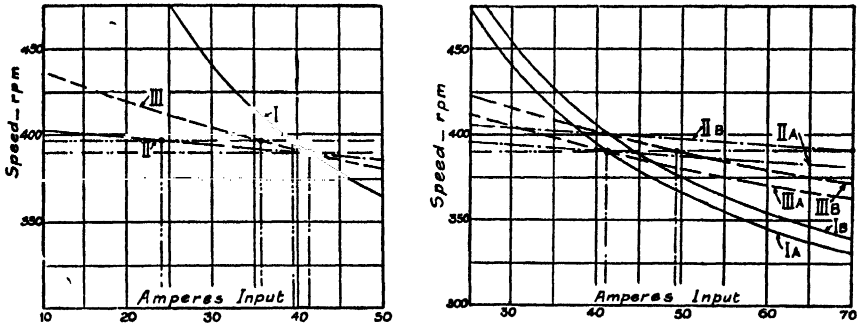


FIG. 13. EFFECTS OF (*a*) DIFFERENCES IN DIAMETERS OF DRIVING WHEELS, (*b*) DIFFERENCES IN SPEED CURVES, ON CURRENT INPUT TO TWO MOTORS, OF VARIOUS TYPES, CONNECTED IN PARALLEL

Summarizing the results for parallel operation, we have

Type of Motor	Reference to Speed Curves Fig. (13)		Diameter of Driving Wheels (in.)		Speed of Armatures (r.p.m.)		Current Input (Amperes)	
	Motor A	Motor B	Motor A	Motor B	Motor A	Motor B	Motor A	Motor B
Series	I$_A$	I$_A$	30	29·5	390	397	41·5	39·5
Series	I$_A$	I$_B$	30	29·5	390	397	41·5	42·3
Series	I$_A$	I$_B$	29·5	30	397	390	39·5	44·5
Shunt	II$_A$	II$_A$	30	29·5	390	397	41·5	24·2
Shunt	II$_A$	II$_B$	30	29·5	390	397	41·5	50
Shunt	II$_A$	II$_B$	29·5	30	397	390	24·2	72
Compound	III$_A$	III$_A$	30	29·5	390	397	41·5	35·8
Compound	III$_A$	III$_B$	30	29·5	390	397	41·5	44
Compound	III$_A$	III$_B$	29·5	30	397	390	35·8	49·5

It is apparent, therefore, that if shunt motors were adopted for electric traction the permissible deviation from the standard speed-curve would have to be considerably smaller than that shown in Fig. 13. Moreover, all the driving wheels of a car or a train would require frequent gauging, as only slight differences in the diameters would be permissible.

These restrictions, however, do not apply with the same force when compound motors are employed on locomotives, and such motors may be preferable to series motors for mainline services on undulating routes. Locomotives with compound motors are in service on the Italian State Railways, the shunt windings of the individual motors of a locomotive being separately excited at low voltage because of the high operating voltage (3,000 V).

Other important items which require consideration in comparing the electrical

performances of series and shunt motors are *voltage rises* and *temporary interruptions in the supply*. The former may occur on low-voltage railways when a heavy current, due to a short circuit on the track, is cleared by the opening of the sub-station circuit breakers, and temporary interruptions in the supply to the motors occur when crossovers and section-insulators are crossed with the controller "on." These operating conditions are peculiar to traction service and therefore require special consideration.

Voltage-rises. When a motor is subjected to a sudden voltage-rise the initial value of the current-rush is determined by the impedance of the motor circuit, and the duration of this current-rush depends upon the rate at which the counter-e.m.f. of the motor is built up. The current-rush will, therefore, have a lower initial value, and will be of shorter duration, in series motors than in compound or shunt motors.

Temporary Interruption of Supply. When the supply is briefly interrupted and restored at full voltage, there is a characteristic difference between the operation of series and shunt motors. With a shunt motor the counter-e.m.f. is maintained—at decreasing value—during the interruption, but with a series motor the counter-e.m.f. ceases when the interruption occurs. Upon the restoration of the supply voltage, the shunt machine will have a certain counter-e.m.f., and the initial value of the current-rush will be determined by the resultant e.m.f. and the impedance of the armature circuit. With the series motor, however, the initial current-rush will depend entirely on the impedance of the motor circuit.

Commutation of Current-rush. The large current due to a current-rush may have to be commutated under unfavourable conditions, e.g. the commutating flux may not follow instantaneously the rapid changes in the armature current, or flux distortion may occur due to weak field. Under these conditions sparking may occur at the brushes, but the operation of the motor may be considered as satisfactory provided that this sparking does not cause a flash-over.

To produce a flash-over, sufficient voltage must exist between adjacent commutator segments so that an arc, once formed between a brush and a segment, will be maintained and extended from segment to segment as the commutator rotates.

Under normal operating conditions the voltage between adjacent segments around the commutator depends on the flux distribution in the air-gap; the voltage between segments in the vicinity of the brushes being influenced largely by armature reaction.

But, under conditions of weak field, un-neutralized armature reaction and slow growth of commutating flux, current-rushes may cause relatively high voltage between the segments in the vicinity of the brushes. This adverse voltage distribution is accentuated by e.m.f.'s induced, by transformer action, in the coils undergoing commutation, due to the rapid changes in the flux linked with these coils.

Ability of Motors to Withstand Current-rushes. The modern series traction motor (without a compensating winding) is able to withstand sudden voltage rises of a large amount, and also brief interruptions in the power supply even when operating with weakened field (50 per cent of full field). Compensating

windings are necessary only in cases where a much greater range of field weakening is required.

Compound motors for trolleybus service can also operate satisfactorily under similar conditions without compensating windings, but motors for 1,500- and 3,000-V locomotives would generally require these windings.

On the other hand a shunt motor if employed on traction service would almost invariably require a compensating winding.

FIG. 14. FRAME OF BROWN BOVERI 1,000 H.P. LOCOMOTIVE MOTOR WITH POLE-FACE WINDING

Note. This motor has the exciting field winding divided into three sections, and five field strengths are obtainable for a given value of armature current.

Compensating Winding. The function of a compensating winding is to neutralize armature reaction. The conductors of such a winding must, therefore, be located in the pole faces and should be distributed in the same manner as those of the armature winding, the ampere-turns being equal to but opposing those of the armature. Practical conditions, however, restrict the compensating winding to about 50 to 60 per cent of the pole pitch, in which case the neutralizing ampere-turns will be supplied jointly by the compensating winding and the commutating-pole winding.

Fig. 14 shows a 1,000-h.p., 1,500-V series motor with compensating winding, the conductors of which occupy 10 slots in each pole face. This motor is designed for a wide range of field control—the minimum exciting ampere-turns being 26 per cent of the full excitation at this current—and each coil of the exciting winding is wound in three sections.

Considerations in Electrical Design. To meet the requirements of immunity from flash-overs and stability when operating with weak field the electrical design, when no compensating winding is employed, must provide for a relatively "strong" field at rated load and a liberal number of commutator segments. In addition solid spool bodies and short-circuited turns (for "anchoring" purposes) in the field coils must be eliminated so that the growth of the flux may not be retarded by eddy currents. In some motors the commutating poles are laminated for the same reason.

The field ampere-turns at rated load are therefore of the order of 1·5 to 2 times the armature ampere-turns, and the number of commutator segments is chosen to give an average voltage per segment between 10 to 13 volts for a 600-V motor and 15 to 18 volts for a 1,500-V motor. In some motors the ampere-turns on the commutating poles at rated load are in excess of the theoretical value in order that the commutating flux may be adequate when current rushes occur.

Although a high efficiency is desirable the economics concerned with the optimum efficiency involve the consideration of the energy required for propulsion, as every ton of additional weight, resulting from an endeavour to improve the efficiency, increases the energy consumption by approximately $(2r + 0{\cdot}0283 V_m^2/D)$ watt-hours per mile (*see* p. 25) which must be offset against the energy saving due to the higher efficiency.

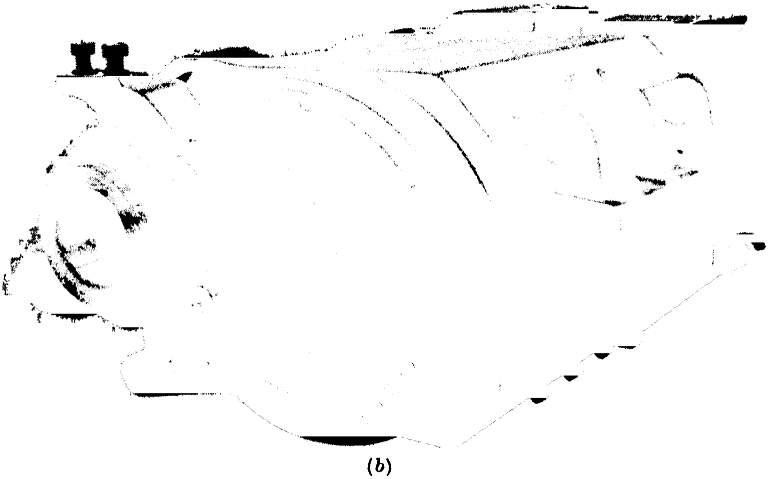
The severe limitations of the overall dimensions and the importance of light weight necessitate a high speed of rotation for the armature. Modern motors are therefore designed to operate at armature speeds, at the 1-hour rating, of from 800 to 1,500 r.p.m. The maximum speed in service may reach 2 to 3 times these values according to the class of service, the limiting speed being governed by the maximum peripheral speed of the core and windings, which is approximately 13,000 feet per minute.

Such high speeds are practicable with roller bearings and "seasoned" commutators. The "seasoning" process is carried out in the factory and consists of cycles of heating, running at high peripheral speed for a brief period, cooling, tightening. By these means the segments are settled to a permanent state in which no "high bars" will develop in service.

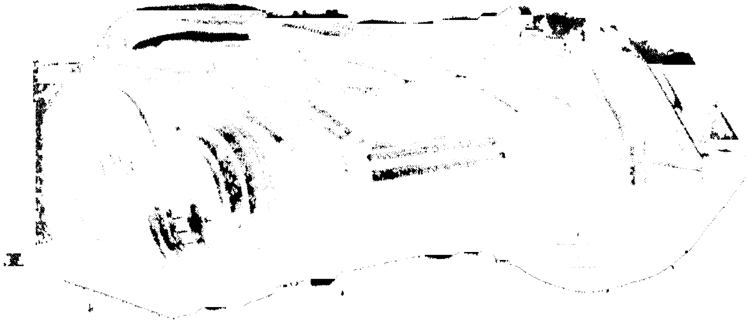
Special Features in Construction. The construction of traction motors differs in many respects from that of general-purpose d.c. industrial motors due to—(i) the necessity of minimum weight combined with adequate mechanical strength and robustness, in consequence of which a gear transmission is necessary to obtain a sufficiently high rotative speed for the armature; (ii) the location of the motor under the car floor (except in some electric locomotives), which position very severely restricts the overall dimensions to definite limiting values determined by the diameter of the driving wheels and the track gauge; (iii) the large centrifugal forces to which the rotating parts are subjected at the maximum speed of the train or car.

In consequence, cast iron must be excluded from the construction and replaced by either (*a*) cast steel or fabricated steel bar and plate for the magnetic portion of the frame and those parts requiring high mechanical strength, or (*b*) pressed or fabricated steel plate and light alloys for other parts not highly stressed.

(a)



(b)



(c)

FIG. 15. TYPICAL TRACTION MOTORS

(a) Metropolitan-Vickers tramway motor with resiliently mounted frame and roller suspension bearings
(b) English-Electric nose-suspended railway motor with sleeve suspension bearings.
(c) Crompton-Parkinson low height railway motor with roller suspension bearings.

CONSTRUCTIONAL DETAILS*

Frames. All classes of traction motors have box frames of octagonal or circular cross section. The frames are usually of cast steel, but in some cases steel plate is employed to obtain more uniform magnetic qualities than is possible with castings. Typical motors are illustrated in Fig. 15.

Suspension bearings and gear-case supports are required on the frames of axle-suspended geared motors. The bearings (usually of the sleeve type) maintain the armature shaft parallel to, and a fixed distance from, the axle and support approximately one half of the weight of the motor, the other support being on the truck frame (*see* Figs. 22, 23). Recently roller bearings have been introduced, and although more costly initially than sleeve bearings they have the advantages of low maintenance, negligible wear and improved gear performance.

The *armature bearings* are located in housings or frame-heads which are spigoted into the ends of the frame. Roller bearings are now employed because

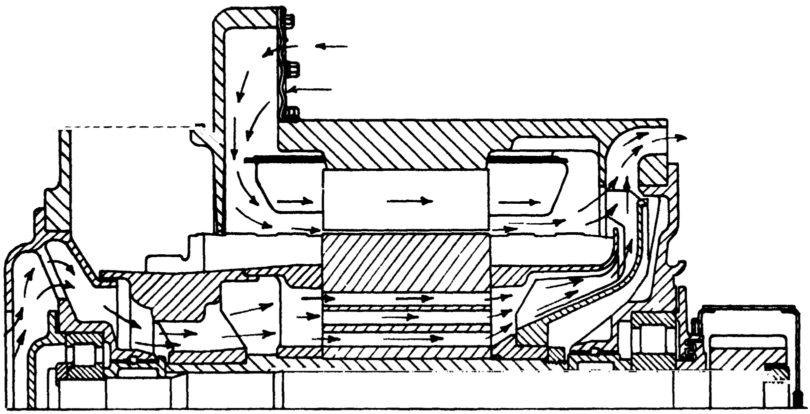


FIG. 16. HALF-LONGITUDINAL SECTION OF RAILWAY MOTOR (METROPOLITAN-VICKERS)

of their reliability at high speeds. Moreover, due to the negligible wear, no variation occurs in the air gap between the armature and pole faces, and when the suspension bearings are also of the roller type, both the parallelism and the distance between armature shaft and axle are rigidly maintained. Sealing glands of the labyrinth type are necessary to retain the lubricant in the bearing and prevent entry of grit. Typical arrangements are shown in Fig. 16.

Poles and Field Coils. The number of poles is limited by space restrictions. Four main poles are suitable for outputs up to about 450 h.p. when the track gauge is standard (4 ft $8\frac{1}{2}$ in.), and six poles for larger outputs. For narrow-gauge locomotive motors, however, six poles may be desirable even for moderate outputs in order to obtain a longer armature core than would be possible with four poles.

The *main poles* are built up of soft steel laminations, and the *interpoles* are

* The details of frames, suspension and gearing apply principally to motors for motor-coaches and tramcars. Motors for locomotives, trolleybuses and battery vehicles are discussed in separate sections later.

usually steel forgings, but in some cases they are laminated in order to reduce eddy currents which would delay the building-up of the commutating flux with rapidly varying currents.

High-reluctance interpoles may be necessary for motors operating under extreme weak-field conditions to ensure that the commutating flux is always proportional to the armature current. The high reluctance is obtained, with the normal air gap at the pole face, by non-magnetic shims or packing between each pole core and its seating on the magnet frame.

The *field coils* are former-wound, bakelized, pressed to size and treated so as to be impervious to moisture. Only class B materials, e.g. mica, asbestos, fibrous glass, are employed for insulation.

For *tapped-field control* each coil of the main-field winding must be wound

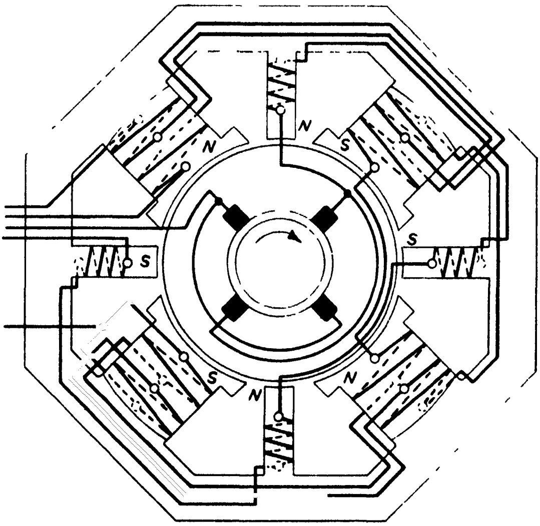


FIG. 17. INTERNAL CONNEXIONS OF TAPPED-FIELD MOTOR

in two or more sections. The corresponding sections of each coil are connected to form a series-group, and the two, or more, groups are connected in series, tappings being brought out from the junctions of the groups as shown diagrammatically in Fig. 17.

Armatures. The conditions under which traction motors operate demand a thoroughly sound mechanical construction of the armature, as well as means for replacing a damaged shaft without disturbing either the winding or the commutator. The core is usually built on a steel spider which is extended to carry the commutator.

Axial ducts are provided in the core, end-flanges and commutator shell for ventilation, the cooling air entering at the commutator end and leaving at the pinion end. With a self-ventilated motor a fan is fitted to the back end-flange of the armature as shown in Fig. 16. In this particular case, which refers to a high-voltage motor, the entering air does not pass over the surface of the commutator, and there is no possibility of brake-shoe dust being deposited on the leakage surfaces of the brushgear insulators and commutator.

To prevent deposits of dust at the back of the commutator and underneath the armature coils, the back vee-ring and the adjacent end-flange of the armature core are either spigoted and sealed, or combined with the commutator shell. Some manufacturers also consider it desirable to make a dust-tight seal between each end-flange and the adjacent laminations. For the same reason the coil projections are covered with canvas as shown in Fig. 20.

Armature Winding. Both wave (two-circuit) and lap (multiple-circuit) windings are employed in modern motors, the choice depending on a number of factors concerned with design. Wave windings would be employed for all tramway, trolleybus and motor-coach motors: lap windings would be employed for the larger railway motors and in cases where the number of commutator segments for a two-circuit wave winding with single-turn coils would result in too high a value for the average voltage per segment. The successful operation of lap windings in traction motors is due to the use of roller-type armature bearings and equalizer connexions at every coil. Wherever possible single-turn coils are employed to obtain good communication and freedom from flash-overs. For this reason it is now the practice to use 250/300-volt motors (two in series)

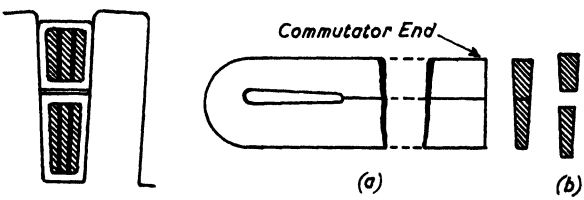


FIG. 18. TAPERED BAR CONDUCTOR (METROPOLITAN-VICKERS)
(a) Shaped and split bar; (b) Cross section of upper and lower portions of slot conductor.

in four-motor equipments for tramcars and individual motor-coaches when rheostatic braking is required for service stops.

Tapered bar conductors and tapered slots are employed in a considerable number of British-built traction motors as, due to the parallel teeth, a larger output can be obtained from a given size of core. Fig. 18 shows a cross-section of a slot with the conductors in position. The manufacturing process for single-turn coils is quite simple—the coil sides are formed to shape in the usual manner from a suitable length of bar of the full depth which is slit longitudinally as indicated at (*a*) in Fig. 18.

Precautions to minimize eddy currents in the conductors may be necessary in certain cases (e.g. high speed, high tooth flux densities, deep slots). The usual method is to split the slot portion of each conductor of the top layer and introduce a double twist or cross-over as shown in Fig. 19. Alternative methods are—(i) laminated conductors, (ii) folded (transposed) bars, and (iii) coils with the bar conductors arranged on the flat, instead of on edge.*

Joints between the coils and the commutator segments need special consideration in view of the high operating temperatures permissible with class B

* The relative advantages and disadvantages of these methods are discussed in a paper by M. J. Baldwin on "Twenty-five Years Progress in the Design of Traction Motors." *Trans. A.I.E.E.*, 68, 132.

and the newer class H insulation (silicone-treated class B materials). The usual soldering process with high-temperature solders is considered to be satisfactory for class B insulation. For the higher temperatures permissible with class H insulation a cold welding process has been developed in which silver is employed as the "flow" metal. The slots in the commutator risers, and the ends of the

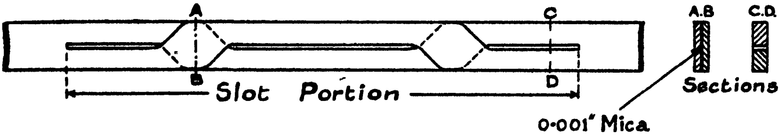


FIG. 19. DOUBLE CROSS-OVER IN ARMATURE BAR

conductors to be inserted therein, are silver plated: the latter are forced into the slots, with shims where necessary, and the top of each slot is sealed with a high melting point cadmium-silver solder.*

The *binding bands* and slot wedges fulfil an important function on account of the large centrifugal forces to which the armature winding may be subjected. The tinned-steel binding wire is applied under considerable tension with the

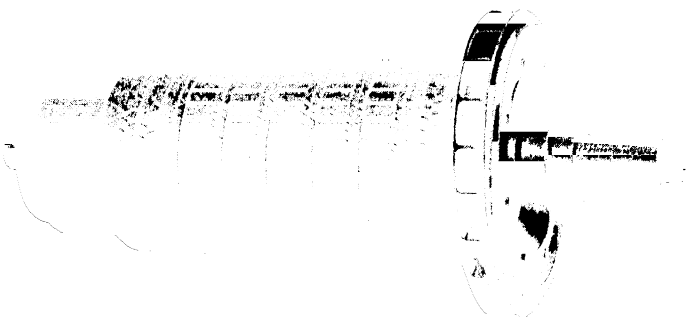


FIG. 20. ARMATURE OF RAILWAY MOTOR SHOWING FAN AND SERRATED CLIPS FOR BINDING BANDS (CROMPTON-PARKINSON)

armature hot and special precautions are taken to ensure tightness. The individual turns are retained in position by clips and sweating. In some cases the clips are continuous and serrated, the serrations, when bent over, enveloping the whole width of the band. Fig. 20 shows a completed armature banded in this manner.

Commutator and Brushgear. The construction of the commutator follows the usual practice, the "arch-bound" method of clamping the segments being employed for high speeds, together with a "seasoning" process to ensure a truly cylindrical surface.

The *brush holders* are rigidly supported from the frame or bearing housing

* For details see paper on "Solderless Commutator Joints for Railway Traction Armatures," by J. R. Reed. *Trans. A.I.E.E.*, 69, 638.

as shown in Fig. 21, but in large six-pole machines they are mounted on a movable ring. Mica is the principal insulation and porcelain caps are fitted to increase the leakage surface.

FIG. 21. METHOD OF MOUNTING BRUSH HOLDERS (ENGLISH ELECTRIC)

Suspension. Two forms of suspension for an axle-mounted motor with single-reduction spur gearing are shown in Figs. 22, 23. In one case, Fig. 22, a lug or "nose," *H*, on the motor frame, remote from the suspension bearings, is supported on a bracket, *F*, fixed to the transom, *B*, of the truck, a strap, *G*, restricting the upward movement With locomotive motors the nose is spring supported.

In the other case, Fig. 23, a suspension-bar, *A*, bolted to the motor frame, rests upon "support" springs, *B*, which are carried on each side-frame, *D*, of the truck and maintained in position by bolts. Reaction springs, *C*, restrict the upward movement of the suspension-bar.

With a cardan-shaft drive—as employed for trolleybuses, battery road vehicles and some modern rail vehicles (tramcars and motor-coaches)—the motor is fixed to the chassis or truck frame, machined lugs or facings on the motor frame being provided for this purpose.

Gearing. A single reduction is usually employed for motor-coaches and locomotives, but a double reduction may be necessary for Diesel-electric shunting locomotives to obtain a suitable speed for the motor armature. Fig. 24 shows an axle-suspended motor with double-reduction gearing.

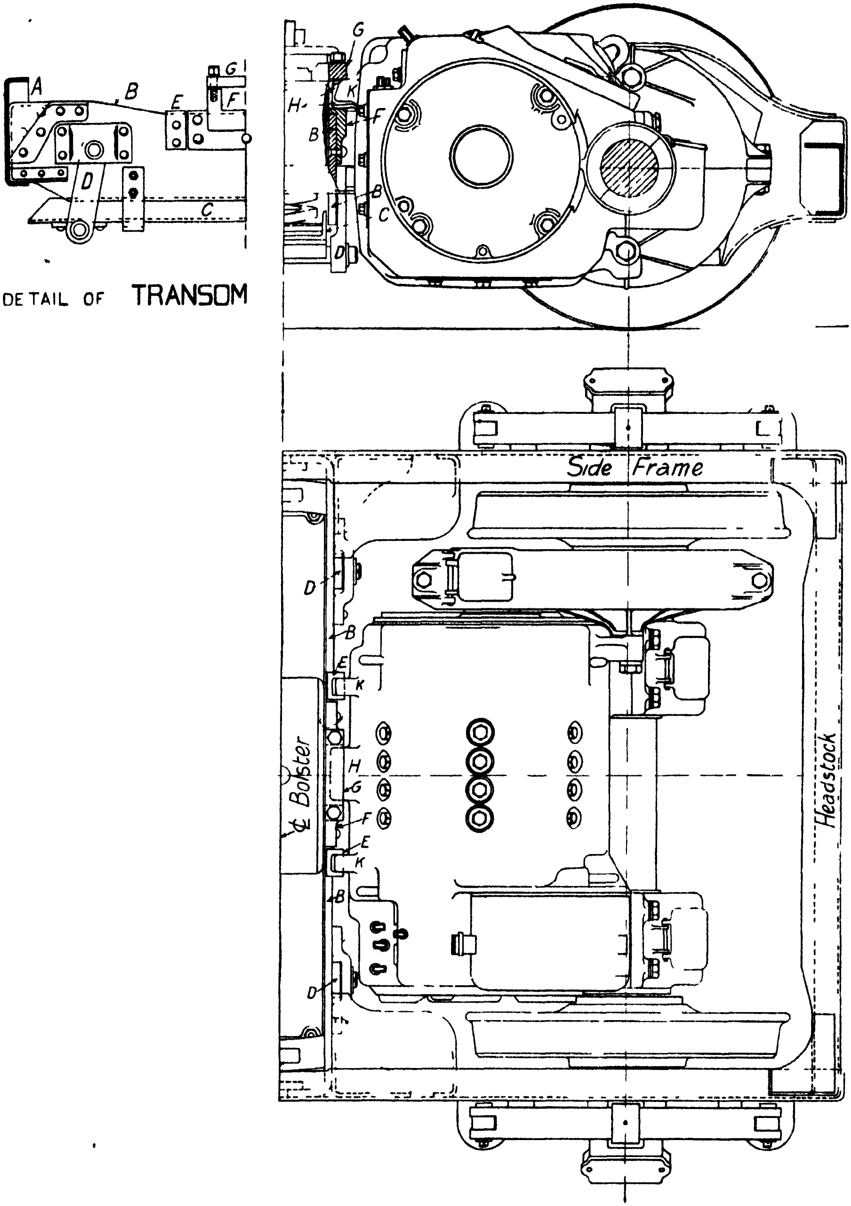


FIG. 22. ARRANGEMENT OF NOSE SUSPENSION FOR RAILWAY MOTOR (B.T.-H.)

A—side frame of truck.
B—transom.
C—spring-plank (from which the springs supporting the bolster are carried).
D—swing-links (for carrying the spring-plank).
E—brackets for supporting safety lugs, *K*.
F—bracket for supporting nose.
G—strap.
H—nose.
K—safety lugs.

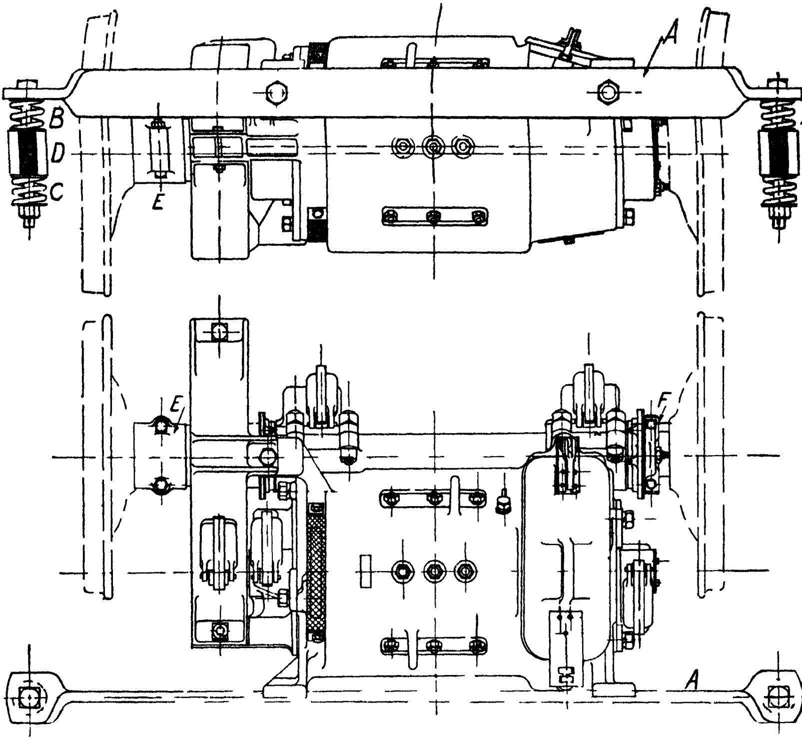


FIG 23. ARRANGEMENT OF BAR SUSPENSION (METROPOLITAN-VICKERS)

A—suspension bar
B—supporting springs
C—reaction springs
D—side frame of truck
E—non adjustable axle collar
F—adjustable axle collar

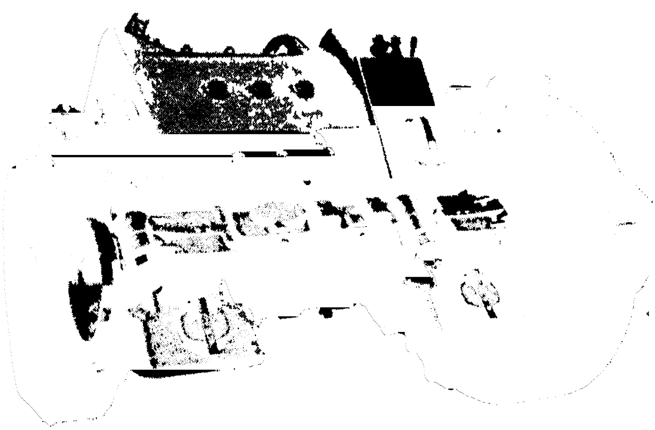


FIG. 24. AXLE-SUPPORTED MOTOR WITH DOUBLE-REDUCTION GEARING (ENGLISH ELECTRIC)

With single-reduction gearing the maximum gear ratio which can be accommodated is governed by (i) the maximum permissible diameter of the gearwheel (which is governed by the diameter of the driving wheels), (ii) the smallest permissible diameter of the pinion, and (iii) the distance between centres of armature shaft and axle. With driving wheels of small diameter the axle must usually be recessed into the motor frame to obtain the desired gear ratio.

Pinions are usually of alloy steel with case-hardened (600 Brinell) and ground teeth.

Gearwheels are of either cast steel or forged alloy steel with hardened and ground teeth.*

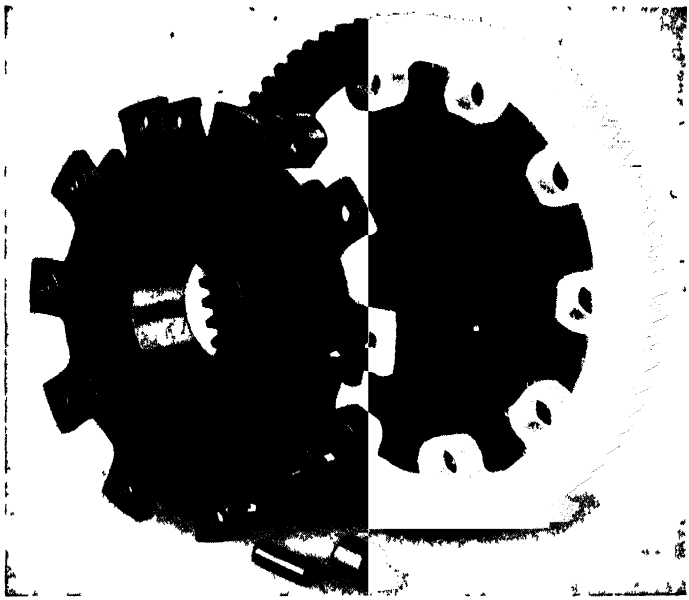


FIG. 25. HUB, TOOTHED RIM, RUBBER BUSH AND PIN OF RESILIENT GEARWHEEL (METROPOLITAN-VICKERS)

Resilient gearwheels, incorporating either springs or rubber bushes between the hub and rim, are now frequently used with axle-suspended motors. The cushioned drive reduces the impact forces (due to irregularities in the track) on the gear teeth and the armature shaft, and prolongs the life of the gears. Rubber bushes, Fig. 25, have sufficient resilience for d.c. motor drives, and have given satisfactory results in service; they are much cheaper and simpler than springs and require no maintenance.

Ventilation. The modern commutating-pole traction motor requires artificial cooling in order to obtain the full advantages of the commutating-pole design, as with natural cooling the rating of the motor would be considerably below that which would be possible from the commutation standpoint. The ventilation may be effected either by means of a fan, fixed to the back of

* Much useful information on traction gears is given in British Standard, No. 235.

the armature, in conjunction with longitudinal ventilating ducts and suitable openings in the frame—such a machine being known as a *self-ventilated motor*—or by means of an external blower in conjunction with suitable air ducts and openings in the frame of the motor—such a machine being known as a *forced-ventilated motor*. The latter system of ventilation is used principally with motors for locomotive service as, in this case, all motors on the locomotive may be ventilated by means of a single blower, which can deliver a large quantity of air at the required pressure.

The advantages of forced ventilation are—(i) the quantity of air may be adjusted, if desired, within limits; (ii) the supply of air to the motors is not affected by the speed of the latter, so that efficient cooling takes place during the whole period that the motors are in service; (iii) a blower is more efficient for ventilating purposes than a fan fixed to the armature shaft, especially when a large quantity of air is required.

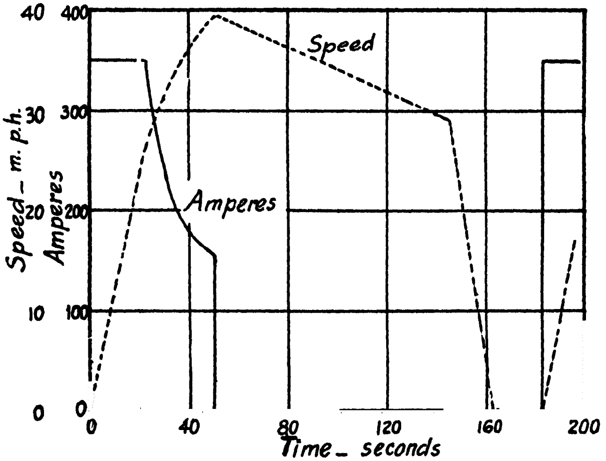


Fig. 26. Duty Cycle for One Motor of a Suburban Train

For tramway, trolleybus and motor-coach railway services, however, a self-ventilated motor possesses advantages over a forced-ventilated motor, as the former is self-contained and the fan adds very little weight to the equipment.

The position of the air inlet openings should be such as to avoid, as far as possible, the direct ingress of road grit and brake-shoe dust. Road grit causes unnecessary wear of the brushes, and brake-shoe dust not only provides a conducting path when deposited upon the surfaces of insulators, but is inflammable when ignited by a spark or a flash-over.

Air filters or cleaners are now often used on trolleybus motors, and on motor-coaches ducts may be provided in the coach body, with flexible connexions to the motor inlets, so that dust-free air may be drawn through louvres in the upper part of the coach body.

Rating

The rating of an electrical machine has usually some connexion with the nature of the load or duty-cycle on which the machine has to operate, e.g. industrial motors which run continuously under practically steady loads are rated on a continuous load basis, the rated load being defined as the output

which the motor will develop continuously with a specified temperature rise. But with trolleybus, tramway and suburban railway services the duty-cycle consists of irregular cycles of variable output, a typical cycle being represented in Fig. 26. The irregularity of the load-cycle is influenced not only by normal and known variables, such as the variable distance between stations, gradients, etc., but also by abnormal and unknown variables such as extra stops due to signal checks, adverse winds, abnormal train resistance, etc.

Hence under these circumstances a rating corresponding to the actual duty-cycle could not be obtained with factory tests. Accordingly, for commercial purposes, a nominal or arbitrary rating is adopted for traction motors, and is based on the one-hour load (at normal voltage) which will produce a temperature rise of 120°C (by resistance) when the motor is tested at the factory on a stand in the manner described in Chapter VII.

This method of rating was introduced in 1902 by the American Institute of Electrical Engineers, and was at that time considered to represent, for commercial purposes, a fair method of comparing the performances of motors by means of a simple factory test, i.e. motors of equal one-hour ratings were considered to be capable of operating similar services with approximately the same temperature rise. But with modern ventilated motors the one-hour rating gives no comparison between the service performances of motors. The one-hour test, however, imposes testing conditions which are sufficiently severe to ensure reliability of the motors in service, and for this reason it is retained at the present day.

The *temperature rise in service* depends principally upon the average losses in the motor under service conditions and the rate at which these losses can be dissipated. The former is influenced by the nature of the service and the design of the motor; the latter is influenced principally by the ventilation, and, with self-ventilated motors, is a function of the armature speed.

With motors built to British and American Standards and having class B insulation the normal maximum temperature limits under the most adverse service conditions are 145°C for the armature winding and 155°C for the field winding, with temporary peak values of 160°C and 170°C respectively, the temperatures being computed from resistance measurements.

Effect of Ventilation on Temperature Rise and Rating. With any electrical machine the rate of increase of temperature—due to the heat produced by the electrical and mechanical losses—depends upon two factors, (i) the rate at which heat is stored in the mass of the machine, (ii) the rate at which heat is dissipated by radiation, ventilation, etc. The temperature becomes constant when the heat produced is entirely dissipated by radiation and ventilation. The ultimate temperature rise at a steady load is, therefore, affected by the ventilation, but the temperature rise at the end of a run of short duration will be influenced largely by the thermal capacity of the machine.

Hence the one-hour rating of a traction motor is influenced more by the thermal capacity of the motor than by the ventilation, but the continuous and service ratings are influenced almost entirely by the ventilation, and will depend upon the volume of air which can be passed through the motor per minute.

The *volume of air* required to dissipate a given loss is easily calculated if the effects of radiation, etc., are ignored. Thus, taking the weight of 1 ft^3 of air to be 0·076 lb (or 34·5 grammes), the specific heat as 0·237, and the calorie to be

equivalent to 4·2 watt-seconds, then the energy expended in heating 1 ft^3 of air θ°C is

$$34{\cdot}5 \times 0{\cdot}237 \times \theta \times 4{\cdot}2 = 34{\cdot}3\theta \text{ watt-seconds.}$$

Effect of Operating Voltage on the Rating. Motors for operating on high-voltage circuits require larger leakage surfaces at the commutator and brush-gear in addition to extra insulation on the armature and field coils. Due to the additional insulation on the coils, and the small size of conductor, the space-factors of the armature and field coils of a high-voltage motor will be lower than those of a similar armature and frame wound for a low voltage, e.g. 600 volts.

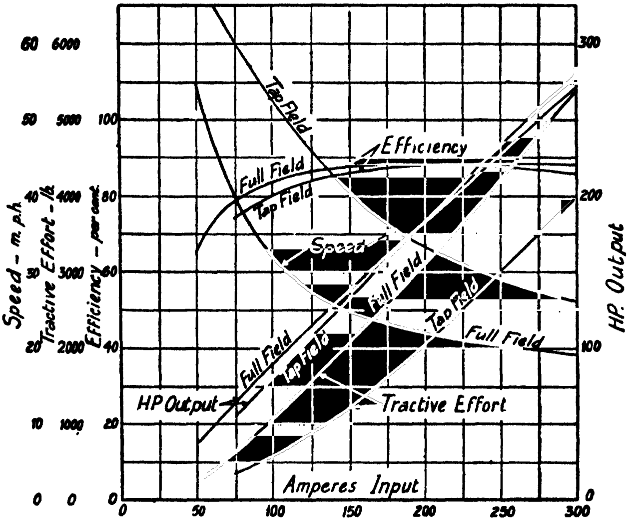


FIG. 27. CHARACTERISTIC CURVES OF RAILWAY MOTOR (B.T.-H.)
250 h.p., 775 volts, 42-in. wheels, 3·18 : 1 gear ratio.

In many cases, however, two 750-volt motors are operated in series on 1,500-volt circuits, each motor being insulated for 1,500 volts, and by the use of high-grade insulation, the rating of these motors can be made equal to that of standard low-voltage motors of similar dimensions.

Effect of Wheel Diameter and Gauge on Rating. With an axle-mounted motor, the size of motor, and therefore the rating, is limited by (i) the diameter of the driving wheels, (ii) the gauge of the track rails. For a given diameter of wheel the maximum vertical dimension of the motor is fixed by (*a*) the clearance between the bottom of the motor and the track, (*b*) the clearance between the top of the motor and the underside of the car floor. In deciding upon the minimum clearance between the bottom of the motor and the track, due allowance must be made for wear of the wheel tyres, as otherwise, with a large motor, the full wear may not be obtained from the tyres.

The *gauge* of the track rails has also a considerable effect on the rating of a motor for a given diameter of driving wheel, and narrow gauges restrict very considerably the output, especially when the wheel diameter cannot be increased to allow the use of a six-pole design.

When the whole distance between the wheel hubs is occupied by the motor and gearing, it is possible, with countersunk or roller armature bearings, and single gearing, to utilize about 75 per cent of this distance for the armature and commutator, the remaining 25 per cent being occupied by the gear, gear-case, and frame-heads.

Characteristic Curves

Typical characteristic curves of a railway motor designed for motor-coach service are given in Fig. 27. These curves show the speed, tractive effort, output and efficiency, at normal voltage, for a given gear ratio and diameter of driving wheels. They have been plotted in accordance with the standard method adopted for traction motors, and the manner in which the various quantities are determined and calculated is considered in detail in Chapter VII.

Locomotive Motors

Except in the case of *light locomotives*, for which the standard geared motor, as used on motor-coach trains, is suitable, the motors are of larger output than those previously discussed, and in consequence modifications are necessary in the methods of transmitting the power to the driving axle and of mounting the motors. Moreover, the service conditions usually involve relatively long running periods, and therefore the motors must be selected on a continuous-rating basis. Again, owing to the facility with which forced ventilation can be applied to the motors and the advantages of this method of ventilation, the larger locomotive motors are always of the forced-ventilated type.

Suspension. Nose suspension is employed on slow and moderate-speed locomotives for motors with ratings up to 500 h.p.

Frame suspension (i.e. mounting the motors on the frame of the truck or locomotive, which is spring supported from the axles) is necessary on high-speed locomotives to obtain good running qualities. In this case flexibility is necessary in the transmission system as transverse relative motion may occur between the armature shafts and the axles. The flexibility is usually obtained by fixing the gear-wheel to a quill or hollow shaft (which surrounds the axle with suitable clearances) and interposing a flexible coupling between the quill and the driving wheels. Details, together with alternative methods, are given in Chapter XVII.

Gearing. Single gearing is suitable for outputs (per axle) up to about 600 h.p. and twin gearing (i.e. a pinion and gear-wheel at each end of the motor) for larger outputs. In the latter case springs are fitted to either the pinions or the gear-wheels to ensure equal division of the load.

Frame-mounted Motors. Fig. 28 illustrates a frame-mounted *twin-geared* motor with resilient pinions. In this case, as the final drive is by connecting rods, the gear-wheels are mounted on a shaft which is carried in bearings on the locomotive frame. Fig. 29 illustrates a frame-mounted motor for a *quill drive* and shows the usual arrangement when the motor and gearing have to be accommodated between the wheel hubs. The quill is carried in bearings on the motor frame, and as the latter is fixed to the truck or locomotive frame, the whole weight of the motor, quill and gearing is spring supported.

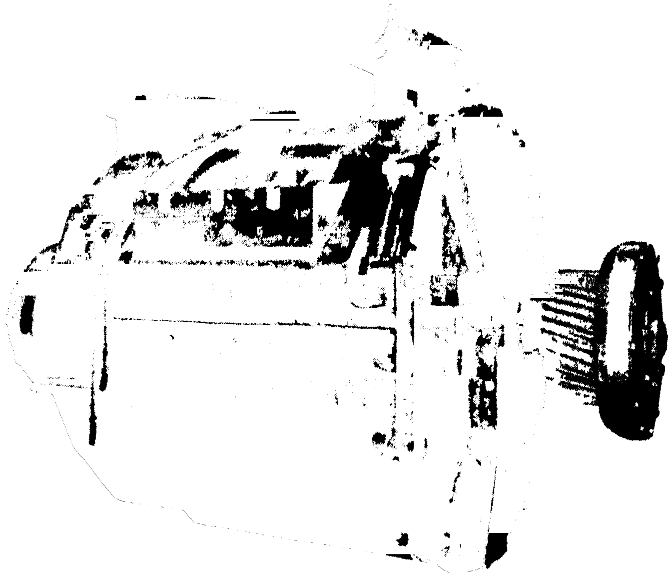


FIG. 28 FRAME-MOUNTED TWIN GEARED MOTOR WITH RESILIENT PINIONS (METROPOLITAN VICKERS)

FIG 29 FRAME-MOUNTED MOTOR FOR QUILL DRIVE (ALSTHOM)

The *twin motor*, with quill drive, provides one method of obtaining a greater output per axle than is possible with a single motor and a quill drive. The arrangement is shown in Fig. 30, and possesses the following advantages—(i) the two armatures may be designed for permanent series connexion on

1,500- and 3,000-volt circuits; (ii) the armatures and frame diameters are smaller than those of the corresponding single motor and in consequence a higher gear ratio is practicable, thereby leading to a higher armature rotative speed and reduced dimensions; (iii) only single, instead of twin, gearing is necessary, and therefore a greater proportion of the distance between the wheel hubs is available for the armature.

An *alternative method*, also particularly suitable when two motors are to be connected permanently in series, is to use two single motors—each rated at one half of the total output required from the axle—and to mount these in *tandem*, with the pinions driving a central gear-wheel on a quill, from which the axle is driven by a universal (Oldham type) coupling. This arrangement allows practically the full width of the locomotive body to be used for the motors and gearing, and enables outputs of 1,000 h.p. or more to be obtained from a single axle.

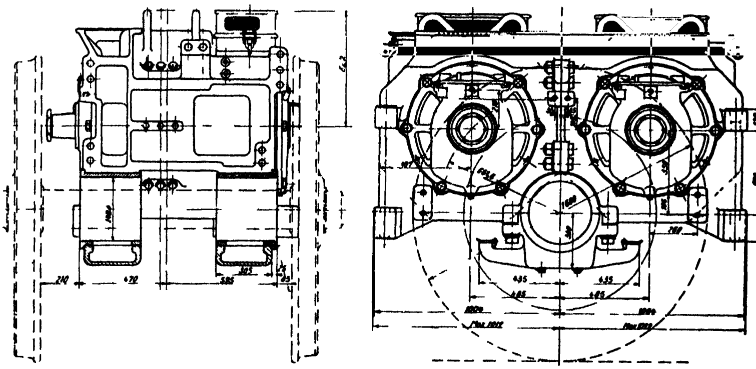


FIG. 30 GENERAL ARRANGEMENT OF TWIN MOTOR AND DRIVING AXLE (OERLIKON)

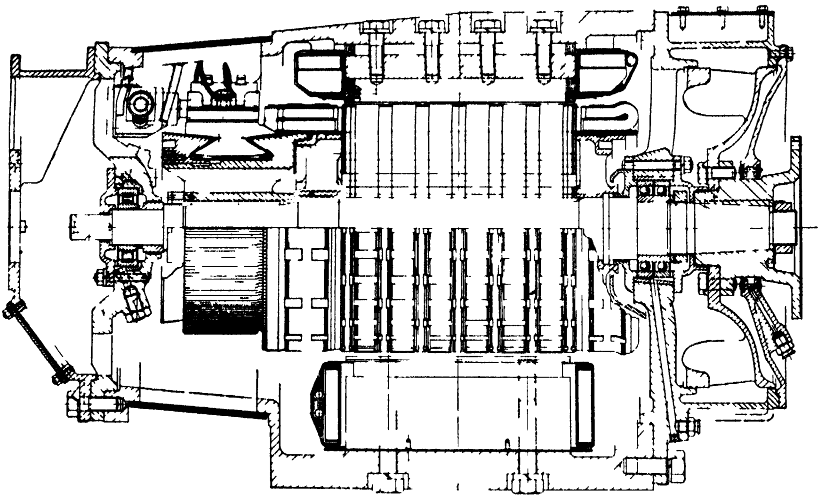
Speed Control. Locomotives for express passenger service and those for mixed services must have a relatively large number of economical running speeds. These speeds may be obtained by grouping the motors of a multi-motor equipment in series, series-parallel and parallel (as explained in Chapters VIII and IX) and by weakening the field, by either tapping or shunting the field winding or a combination of both. If shunting is employed to obtain a moderate or large amount of field weakening the shunt must be inductive, with a time-constant approximating to that of the field winding, in order to ensure the correct division of current under both normal and transient conditions.

In the motor illustrated in Fig. 14, the exciting field winding is divided into three sections, and five speeds are obtained, for a given armature current and voltage, by field tapping and shunting.

TROLLEYBUS MOTORS

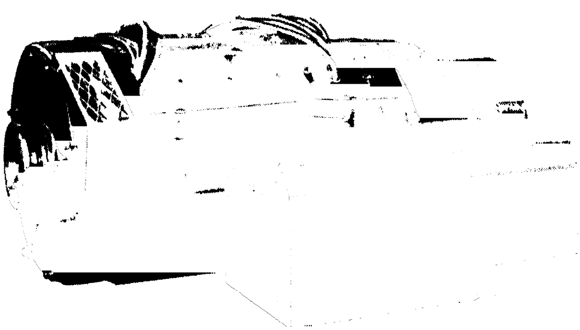
Modern trolleybuses are equipped with a single-driving motor* with a 1-hour

* In Great Britain the *laden* weight of trolleybuses is limited by statutory regulations to definite values depending on the number of axles (*see* Chapter XV). Hence lightness in the electrical equipment is essential, and if a two-motor equipment were used a reduction in the seating capacity of at least two passengers would be necessary in comparison with the same vehicle with a single-motor equipment.



(*a*)

(*b*)



(*c*)

FIG. 31A. TROLLEYBUS MOTORS

(*a*) Longitudinal section (Metropolitan-Vickers)
(*b*) Exploded view showing fan, end shields, frame with armature, brush-gear (Crompton-Parkinson)
(*c*) Motor with overhung generator and cyclonic dust catcher for incoming ventilating air (G E C)

rating between 100 and 140 h.p. (corresponding to a vehicle speed of about 12 m.p.h.). The motor is fixed to the chassis of the vehicle (which is spring supported from the road wheels) and the power is transmitted through a propeller shaft to a worm-driven differential gear in the back axle, this method of power transmission being similar to that adopted in internal-combustion-engined vehicles (except that, on the trolleybus, there is no change-speed gearing). The frame of the motor can therefore be made lighter than that of a corresponding axle-suspended motor, and with the moderately high armature speed, of 1,100 to 1,250 r.p.m. at the 1-hour rating, the weight per rated h.p. is of the order of 10 lb. Typical motors are shown in Fig. 31A.

Due to the rubber tyres and the quietness of the worm drive, precautions are taken to ensure quiet operation of the motor, e.g. magnetic hum is reduced to a low level and the ventilating fan is of the aphonic type. The fan is located outside the bearing housing at the driving end (in order to obtain a large diameter), and the incoming air usually enters through a filter or cyclonic dust catcher, one form of which is shown diagrammatically in Fig. 31B.

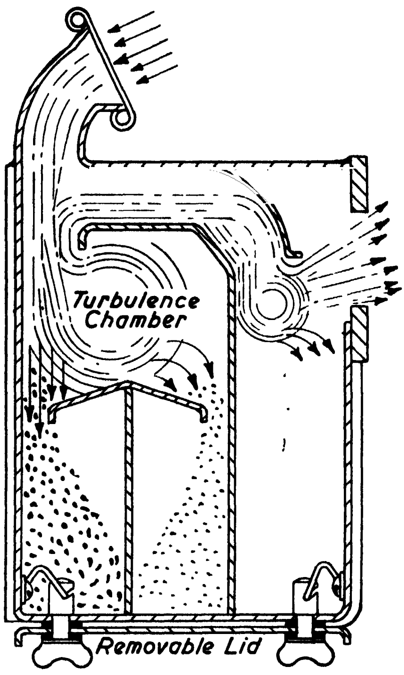


FIG. 31B. CROSS SECTION OF CYCLONIC AIR FILTER SHOWING AIR FLOW AND DUST-SETTLING COMPARTMENTS (G.E.C.)

The main field windings are usually compound, as a straight series winding may cause difficulties in rheostatic braking, because the mechanical weakness of the worm, differential gear and axle shafts imposes a limit to the braking torque. The shunt and series windings are proportioned according to the type of braking characteristic desired. For example, in a motor intended for regenerative braking the ampere-turns supplied by the shunt winding at normal voltage are greater than those of the series winding at rated current; while in a motor intended for rheostatic braking the shunt ampere-turns at normal voltage are relatively weak in comparison with the series ampere-turns at rated current (motor operation). In both cases the windings act cumulatively for motor operation and differentially for generator operation.

Typical characteristic curves are given in Fig. 32, which show the wide range of speed control, during motor operation, by field weakening, and the limitation of the braking torque during generator operation.

COMMERCIAL BATTERY VEHICLE MOTORS

Battery vehicles are usually equipped with batteries of either 30 or 36 lead-acid cells, the number of cells and their ampere-hour capacity depending on the vehicle and its nominal load.

A typical series motor suitable for a range of vehicles carrying loads from

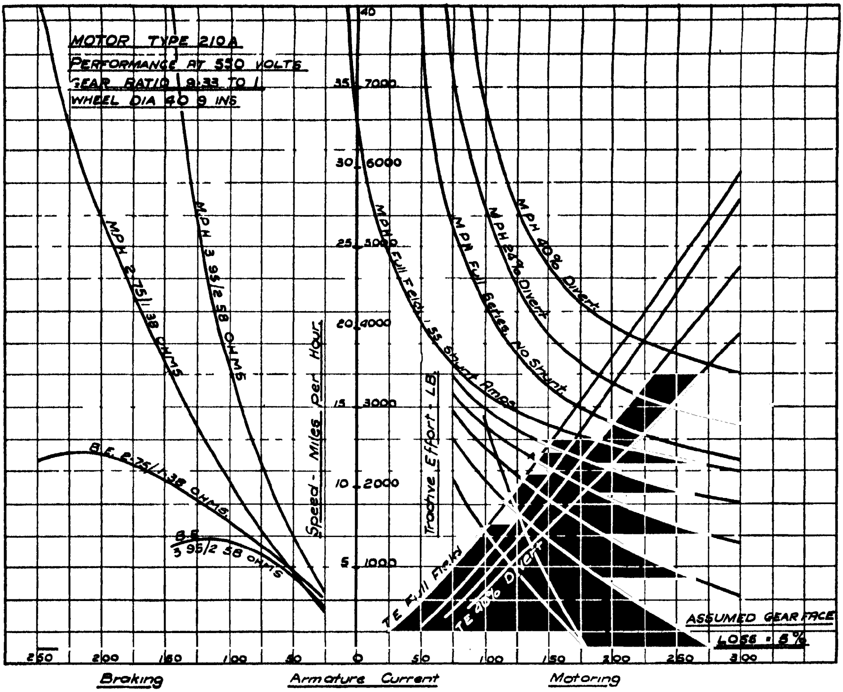


FIG. 32. CHARACTERISTIC CURVES OF COMPOUND TROLLEYBUS MOTOR (METROPOLITAN-VICKERS)

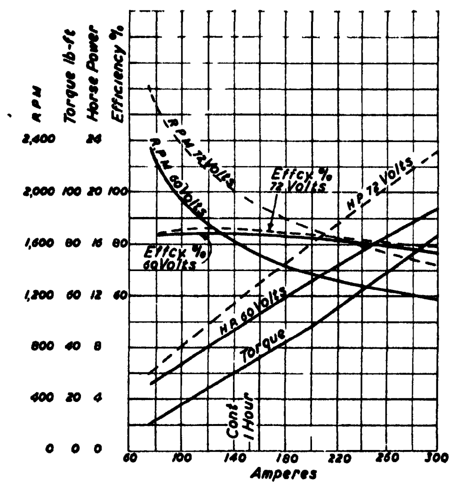


FIG. 33. CHARACTERISTIC CURVES OF B.T.-H. BATTERY-VEHICLE MOTOR

12 cwt to 2 tons has a 1-hour rating of $12\frac{1}{4}$ h.p. at 72 volts, and $10\frac{1}{4}$ h.p. at 60 volts. Characteristic curves are shown in Fig. 33.

Two such motors connected in series and supplied from a 72-cell battery are employed for vehicles carrying loads of 6 tons.

The motor is of the self-ventilated type with class B insulation. The frame and frame-heads are of fabricated steel plate, the frame being cylindrical in shape with machined facings for bolting to longitudinal members of the chassis. The drive is by cardan shaft to the back axle. When two motors are employed each armature shaft is geared to a common gear-wheel which is coupled to the cardan shaft.

CHAPTER V

SINGLE-PHASE TRACTION MOTORS

THE development of single-phase motors for electric traction has been confined exclusively to machines of the commutator type, as the single-phase induction motor is incapable of exerting a large torque at starting. Although in the early days of single-phase traction commutator motors of several types (e.g. compensated series, compensated repulsion, brush-shifting repulsion, series-repulsion) were developed, only the compensated-series type has survived. This type of motor has been adopted as standard for the extensive network of the Swiss Federal Railways and for other electrifications in Scandinavia, Austria and Germany; it is also in use in America on the Pennsylvania Railroad and the New York, New Haven and Hartford Railroad.

These applications are all of low frequency ($16\frac{2}{3}$ c/s and 25 c/s) due to factors concerned with the design and performance of the traction motors at the time when these railways were electrified. Recent development work on series motors has shown that they can be built (with limitations) to give satisfactory service at the standard industrial frequency of 50 c/s, thus enabling entirely new long-distance electrifications to be planned for this frequency throughout the whole system.

THEORY OF SINGLE-PHASE SERIES MOTOR

The simplest form of single-phase series motor consists of a laminated magnetic circuit and an armature, with commutator and brushes which closely resembles the armature of a low-voltage d.c. motor. The main excitation is supplied by a field winding which is connected in series with the armature.

If such a motor is supplied at standstill with alternating current of low frequency a pulsating torque is developed, and the relationship between the mean value of the torque and the root-mean-square value of the current is almost identical with that obtained when the motor is supplied with direct current.

The similarity between the performance with direct and alternating current follows from the fundamental principles of electromagnetic commutator machines. Thus in any electromagnetic machine the magnitude of the torque developed at any instant is directly proportional to the product of the values, at that instant, of the flux and armature current, while the direction of the torque depends solely upon the relative directions, in space, of these quantities. With series excitation, an unsaturated magnetic circuit, and an alternating supply, the flux is practically in phase with, and is proportional in magnitude to, the armature current. Hence the torque at any instant is approximately proportional to the square of the current.

RELATIONSHIP BETWEEN TORQUE AND CURRENT. The relationship between the mean value of the torque and the root-mean-square (r.m.s.) value of the current is readily deduced for the simple case when both current and flux follow sine laws. Thus if the current is given by $i = I_m \sin \omega t$ and the flux by $\Phi = \Phi_m \sin \omega t$, the instantaneous torque $\mathfrak{T}_{inst}$, is

$$\begin{aligned}\mathfrak{T}_{inst} &= k\Phi i = kI_m\Phi_m \sin^2 \omega t \\ &= \tfrac{1}{2}kI_m\Phi_m(1 - \cos 2\omega t) \\ &= \tfrac{1}{2}kI_m\Phi_m - \tfrac{1}{2}kI_m\Phi_m \cos 2\omega t\end{aligned}$$

where k is a constant.

Thus the torque is pulsating, and may be resolved into a steady component (equal to $\frac{1}{2}kI_m\Phi_m$) and an alternating component (equal to $\frac{1}{2}kI_m\Phi_m \cos 2\omega t$) having a frequency double that of the supply current. The mean value of the latter, taken over a period of the supply current, is zero.

Hence the mean torque over a period is

$$\mathfrak{T}_{mean} = \tfrac{1}{2}kI_m\Phi_m$$

In the case of a purely unsaturated magnetic circuit the flux is directly proportional to the current, and the mean torque is

$$\mathfrak{T}_{mean} = k'I^2$$

where $I (= I_m/\sqrt{2})$ is the r.m.s. value of the current.

The *function of the commutator* is the same in both single-phase and d.c. motors, i.e. to enable each motor to exert a continuous torque. Thus with a.c. operation the flux and armature current both alternate at the same frequency, and the torque exerted by any particular armature conductor when under a given pole face is always in the same direction. When this conductor moves to a position under a pole face of opposite polarity, the direction of the current in it is reversed (due to the action of the commutator and bushes), and the torque is in the same direction as before. Therefore the commutator performs the same functions whether the supply is direct or alternating, and is as necessary for one case as for the other.

Electrical Conditions at Starting. Two important conditions occur at starting, (i) the power factor is extremely low, (ii) the armature coils which are short-circuited by the brushes on the commutator will probably become overheated and vicious sparking will occur if the segments connected to these coils break contact with the brushes (due to any slight movement of the armature).

The low power factor is due to the inductances of the armature and field windings. The inductance of the former is due to the magneto-motive force produced by the current in the armature, the direction of this magneto-motive force being along the neutral, or brush, axis. Since this magneto-motive force is not essential for the production of torque in the motor it may be neutralized without affecting the relationship between torque and current.

In order to neutralize the magneto-motive force of the armature a stationary winding, producing an equal, but opposing, magneto-motive force, must be placed as closely as possible to the armature winding, and must be distributed in space in the same manner as the armature winding. Such a winding—called a neutralizing or *compensating winding*—is, therefore, always provided in commercial motors and is located in the pole faces, as shown in Fig. 42: it may be connected in series with the armature winding (i.e. conductively excited), or, alternatively, it may be short-circuited upon itself (i.e. inductively excited). In the latter case the neutralizing ampere-turns are derived by transformer action from the armature winding, but, due to inevitable losses and to the presence of an air gap between the two windings, the neutralizing ampere-turns so obtained will always be lower than the armature turns, and, therefore, a portion of the armature magneto-motive force will remain un-neutralized. The method, however, possesses the advantage that the number of turns in the winding do not require to be so carefully chosen as for the conductively-excited winding. But with present-day motors the conductively-excited winding is preferred on account of the better performance of the motor.

The overheating of certain armature coils and the vicious sparking at the brushes are due to the relatively large circulating currents which are induced by the alternating main flux in the coils short-circuited by the brushes, these coils and the main field coils acting as the secondary and primary windings of a transformer. The conditions are represented in Fig. 34, which shows the relative directions, at a given instant, of the currents and fluxes for a ring armature winding with the brushes in the neutral position. It will be observed

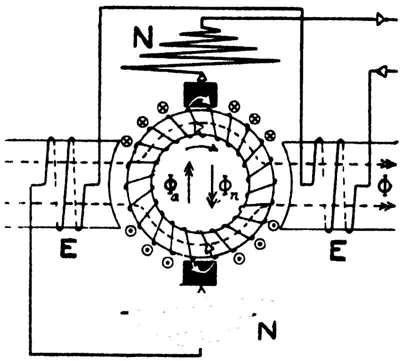


FIG. 34. DIAGRAM SHOWING DIRECTIONS OF TRANSFORMER E.M.F'S. AND CIRCULATING CURRENTS IN ARMATURE OF SINGLE-PHASE SERIES MOTOR

The fluxes which would be produced by the separate action of armature and compensating windings are denoted by Φ_a and Φ_n respectively.

that the armature coils occupying the neutral zones are traversed by the whole of the main flux, and, therefore, the coils which are short-circuited by the brushes will be the seat of relatively large circulating currents. The steps which must be taken to reduce these currents in commercial motors are considered in the discussion of commutation which is given later.

Electrical Conditions when Running. These involve the consideration of (i) the relationship between speed and torque, (ii) power factor, (iii) commutation.

The relationship between speed and torque is best obtained by deriving first the relationships between (i) torque and current, (ii) speed and current. The relationship between torque and current has already been obtained, and the relationship between speed and current is readily obtained from the fundamental principles governing the action of electromagnetic machines. Thus, in any rotating armature supplied with energy through brushes and commutator, the e.m.f. generated in the armature winding due to its rotation—this e.m.f. will be called the *dynamic e.m.f.*—together with any other internal e.m.f's. due to the current and the main flux must, at every instant, balance the external e.m.f. applied to the brushes.

With a d.c. supply only two internal e.m.f's. have to be considered, (i) the dynamic e.m.f. (usually called the "back-e.m.f." or "counter-e.m.f." in direct-current motors), and (ii) the voltage drop due to the resistance of the armature circuit. But with an alternating-current supply other e.m.f's. are present, which are due to (iii) the alternations of the main flux through the armature winding, and (iv) the alternations of the current in the armature conductors. The former are called *static e.m.f's.* and the latter *inductive e.m.f's.*

The static e.m.f's. due to the alterations of the main flux through those armature coils which form the electrical circuits between the brushes cancel out when the magnetic circuits are symmetrical (i.e. when the flux from each pole divides equally in passing through the armature core, as indicated in Fig. 34) and the axis of the brushes is perpendicular to the axis of the main flux, these conditions being represented diagrammatically in Fig. 34. But in those coils which are short-circuited by the brushes the e.m.f's. induced by the main flux are active and set up circulating currents in the closed circuits.

The inductive e.m.f's. due to the alternations of the current in those armature coils which form the circuits between the brushes are almost negligible in commercial motors which are provided with pole-face windings for neutralizing the magneto-motive force of the armature.

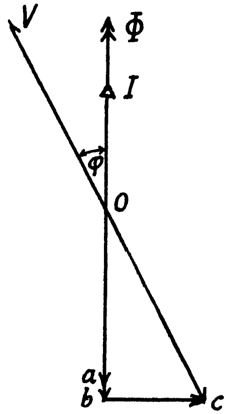


FIG. 35. VECTOR DIAGRAM FOR SERIES MOTOR

The dynamic e.m.f. is, therefore, the principal internal e.m.f. in the armature circuit (i.e. that portion of the armature winding between the brushes) of a commercial single-phase motor when the armature is rotating.

The r.m.s. value of this e.m.f. is given by the equation

$$E = (2/\sqrt{2})p\Phi_m Nn \times 10^{-8} \quad . \quad . \quad . \quad . \quad (15)$$

where p denotes the number of poles, N the number of turns per circuit of the armature winding, n the speed in revolutions per second, and Φ_m the maximum or crest value of the flux per pole, which is assumed to vary sinusoidally with respect to time.

Hence the speed in revolutions per second is given by

$$n = E \times 10^8/[(2/\sqrt{2})p\Phi_m N]$$

which is of similar form to the speed equation for a d.c. motor.

The r.m.s. value of the static e.m.f. induced in the exciting winding by the alternations of the main flux is

$$E_s = (2\pi/\sqrt{2})fN_f\Phi_m \times 10^{-8} \quad . \quad . \quad . \quad . \quad (16)$$

where f denotes the frequency of the supply current, N_f the number of turns in the exciting winding, and Φ_m has the same significance as in equation (15).

Power Factor. The conditions under which a high power factor may be obtained can be determined by a consideration of the vector diagram for the motor. For the simplest case of a motor in which the armature reaction is neutralized, and only resistance losses in the armature and field circuits are present, the vector diagram is shown in Fig. 35. In this diagram the flux is represented by $O\Phi$ and is the vector of reference, the current—which is in phase with the flux—by OI, and the terminal voltage—which leads the current by the angle ϕ—by OV. The terminal voltage balances the vector sum, Oc, of the internal e.m.f's., of which the dynamic e.m.f. is represented by Oa, the voltage drop due to the total resistance of the motor by ab, and the inductive e.m.f. induced in the field (excitation) winding by bc.

Hence for a high power factor (i.e. for ϕ to be a small angle) the inductive e.m.f. must be small in comparison with the dynamic e.m.f.

If resistance losses are ignored we have

$$\tan\phi = \frac{\text{inductive e.m.f.}}{\text{dynamic e.m.f.}}$$

$$= \frac{(2\pi/\sqrt{2})fN_f\Phi_m \times 10^{-8}}{(2/\sqrt{2})pnN\Phi_m \times 10^{-8}}$$

Whence

$$\text{Power factor} = \cos\phi = (1 + \tan^2\phi)^{-\frac{1}{2}}$$

$$= \sqrt{\frac{1}{1 + [\frac{1}{2}\pi(n_s/n)(N_f/N)]^2}} \quad . \quad . \quad . \quad . \quad (17)$$

where n_s is the synchronous speed ($= f/\frac{1}{2}p$ rev. per sec), f the frequency of supply, p the number of poles, n the speed of rotation of the armature, N_f the *total* number of turns in the exciting winding, N the number of turns per circuit of the armature (i.e. the number of turns in series between the brushes), Φ_m the crest value of the flux per pole, the distribution of which is assumed to be sinusoidal.

Hence for a given motor the power factor at the highest speed (i.e. at light load) will approach unity and will decrease as the speed decreases.

Transformer e.m.f. The commutating conditions in the simple series motor are considerably more complicated than those in a direct-current motor, as, in addition to the e.m.f's. induced in the commutated coils by the reversal of the armature current, there is a static e.m.f.—called the "transformer e.m.f."—which is induced in these coils by the alternations of the main flux. This e.m.f., for a coil of N_c turns, is given by

$$E_t = (2\pi/\sqrt{2})N_c f\Phi_m \times 10^{-8} \quad . \quad . \quad . \quad (16a)$$

Hence, as N_c is unity in traction motors, E_t is proportional to $\Phi_m f$.

Effects Caused by Circulating Currents. Since the magnitude of the transformer e.m.f. in a given motor is proportional to the main flux its largest values occur at heavy loads (i.e. at high current inputs), so that under these conditions severe sparking is liable to occur, as the coils undergoing commutation have the circulating currents due to the transformer e.m.f. superimposed upon the current which is being commutated.

The circulating currents, however, produce other effects which are detrimental to the performance of the motor. Thus, since the coils occupying the neutral zone are in the position of maximum mutual inductance with respect to the exciting winding, and the circulating currents are produced by induction, the ampere-turns set up by the short-circuited coils act in opposition to the ampere-turns producing the main flux (*see* Fig. 34). This reaction results in a weakening of the main flux as well as a phase displacement between this flux and the armature current, thereby reducing the torque. These effects are shown in the vector diagram of Fig. 36, in which OY represents the main flux, OB the ampere-turns producing this flux, and OF the transformer e.m.f. (which is in time quadrature with the flux). If we disregard any other e.m.f's. that may be induced in these coils, then the circulating currents produced by the transformer e.m.f. can be represented by OG, in phase with OF. The reaction ampere-turns due to the circulating currents, are represented by OC, and act along the axis

of the field winding. Therefore, OB represents the resultant of the field ampere-turns and the reaction ampere-turns. Hence the field ampere-turns are represented by OA, and the main current by OI, which leads the flux by the angle θ.

Therefore the circulating currents are not only detrimental to commutation, but they affect adversely the output and efficiency. They also lead to increased losses and heating, and necessitate more ampere-turns to be supplied by the main field winding than are required for magnetic purposes.

Reduction of Circulating Currents. When the armature is rotating the statically-induced transformer e.m.f. in the coils undergoing commutation may be neutralized by an opposing dynamically-induced e.m.f. (called the "commutating e.m.f.") if a commutating flux of the correct density and time-phase is provided in the neutral zone. Under these conditions there would be no circulating currents.

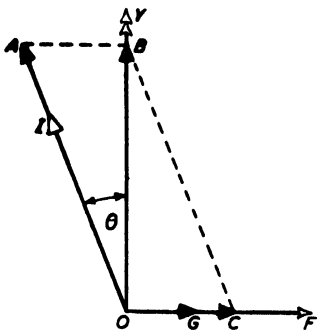


FIG. 36. EFFECTS DUE TO CIRCULATING CURRENTS

If complete neutralization of the transformer e.m.f. is to be obtained over a range of speeds the magnitude of the commutating flux must be proportional to the quotient (main flux/speed). This condition follows from the fact that the magnitude of the transformer e.m.f. is proportional to the main flux, and the commutating e.m.f. is proportional to the product of armature speed and commutating flux. But the speed is approximately proportional to the quotient (terminal voltage/main flux). Hence, the commutating flux would have to be proportional to the quotient [(main flux)^2/terminal voltage], a condition which would be difficult to satisfy in practice.

In a commercial motor it is only practicable to neutralize the transformer e.m.f. over a limited range of loads. But, what is of greater importance, this voltage cannot be neutralized to the slightest degree at starting.

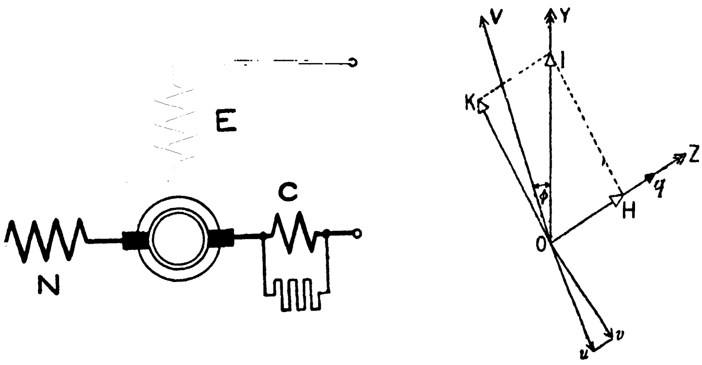
Circulating Currents at Starting. The magnitude of the circulating currents at starting is proportional to transformer e.m.f. per coil $\times$ number of coils short-circuited by a brush $:$ resistance of path of circulating currents. This path includes the contact surface of the brushes, the appropriate armature coils and their connexions to the commutator segments.

Hence for a given transformer e.m.f. per coil the circulating currents can be limited by the use of narrow brushes of high contact resistance, together with, if necessary, high-resistance connexions, or "resistive leads," between the coils and commutator segments. The latter method has limited applications in practice and is not employed in modern $16\frac{2}{3}$-c/s European motors. High-resistance connexions, however, are employed in some recently-developed 50-c/s motors, and also in some American 25-c/s motors for motor-coach trains.

Reactance Voltage of Commutation. In addition to the transformer e.m.f. the coils undergoing commutation have induced in them an e.m.f. (called the "reactance voltage of commutation") due to the change in the flux linked with the slot portions and end connexions of these coils when the current is reversed or commutated. This flux, in the case of a particular coil is directly proportional

to the current in that coil, and therefore the induced e.m.f. is proportional to change of current during commutation/time of commutation. When the time of commutation—which is constant at constant speed—is small in comparison with the time of one cycle of the supply current, the current in a particular coil during commutation changes from, say, $+ i$ to $- i$, where i is the current in the armature winding at the particular instant at which commutation commences, and, for an a.c. commutator motor, may have any value between zero and the crest value of the armature current for the particular operating conditions. Therefore the induced e.m.f. is proportional to i, and the "reactance voltage" is proportional to, and in phase with, the supply current.

Hence the reactance voltage and the transformer e.m.f. have a phase difference of 90°.



(a) (b)

FIG. 37. CIRCUIT AND VECTOR DIAGRAMS FOR SHUNTED COMMUTATING-POLE WINDING

Commutating Poles. Although these poles are ineffective in reducing sparking at starting and only partially effective at low armature speeds, their use enables the circulating currents to be reduced to low values over a range of speeds and loads, so that good commutation is obtained for normal running conditions. Other advantages are higher efficiency and a greater output from a given size of armature core.

For ideal commutation (i.e. no circulating currents) the flux density at the face of the commutating pole must be such that the e.m.f. generated in the armature conductors by their motion through this flux balances, in both magnitude and phase, the vector sum of the transformer e.m.f. and the reactance voltage. To satisfy this condition the phase of the commutating flux must be displaced from the main flux by about 30° to 50° (lagging).

The ampere-turns to be supplied by the commutating-pole winding must balance any un-neutralized armature ampere-turns and provide the requisite magneto-motive force for passing the commutating flux through the pole, air gap, and armature teeth.

Excitation of Commutating Poles. The simplest method of obtaining the phase displacement of the commutating flux is to provide a series winding and shunt this with a non-inductive resistor. The circuit and vector diagrams are shown in Fig. 37.

In the vector diagram (which represents the conditions for ideal commutation) the magnitude and phase of the ampere-turns required to produce the commutating flux *OZ* is represented by *Oq*, while *OY* represents the main flux and *OI* the main current. If the armature ampere-turns are completely compensated, then the ampere-turns on the commutating pole must be equal to *Oq*. The current in the commutating-pole winding must be in phase with *Oq*; and if *OH* represents the current required, then *OK*—the vector difference of *OI* and *OH*—represents the current in the shunt.

The voltage drop *Ou* in the commutating-pole circuit is obtained by compounding *Ov* (which represents the e.m.f. induced in the commutating-pole

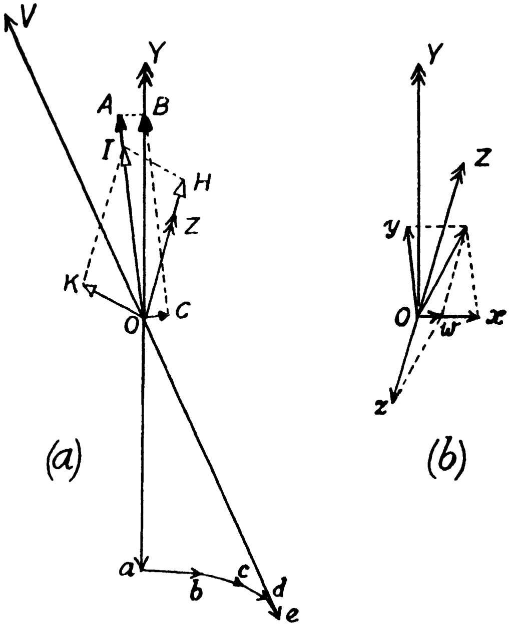


FIG. 38. VECTOR DIAGRAM FOR COMMERCIAL SERIES MOTOR

winding) with *vu* (which represents the internal voltage drop due to the resistance of the winding). The resistance value of the shunt is then obtained by dividing *Ou* by the shunt current *OK*.

This method has given very satisfactory results in practice. The loss in the shunt-resistor at rated load is about 1 per cent of the output for modern low-frequency ($16\frac{2}{3}$ c/s) motors, but may be from 4 to 9 times this amount for industrial-frequency (50 c/s) motors.

Vector Diagram for a Commercial Series Motor. The vector diagram must include the effects of losses, magnetic saturation, magnetic leakage, and circulating currents, all of which vary with the load. Hence the diagram can be drawn to represent only one operating condition. The process, however, involves trial and error as the magnitudes and phases of some quantities have to be assumed.

When the terminal voltage corresponding to a particular value of flux and

speed (or dynamic e.m.f.) is to be determined the diagram is constructed as *follows.*

A value is assumed for the main flux (*OY*, Fig. 38*a*). The saturation ampere-turns required for this flux are represented by *OB*. The reaction ampere-turns *OC* (due to circulating currents) are assumed—in magnitude and phase—in order that the field ampere-turns *OA* and the main current *OI* may be determined.

The magnitude and phase of the current in the commutating-pole winding is now determined from data of the winding and the shunting resistor. Thus, in Fig. 38*a*, *OK* represents the current in the shunt resistor, and *OH* the current in the commutating-pole winding. The commutating flux is, therefore, represented by *OZ*, which is in phase with *OH*.*

Next the magnitude and phase of the reaction ampere-turns, *OC*, must be checked. The voltage producing the circulating currents is the resultant of (i) the transformer e.m.f., (ii) the reactance voltage, and (iii) the commutating e.m.f. These e.m.f's. can all be calculated and are represented, in Fig. 38*b*, by *Ox* (transformer e.m.f.), *Oy* (reactance voltage), and *Oz* (commutating e.m.f.). Hence *Ow* is the resultant voltage producing the circulating currents, and these currents may be considered to be in phase with *Ow*. The reaction ampere-turns, *OC*, Fig. 38*a*, due to the circulating currents are, therefore, in phase with *Ow*, and their magnitude can be determined from the magnitude of *Ow* and data of the armature winding.

The vector diagram is completed by determining the terminal voltage of the motor, which is obtained from the vector polygon of internal e.m.f's. in the main circuit of the motor. Thus, in Fig. 38*a*, *Oa* represents the dynamic e.m.f. generated in the armature by its rotation in the main flux; *ab*, *bc*, *cd*, *de*, the e.m.f's. due to the impedances of the exciting, neutralizing, commutating-pole and armature windings taken in order. *Oe* is therefore the resultant internal e.m.f. and is balanced by the terminal voltage *OV*.

General Remarks on Single-phase Series Motors

In comparison with d.c. traction motors, single-phase series motors have a large number of poles, low flux per pole, small air gaps and moderate flux densities in the magnetic circuits. Hence relatively few exciting ampere-turns per pole are required and only a small radial depth of iron is necessary in the stator core. Pole-face, or compensating, windings for neutralizing armature reaction, and commutating poles are necessary to obtain good performance.

Flux per Pole. Considerations of the permissible sparking at starting, due to the commutation of the circulating currents, limit the transformer e.m.f. in the armature coils to about 3 volts per coil, assuming a simple lap winding with single-turn coils and no high-resistance connexions. Inserting this value in equation (16*a*), we obtain $\Phi_m = 67{\cdot}6/f$ megalines per pole, where f is the supply frequency. Hence for a frequency of $16\frac{2}{3}$ c/s the flux is limited to about 4 megalines per pole, but for a frequency of 50 c/s the limit is 1·35† megalines under the assumed conditions.

* If any armature ampere-turns are un-neutralized by the compensating winding they must be balanced by a component of the commutating-pole ampere-turns.

† A higher flux under running conditions may be employed if the field winding is shunted at starting and at low speeds (e.g. up to about 6 m.p.h.).

Number of Poles. With the limited flux per pole a large number of poles is necessary to obtain sufficient total flux for the requisite torque (since torque is proportional to the product of total flux and total armature ampere-conductors). Low-frequency motors for motor-coaches have either 6 or 8 poles, and modern locomotive motors have from 10 to 14 poles. Industrial-frequency locomotive motors of 700 h.p., with duplex armature winding, have 16 poles.

Air Gap. A small air gap (2 to 3 mm) and moderate flux densities are necessary from considerations of power factor.

Power Factor Considerations. We have already seen that for a high power factor the inductive e.m.f. of the motor must be low in comparison with the dynamic e.m.f. A magnetic circuit of low reluctance, together with a compensating winding to neutralize armature reaction, satisfies the condition for a low inductive e.m.f. To obtain a low ratio for the inductive and dynamic e.m.f's. equation (17) shows that (i) the speed of rotation of the armature must be high in relation to the so-called "synchronous speed," and (ii) the number of turns per circuit in the armature winding should be large in comparison with those in the exciting (main field) winding.

These conditions necessitate a high speed of rotation, a low frequency, a relatively large number of poles and the maximum practicable number of armature conductors (which in the present case is governed solely by heating considerations).

In modern low-frequency motors the speed at rated load is from seven to eight times the "synchronous" speed, and the number of turns per circuit of the armature winding is from two to three times the number of turns in the field winding. Hence, theoretically, the power factor at rated load, as given by equation (17), is

$$\cos\phi = [1 + (\tfrac{1}{2}\pi \times \tfrac{1}{8} \times \tfrac{1}{2})^2]^{-\frac{1}{2}} = 0{\cdot}994$$

but in practice—due to the leakage reactances of the field and armature windings—the power factor would be about 0·96 to 0·97.

The low reluctance of the magnetic circuit and the neutralization of armature reaction result in relatively few ampere-turns per pole for the field (exciting) winding. In consequence the ratio (field ampere-turns per pole/armature ampere-turns per pole) is of the order of 0·4 to 0·6, whereas for a d.c. traction motor it is of the order of 1·5 to 2.

Thus considerations of power factor and commutation require the single-phase series motor to be designed on radically different lines to the d.c. traction motor.

Operating Voltage. The limitations imposed by the commutating conditions at starting have a direct influence on the operating voltage, which is restricted to low values, due to the low flux per pole and the limited number of commutator segments which can be accommodated between the closely-spaced brushes.

Thus for a motor with p poles, a simple lap winding with N turns per circuit and single-turn coils, the number of commutator segments is equal to pN. If τ_c is the pitch of the segments in inches and v_c is the peripheral speed of the commutator (ft/min) corresponding to the speed n (r.p.s.) at rated load, we have $pNn = 12v_c/60\tau_c$.

Making this substitution for pNn in the equation for the dynamic e.m.f. in the armature, we have

$$\begin{aligned} E &= (2/\sqrt{2})\Phi_m pNn \times 10^{-8} \\ &= (2/5\sqrt{2})\Phi_m(v_c/\tau_c) \times 10^{-8} \end{aligned}$$

Now Φ_m is limited to a definite value as already shown, and if v_c is chosen at 6,000 ft/min (30 metres per sec) at rated load, τ_c at 0·18 in. (4·6 mm), the dynamic e.m.f. for $16\frac{2}{3}$ c/s (for which, with $E_t = 3V$, $\Phi_m = 4 \times 10^6$) is

$$(2/5\sqrt{2}) \times 4 \times 10^6 \times (6{,}000/0{\cdot}18) \times 10^{-8} = 380 \text{ V}$$

The corresponding terminal voltage of the motor would be about 410 V, which is approximately the operating voltage of locomotive motors for this frequency.

For the industrial frequency of 50 c/s and the same values for E_t, v_c, τ_c, the dynamic e.m.f. would be 127 V. But higher values (180 to 200 V) are possible if a duplex armature winding is employed or if the limits of E_t and v_c are increased and τ_c is reduced to 0·15 in. (3·8 mm).

Output per Pole. If losses are ignored the output (P watts) is given by

$$P = EI = p\Phi_m NIn \times (2/\sqrt{2}) \times 10^{-8}$$

whence
$$P/p = \Phi_m NIn \times (2/\sqrt{2}) \times 10^{-8}$$

Now NI is equal to the total armature ampere-turns, and may be expressed in terms of the armature diameter (D) and the specific electric loading (Q)—i.e. ampere conductors per unit length (cm or in.) of armature periphery—thus $NI = \frac{1}{2}\pi DQ$. Also Φ_m can be expressed in terms of the transformer e.m.f. frequency, and other constants of the armature, as shown on p. 62. Effecting these substitutions, we have

$$\frac{P}{p} = \left(\frac{\sqrt{2}}{2\pi} \cdot \frac{E_t}{fN_c \times 10^{-8}}\right) \tfrac{1}{2}\pi DQn \times \frac{2}{\sqrt{2}} \times 10^{-8}$$

When the number of turns per armature coil (N_c) is unity

$$P/p = \tfrac{1}{2}E_t DQn/f$$

But if v denotes the peripheral speed of the armature in feet per minute $v = 60n\pi D/12$, when the diameter of the armature is expressed in inches, so that $nD = v/5\pi$. Hence the output equation becomes

$$P/p = vQE_t/31{\cdot}4f \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Q being expressed in ampere-conductors per inch of armature periphery.

Now Q and v have definite limiting values, the former being governed by considerations of heating and the latter by mechanical considerations.* Hence, with limiting values for v and Q, the output per pole is directly proportional to the transformer e.m.f. and inversely proportional to the frequency.

The permissible value of E_t is governed by the magnitude of the circulating currents allowable under starting conditions. For a given frequency the output is, therefore, restricted by the transformer e.m.f.

The importance of a low frequency is again emphasized by equation (18). The adoption of a specially-low frequency—such as $16\frac{2}{3}$ c/s—instead of 25 c/s, therefore, results in improved power factor, higher efficiency, improved commutation, better starting performance, and fewer poles for a given output.

* The peripheral speed at rated load may be of the order of from 7,000 to 8,000 ft per minute, the limiting peripheral speed at light loads being of the order of 13,000 ft per minute. The limiting value of Q for forced ventilation is of the order of 500 to 600 ampere-conductors per cm (1,250 to 1,500 ampere-conductors per in.).

These advantages have been recognized fully by Continental manufacturers of single-phase traction equipment, and the standardization of a frequency of $16\frac{2}{3}$* c/s, instead of 25 c/s, for single-phase railways in Europe is the principal reason why single-phase traction has made such-progress there.

Losses and Efficiency. The low operating voltage results in a relatively large current at the rated load of the motor, e.g. for a 600 h.p. motor the full-load current is approximately 1,250 A at $16\frac{2}{3}$ c/s and 2,800 A at 50 c/s. These currents necessitate a large number of brushes on the commutator and relatively high losses at the surface of the latter, e.g. about 7·5 kW at the low frequency and 18 kW at the higher frequency.

The I^2R losses in the armature and stator windings will be comparable to, or slightly greater than, those in the armature and field windings of the corresponding d.c. motor.

The core loss will be slightly higher than that for the corresponding d.c. motor due to the loss in the stator core and the higher frequency of magnetic reversals in the armature core.

The windage and ventilation losses will also exceed those for the corresponding d.c. motor, and in addition there is the loss in the shunt resistor for commutating-pole winding.

The full-load efficiency of a low-frequency motor will therefore be somewhat lower than that of a d.c. motor of similar output, and the difference will become greater as the frequency increases. Typical values for a 600-h.p. locomotive motor (excluding gear losses) are 90·5 per cent for $16\frac{2}{3}$ c/s, 85 to 87 per cent for 50 c/s and 94 per cent for a d.c. motor.

Loss in Shunt Resistor. This loss (P_R) is equal to the product of the voltage (E_{cp}) at the terminals of the commutating pole winding and the current (I_s) in the shunt. If ϕ_c is the phase difference between the main current (I) and the current in the commutating-pole winding, $I_s = I \sin\phi$. Also $E_{cp} = (2\pi/\sqrt{2})\, \Phi_c N_{cp} f \times 10^{-8}$, where Φ_c is the commutating-pole flux and N_{cp} the number of turns on each commutating-pole.

Expressing the loss as a fraction of the output ($P \simeq EI$) we have

$$\frac{P_R}{P} = \frac{E_{cp} I \sin\phi}{EI} = \frac{(2\pi/\sqrt{2}) p \Phi_c N_{cp} f \times 10^{-8} \sin\phi_c}{(2/\sqrt{2}) p \Phi_m N n \times 10^{-8}}$$

$$= \pi \frac{\Phi_c N_{cp} f}{\Phi_m N n} \sin\phi_c$$

Now $N_{cp} = F_{cp}/I \cos\phi_c$, where F_{cp} denotes the ampere-turns due to the current in the commutating-pole winding, and $\Phi_m = 10^{8}(\sqrt{2}/2\pi)E_t/f$, for single-turn coils. Also $INn = \frac{1}{2}Q\pi Dn = \frac{1}{2}Qv_a$, where Q is the specific electric loading (ampere-conductors per unit length of armature periphery), D, the diameter of the armature, and v_a, the peripheral speed. Whence

$$\frac{P_R}{P} = \frac{4\pi^2}{10^8\sqrt{2}} f^2 \frac{\Phi_c F_{cp}}{v_a Q E_t} \tan\phi_c$$

i.e. the percentage loss in the shunt resistor is proportional to the square of the frequency if the other factors are constant.

* The frequency of $16\frac{2}{3}$ c/s has been chosen in order to give a convenient ratio for the number of poles in frequency changers (which must be employed in the sub-stations when the traction supply is taken from an industrial supply system of standard frequency, i.e. 50 c/s).

Comparison of Low-frequency and Industrial-frequency Motors. The present day *low-frequency* European motor is the outcome of a lengthy period of development and progress, and its successful application to powerful locomotives for service on the Gotthard, Simplon and Lötschberg routes in Switzerland—with their long and heavy (2·7 per cent) gradients—is due to the adoption of the low frequency of $16\frac{2}{3}$ c/s. Outputs per pole of 55 to 60 kW are obtainable without excessively high peripheral speeds for the commutator, and motors of 1,000 h.p. can be built with a single armature. Maintenance of the commutator has progressively decreased due to improvements in construction, better ventilation and the choice of a suitable brush having high contact resistance and good polishing qualities.

The *industrial-frequency* motor is of recent development and its present success is due to the application of the duplex armature winding with equipotential connexions, Fig. 46, which has enabled the flux per pole to be twice that possible with a simple lap winding, so that outputs per pole of 30 to 35 kW are practicable. This winding also enables the permissible thickness of the brushes to be increased and the length of the commutator to be proportionally reduced.

Alternatively a single lap winding with high resistance connexions between the coils and commutator segments is employed, and although the output per pole is of the order of 15 kW, this is, to some extent, compensated by the higher operating voltage (up to 240 V) which is possible under these conditions.

The development of split or sandwich brushes—in which two brushes, each of half the thickness of an ordinary (solid) brush, are cemented together with an insulating cement—has enabled the commutation at starting and at high speeds to be improved, due to a reduction in the magnitude of the circulating currents, as the path includes the double length of the brush as well as the increased contact resistance. In consequence the wear of the commutator caused by sparking has been reduced as well as that of the brushes due to mechanical and electrical causes. Estimations of the average radial wear of the brushes, based on service experience, are about 0·3 mm per 1,000 km (0·019 in. per 1,000 miles), which is about twice that for $16\frac{2}{3}$ c/s motors and about three to four times that for d.c. motors.

The industrial-frequency motor, however, has to operate at lower flux densities than a $16\frac{2}{3}$ c/s motor. Hence its speed/torque characteristic is steeper than that of a low-frequency motor*: in consequence fewer voltage-steps will be necessary during starting, and the division of load between parallel-connected motors will be less affected by wear of the wheels.

The higher frequency also results in higher leakage reactances and a relatively low power factor (about 0·89 at rated load).

The weight per h.p. is greater for the higher frequency because of the larger dimensions of the motor, but as a transformer forms an essential part of the equipment for both frequencies, the overall weight of motors and transformer is approximately the same in both cases.

Operation of Single-phase Motors on d.c. Circuits. As the a.c. series motor is identical in principle with the d.c. series motor, it follows that the former

* Typical percentage values of speed and torque are

Torque (% of 1-hour rating) . . .	50	75	100	125	150
Speed, $16\frac{2}{3}$ c/s (% of 1-hour rating) . .	135	113	100	93	88
Speed, 50 c/s (% of 1-hour rating) . .	150	120	100	88	78

type of motor is capable of operating on d.c. circuits. The neutralizing (or compensating) winding, however, must be excited conductively from the main circuit, as this winding is essential to satisfactory operation with direct-current (on account of the high armature ampere-turns).

The only example of dual operation of single-phase motors is in America, where certain trains of the New Haven Railroad have to operate over the 650-V d.c. lines of the New York Central Railroad.

Constructional Details

Stator. The slots for the compensating winding are of the partially-closed type, but those for the exciting and commutating-pole windings are of the

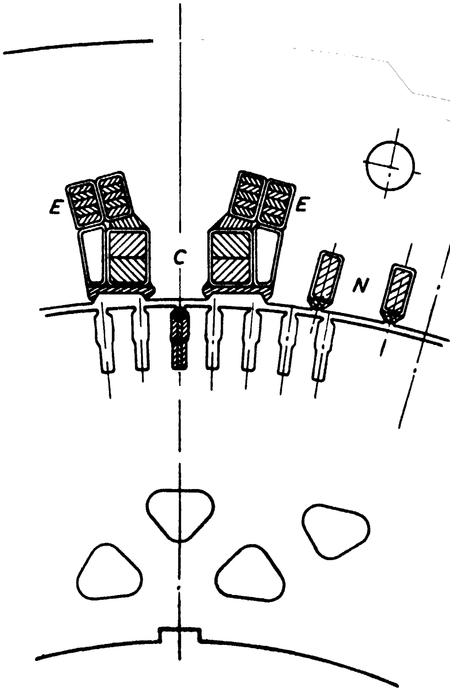
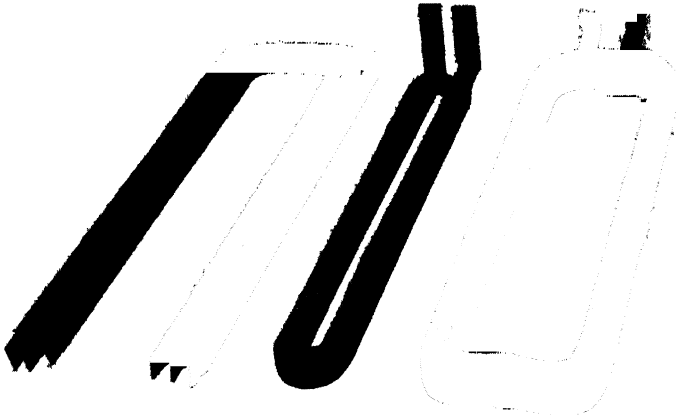


Fig. 39. Stator and Rotor Slots with Conductors in Position (Oerlikon)

C, commutating-pole, *E*, slots for exciting winding, *N*, slots for compensating winding.

open type, to reduce magnetic leakage and to enable former-wound coils to be used. Typical slots are shown in Fig. 39. The tapered form of the commutating pole enables the coils of both the exciting and commutating-pole windings to be inserted through the slot openings; it is also advantageous in preventing magnetic saturation at the base of the pole.

Typical stator coils are illustrated in Fig. 40. The coils of the compensating winding are of the concentric hair-pin type; the straight portions are pushed through the slots and the projecting conductors are then bent to shape and connected to give the desired polarities as shown in the development diagram of Fig. 41. Wound stators of $16\frac{2}{3}$ and 50 c/s motors are shown in Fig. 42.



(a) (b) (c)

FIG 40 COILS OF COMPENSATING (a), COMMUTATING POLE (b) AND EXCITING (c) WINDINGS (OERLIKON)

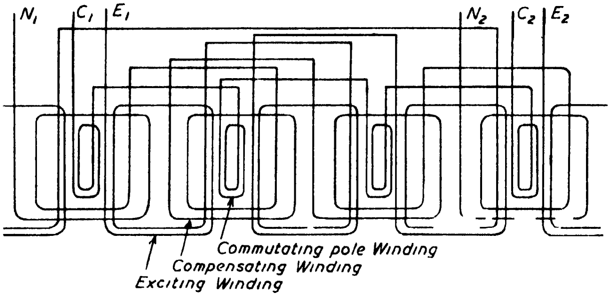
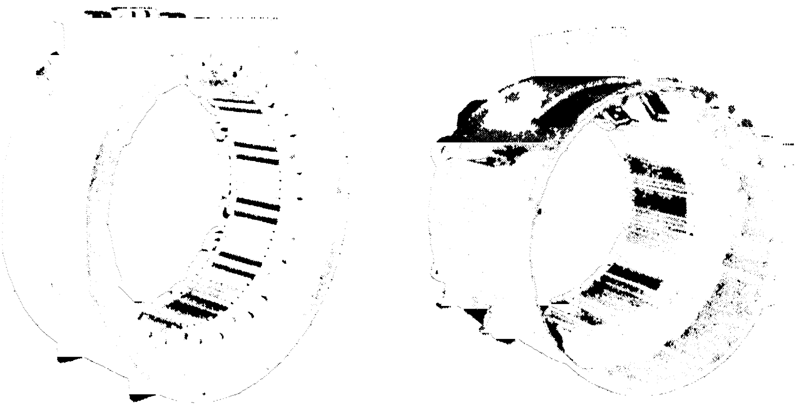


FIG 41 DEVELOPMENT OF STATOR WINDING (4 POLES)



(a) (b)

FIG 42 WOUND STATORS OF OERLIKON MOTORS

(a) 960 h p, 16⅔ c/s (b) 720 h p 50 c/s

A series-parallel arrangement of the coils of the stator winding may be necessary in large motors to obtain the required ampere-turns.

The frame is usually fabricated from steel plate and rolled sections to obtain lightness and strength. Fig. 43 shows an exterior view, and Fig. 44 shows sectional views, of locomotive motors.

Armature. The armatures of European motors have partially-closed slots to obtain a low reluctance for the air gap. In some motors the slots are stepped to obtain approximately uniform flux density throughout the teeth. The

Fig. 43. Brown-Boveri 1,000-h.p., $16\frac{2}{3}$ c/s Motor

One of the cross-arms of the cardan-shaft disc drive (*see* Fig 210) is shown coupled to the hollow armature shaft.

slots are normally skewed by the amount of one slot pitch to prevent flux pulsations due to the motion of the slots and teeth.

The armature winding of low-frequency motors is of the simple lap type with equalizing or equipotential connexions at every coil or alternate coil. These connexions (Fig. 45) provide low resistance paths between points which should normally be at equal potentials. Hence if there are any inequalities in the e.m.f's. of the several circuits, due to unbalanced magnetic circuits, the resulting circulating currents are confined to those coils in which the unbalance occurs.

If such a winding were employed for industrial-frequency motors the output per pole would be severely restricted as already shown. A duplex winding (i.e. two singly re-entrant windings with a single commutator, the segments of which are connected alternately to the coils of each winding)

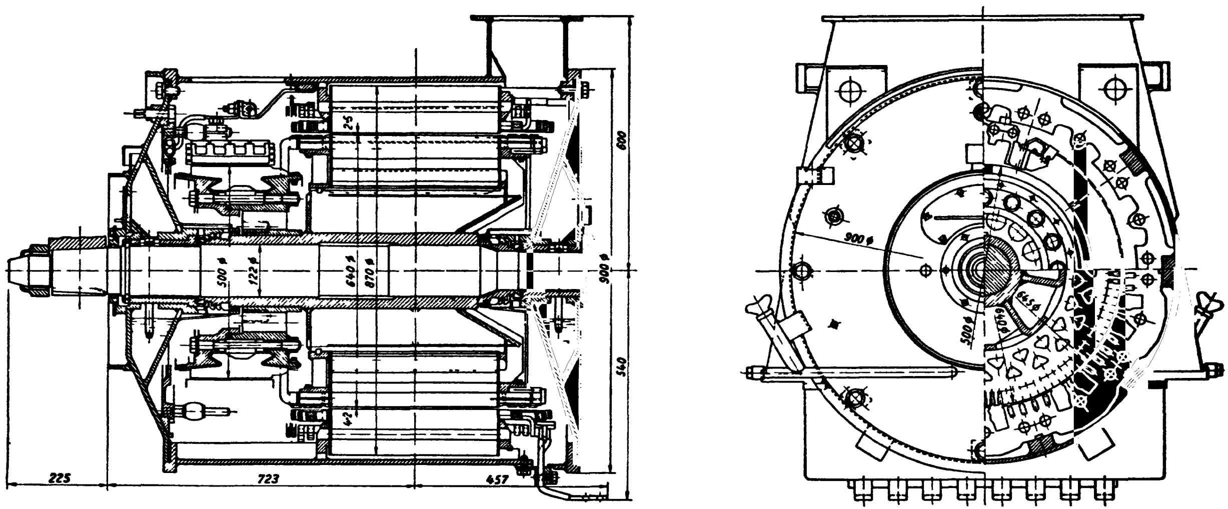


FIG 44 LONGITUDINAL AND CROSS SECTIONAL VIEWS OF OERLIKON 750 H P., $16\frac{2}{3}$ C S MOTOR

would enable higher outputs per pole to be obtained for the same circulating currents because the turns of each winding are commutated separately.

If, however, the equipotential points of *each* winding are interconnected in

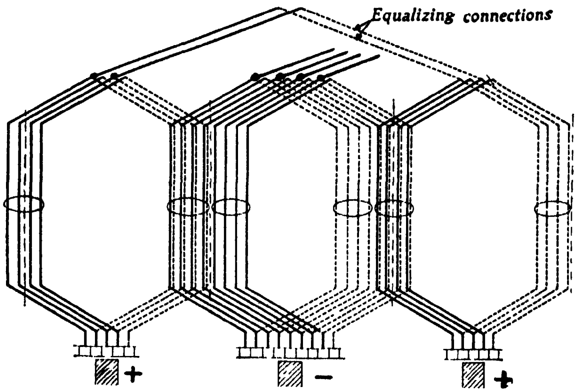


FIG. 45. EQUALIZING CONNEXIONS APPLIED TO LAP WINDING IN WHICH EVERY ALTERNATE COIL IS EQUALIZED

the manner shown in Fig. 46 (the interconnexions being taken through the air spaces between the spider and the core) the commutating conditions become equivalent to those of a half-turn winding. Hence the transformer e.m.f. per coil may be twice that of a simple lap winding, and therefore the flux and the

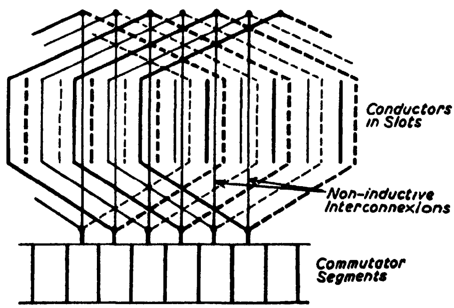


FIG. 46. DEVELOPMENT OF DUPLEX LAP ARMATURE WINDING WITH EQUIPOTENTIAL INTERCONNEXIONS

output per pole may be doubled. This type of winding is used in Oerlikon motors.

Commutator. Although the V-ring type of commutator has been successfully applied to both low-frequency and industrial-frequency motors, the Pollock type, recently developed by the B.T.-H. Co.* is better suited to withstand the operating conditions in these motors on account of its freedom from distortion with varying temperatures, and the higher peripheral speeds which are practicable.

* See *B.T.-H. Activities*, Vol. 19, p. 301.

The construction is shown in Fig. 47. The segments A with their troughs of insulation are retained by steel bars B, which extend almost the whole length of the segments. These bars are welded to steel rings, or diaphragms, C, certain

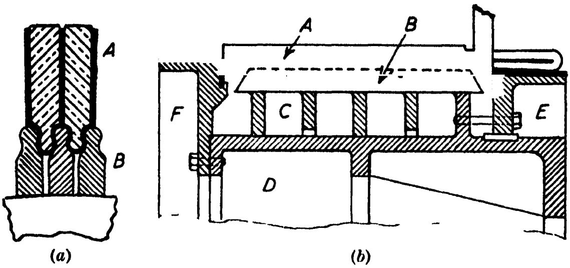


FIG. 47. CONSTRUCTION OF POLLOCK TYPE OF COMMUTATOR (B.T.H.)
(a) Detail of segments and retaining bar, (b) Longitudinal section

of which are a light shrink fit on the shell D. The interior of the commutator is sealed by the insulated end diaphragms, E, F, which allow for expansion of

FIG. 48. BRUSHGEAR FOR 720-H.P., 50-C/S MOTOR (OERLIKON)

the segments and provide creepage distances at the ends. The back flange may also support the equalizing connexions.

Brushgear. On account of the large number of poles and the close spacing of the brushes around the commutator, a compact design of brush-holder is essential. Typical brushgear is shown in Fig. 48.

American Motors. The standard frequency for single-phase railway electrification in America is 25 c/s. The earliest motors (Westinghouse) were of the compensated non-commutating-pole type with resistive connexions in the armature. Later motors (Westinghouse and General Electric) followed European practice, but, due to the higher frequency, suffered from the disadvantages of a larger number of poles (16 for a 625-h.p. motor) and a lower operating voltage (230 V).

Recent motors for motor-coach service* show divergent views on development. In one case a 4-motor equipment is employed, with self-ventilated, frame-mounted motors, each rated at 125 h.p., 190 V, and having 6 poles, a compensating (pole-face) winding, a shunted commutating-pole winding and an armature winding without high-resistance connexions.

In the other case a two-motor equipment, with forced ventilation, is employed. Each motor is rated at 225 h.p., 328 V, and has 8 poles, a shunted

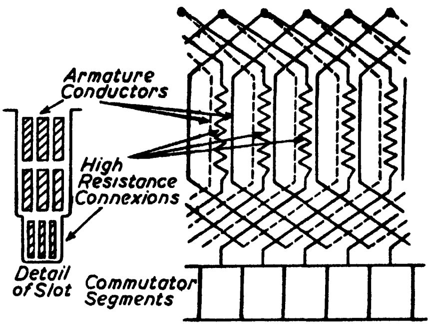


FIG. 49. DEVELOPMENT OF ARMATURE WINDING WITH HIGH-RESISTANCE CONNEXIONS BETWEEN COILS AND COMMUTATOR SEGMENTS

commutating-pole winding but no pole-face winding. Resistive connexions are employed in the armature, and this feature enables the flux per pole and the operating voltage to have higher values than would otherwise be practicable.

The resistive connexions, of high-resistance alloy, occupy the bottom of the slots in the armature core, and connect the commutator segments to the junctions of the coils at the back end of the armature, as indicated in Fig. 49. Although such connexions increase the heating and cost of the armature, and are a possible source of trouble if overheating occurs, their employment in the present case is considered to be justified by the higher flux per pole.

The omission of the compensating winding simplifies construction and maintenance, but requires additional ampere-turns in the commutating-pole winding and results in a lower power factor.

CHARACTERISTIC CURVES

As a transformer forms an essential part of all single-phase motor equipments, a number of operating voltages for the motors can be readily obtained by providing tappings on one of the windings of the transformer. The characteristic curves for a particular motor therefore usually include speed/current curves

* For details see papers by F. C. Kreitler and H. G. Jungk, *Trans. A.I.E.E.*, Vol. 70, pp. 243, 648.

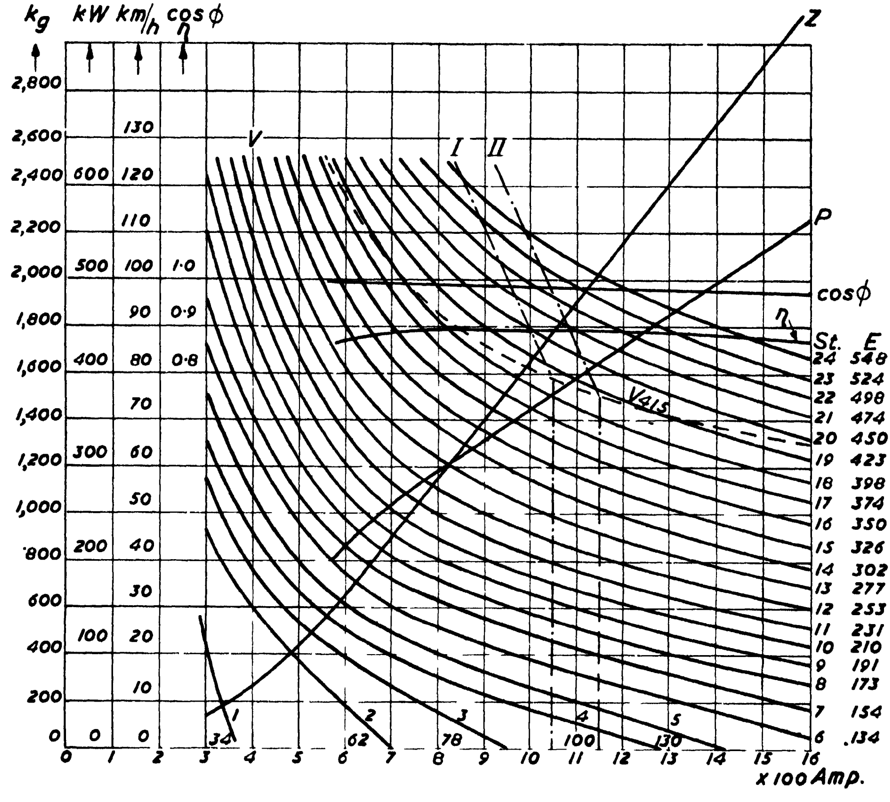


FIG. 50. CHARACTERISTIC CURVES FOR 575-H.P., 415-V, 16⅔-c/s MOTOR (OERLIKON)

Driving wheels, 1,050 mm (41 in) dia , gear ratio 2 85 : 1.
Numbers in column headed *St* refer to position or step of controller, and corresponding numbers in adjacent column, headed *E*, give the voltages at terminals of motor with 15 kV at primary terminals of transformer and a motor current of 1,050 A, except steps 1–3 which refer to armature stationary.

for each operating voltage, together with tractive-effort, efficiency and power factor curves for rated (1-hour rating) voltage.* Typical characteristics for $16\frac{2}{3}$ c/s and 50 c/s motors are shown in Figs. 50, 51, and are based on a copper

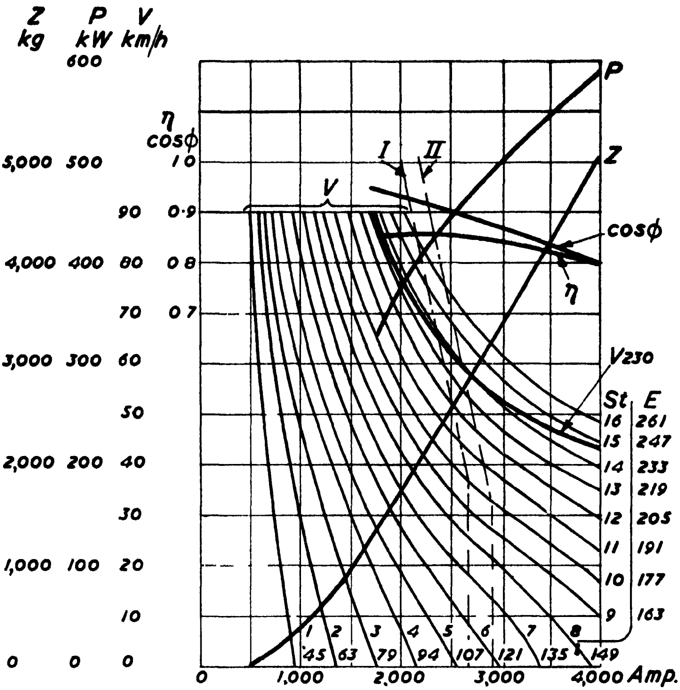


FIG. 51. CHARACTERISTIC CURVES FOR 720 H.P., 230-V, 50-c/s MOTOR (OERLIKON)

Driving wheels, 1,400 mm (51 in.) dia., gear ratio 2.68 : 1
Numbers in column headed *St* refer to position or step of controller, and corresponding numbers in adjacent column, headed *E*, give the voltages at terminals of motor with 20 kV at primary terminals of transformer and a motor current of 2,400 A, except steps 1–4 which refer to armature stationary

temperature of 110 C. In each case the power curve (P) gives the output (in kW) at the driving wheels with normal voltage (1-hour rating) at the motor terminals, and the efficiency curve includes the conventional allowances for losses in the gearing (*see* Appendix). The chain-dotted lines I, II, give the currents, at various speeds, corresponding to the temperature limits of the continuous and 1-hour ratings respectively.

* The speed curve for rated (1-hour) voltage is given for constant (rated) voltage at the motor terminals, but the curves for other voltages are given for constant voltage at the primary terminals of the transformer.

The rated voltage usually corresponds to a speed, with continuously rated current, of about 60 to 70 per cent of the maximum service speed.

CHAPTER VI

POLYPHASE TRACTION MOTORS

THE only type of polyphase motor which has been applied to electric traction is the three-phase induction motor. This motor possesses a "shunt" (or constant-speed) characteristic, and if economical speed control is required special windings or a second motor, or both, are necessary. For traction purposes, however, the motor possesses the important advantage that *regenerative braking* on gradients can be obtained with ideal simplicity, as the machine becomes a generator, feeding back into the supply system, when the speed rises above synchronous speed. The braking speed-torque characteristic is also of the "shunt" type, and therefore the speed of a train descending a uniform gradient remains constant (assuming constant frequency of the receptive supply system) at a value only slightly higher than the synchronous speed.

These characteristics make the three-phase induction motor ideal for railways with long and steep gradients. The robust construction of the motor, absence of a commutator, and high efficiency are also factors which are favourable to its applications for traction service. But the limited speed control and the double trolley wires are serious disadvantages, which, however, may be overcome by the split-phase multi-frequency system in which single-phase distribution at industrial frequency is employed and each locomotive is equipped with a phase converter and polyphase induction motors.

The principal electrifications on the three-phase system, together with the development of suitable motors and control systems, have taken place in Italy, and at the present time about 1,000 miles of track in the Genoa district are in operation at 3,300 V, $16\frac{2}{3}$ c/s. This system, however, may ultimately be replaced by the 3,000-volt d.c. system, which is now the standard system for main lines in Italy. But, with the recent developments in the split-phase multi-frequency system, the application of three-phase induction motors to traction is still of sufficient importance to warrant a discussion of their characteristics and economical methods of speed control.

Fundamental Relationships. The fundamental relationships between speed, torque, and output are given by the following expressions—

(i) Speed $n = n_s(1 - s) = f(1 - s)/\frac{1}{2}p$ . . . . (19)

(ii) Torque $\mathfrak{T} = K\Phi I_2 \cos\phi_2 = K\Phi s E_2 R_2/(R_2^2 + s^2 X_2^2)$

$= K_1 \Phi^2 s R_2/(R_2^2 + s^2 X^2_2)$ . . . (20)

(iii) Power input to rotor $P_2 = E_2 I_2 \cos\phi_2$ . . . . (21)

(iv) Rotor I^2R loss—$P_R = sE_2 I_2 \cos\phi_2 = sP_2$ . . . (22)

(v) Mechanical output $P_M = (1 - s)P_2 = (1 - s)E_2 I_2 \cos\phi_2$

$= K_2(1 - s)\mathfrak{T}$ . . . . (23)

(vi) Electrical efficiency of rotor $\eta_2 = 1 - s$ . . . . (24)

where n is the speed of the rotor in revolutions per second $(= f/\frac{1}{2}p)$

n_s the synchronous speed of the revolving field in revs. per sec,

f the frequency of the supply current,

p the number of poles,
s the slip $(= (n_s - n)/n_s)$,
$\mathfrak{T}$ the gross torque,
Φ the flux per pole,
E_2 the e.m.f. induced in the rotor at standstill,
I_2 the current in the rotor,
ϕ_2 the phase difference between e.m.f. and current in rotor,
R_2 the resistance per phase of the rotor,
X_2 the reactance per phase of the rotor at standstill,
and K, K_1, K_2 are constants.

At starting ($s = 1$) the torque is given by

$$\mathfrak{T}_s = K_1\Phi^2R_2/(R_2{}^2 + X_2{}^2) \quad . \quad . \quad . \quad (20a)$$

and at normal speed the torque is approximately given by

$$\mathfrak{T}_p = K_1\Phi^2 s/R_2 \quad . \quad . \quad . \quad . \quad . \quad (20b)$$

since, owing to the small value of the slip under these conditions—which is of the order of 0·02—the reactance (sX_2) of the rotor circuit is usually very small in comparison with the resistance of this circuit.

Speed Control. Considering only supply systems of constant frequency equation (19) shows that there are two methods of regulating the speed of an induction motor, (i) by varying the slip, (ii) by changing the number of poles. The speed variation obtained by the separate use of either of these methods is limited, but by the combination of the two methods a large speed variation can be obtained.

When speed regulation at constant torque is to be obtained by variation of the slip, the electrical power expended in the rotor circuit is proportional to the slip (Equation 22), but this energy need not be wasted in rheostats, as it can be utilized in the form of mechanical or electrical energy by means of either another induction motor or a commutator machine connected in cascade with the motor to be regulated. In each case the slip energy must be delivered either to the shaft of the main induction motor or to the supply system, but in the latter case a motor-generator set is required, in addition to the auxiliary or cascade machine, to convert the slip energy into electrical energy having the same frequency and voltage as the supply system. Although each system has been applied to industrial plants, only the cascaded-induction-motor system is suitable for electric traction. Its application thereto is due principally to the relatively small size of motors of large output (1,000 to 2,000 h.p.), and the simplicity of a locomotive with two such motors and a collective drive.

Cascaded-induction Motors. The cascade connexion of the main or primary motor (the speed of which is to be regulated) with another induction motor (called the secondary motor) requires the two rotors to be mechanically coupled or geared together. For example, the rotors may be mounted on the same shaft or on separate shafts, but in the latter case the shafts must be geared or mechanically connected together. The rotor of the primary motor must be provided with slip rings, and the rotor of the secondary motor may be of a similar type, or of the squirrel-cage type.

In the cascade connexion the stator winding of the primary motor is connected to the supply. The slip-rings are connected to the stator winding of the secondary motor, and the rotor winding of this motor is connected to a rheostat, as indicated in Fig. 52.* The rheostat enables the speed of the set to be regulated up to the cascade synchronous speed, which, if each motor has the same number of poles, is equal to one-half of the synchronous speed of the primary motor, assuming the field in each motor to rotate in the same direction.

Thus, consider, the set running light, with the rotor of the secondary motor short-circuited, and let p_1, p_2 denote the number of poles in the primary

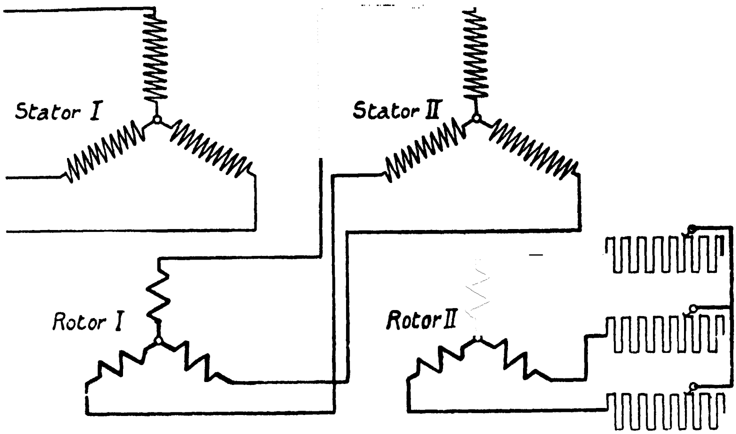


FIG. 52. CASCADE CONNEXION OF TWO INDUCTION MOTORS

and secondary motors respectively. Then, if f is the frequency of the supply system and s is the slip in the primary motor corresponding to cascade synchronous speed, the speed of this motor will be $f(1 - s)/\frac{1}{2}p_1$ revolutions per second. The frequency supplied to the secondary motor is fs_2, and therefore the synchronous speed of this motor will be $fs/\frac{1}{2}p_2$ revolutions per second.

Since the two rotors are mechanically coupled together, the ratio of their speeds is constant, and, if both rotors are fixed to the same shaft, then $fs/\frac{1}{2}p_2 = f(1 - s)/\frac{1}{2}p_1$, whence $s = p_2/(p_1 + p_2)$.

If each motor has the same number of poles, the minimum slip of the primary motor cannot have a value less than 50 per cent, i.e. the cascade synchronous speed is one-half of the synchronous speed of the primary motor. Generally the *cascade synchronous speed*, in revolutions per second, is given by

$$n_c = f/\tfrac{1}{2}(p_1 + p_2) \qquad . \quad . \quad . \quad . \quad . \quad (25)$$

The *mechanical outputs* of the cascaded motors when running with normal slip in the secondary motor are approximately proportional to the numbers of poles in the motors. Thus if P_2' is the power supplied to the rotor of the primary motor, and s_1 is the slip, the mechanical output of this motor is $P_M' = (1 - s_1)P_2'$. Hence, if the I^2R losses in the rotor and stator circuits of

* It is not essential that the rotor of the primary motor be connected to the stator of the secondary motor, as the two rotor windings may be connected together and the rheostats connected to the stator winding of the secondary motor.

the primary and secondary motors are ignored, the power supplied to the rotor of the secondary motor is equal to

$$P_2' - P_M' = s_1 P_2',$$

and, therefore, the mechanical output of this motor is

$$P_M'' = (1 - s_2)s_1 P_2'$$

where s_2 is the slip of the secondary motor.

Whence $$\frac{P_M''}{P_M'} = \frac{(1 - s_2)s_1}{1 - s_1} = \frac{n/n_c}{n/n_s} \cdot \frac{n_s - n}{n_s} = \frac{n_s - n}{n_c},$$

where n_s, n_c are the synchronous and cascade synchronous speeds respectively, and n is the speed of the set.

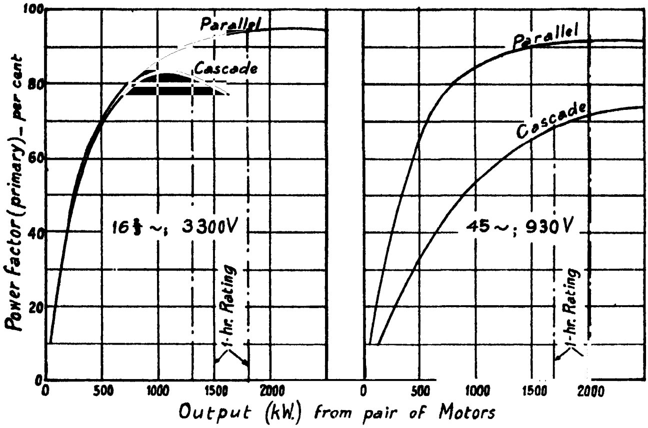


FIG. 53. POWER-FACTOR CURVES FOR CASCADE AND PARALLEL OPERATION OF 1,000-KW, 6-POLE, INDUCTION MOTORS (BROWN BOVERI)

If, however, the slip, s_2, is small, n is approximately equal to n_c, and in this case

$$\frac{P_M''}{P_M'} = \frac{n_s - n_c}{n_c} = \frac{p_2}{p_1} \quad . \quad . \quad . \quad . \quad . \; (26)$$

The *principal objections to speed regulation by the cascade system* are—(i) two motors are required, which, if they are to operate in parallel (at speeds between cascade synchronous speed and full speed), must have the same number of poles, while the stator windings of the secondary motor must be capable of operating with the full supply voltage and also with the rotor voltage (corresponding to cascade synchronous speed) of the primary motor; (ii) the low power factor of the combination.

The low power factor is due to the magnetizing current of the secondary motor being superimposed on that of the primary motor. It is necessary, therefore, to design each motor for a low magnetizing current and low reactance, the attainment of which is facilitated by a low frequency of supply (e.g. 16⅔ c/s).

The effect of frequency on the power factor of cascaded motors is shown in the curves of Fig. 53, which refer to large (1,300 h.p.) 16⅔- and 45-c/s, three-phase, locomotive motors.

Speed Control by Pole-changing. The number of poles may be changed either by providing independent windings (one for each number of poles), or by changing the connexions between the coils of a *single* winding (called a pole-changing winding). The first method gives a free choice of the numbers of poles, but requires deep slots and is inefficient in the utilization of materials. The second method utilizes the materials efficiently but the choice of the numbers of poles is limited, only ratios of 2 : 1, 1·5 : 1, 1·33 : 1 being possible.

Usually only two numbers of poles are obtainable by each method, but by their combination (i.e. if each independent winding is a pole-changing winding) four different numbers of poles may be possible (e.g. 4 and 8, 6 and 12). Special pole-changing windings, however, have been devised to provide three different numbers of poles from a single winding by changing the number of phases in the winding as well as the number of poles (e.g. 6 and 8 poles, 3 phases; 12 poles, 2 phases).

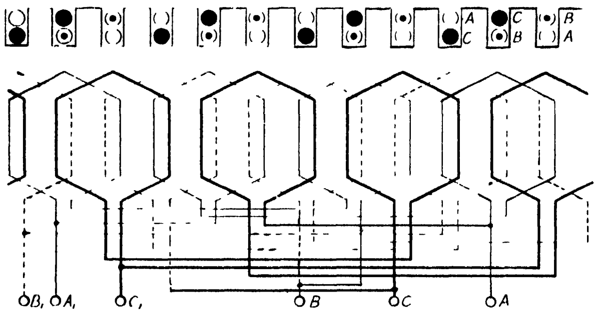


FIG. 54. CONNEXIONS AND DEVELOPMENT OF FRACTIONAL-PITCH POLE-CHANGING WINDING TO GIVE 4 AND 8 POLES

A, B, C supply terminals for 8 poles.
A_1, B_1, C_1 —supply terminals for 4 poles with A, B, C short-circuited.

Simplest Pole-changing Winding. This winding gives two numbers of poles in the ratio of 2 : 1, and involves only three additional terminals on the motor when the rotor is of the squirrel-cage type. It has a large application in industry due to the small number of external connexions and the simplicity of the switchgear. But with a two-motor traction equipment (slip-ring rotors) a 1 : 2 speed ratio can be obtained, with a better overall performance of the motors and greater simplicity, by cascading.

Fig. 54 shows the scheme of connexions for an 8/4-pole winding, the change in the number of poles being effected by reversing the directions of current in *alternate* coils of each phase. Thus for 8 poles the phases are delta connected and all coils of a phase are in series: for 4 poles the phases are star connected and the two halves of each phase are in parallel, their mid-points being connected to the supply. The fractional coil pitch ($\frac{2}{3}$ for 4 poles, $1\frac{1}{3}$ for 8 poles) is so chosen that the differential, or coil-pitch factor in the e.m.f. equation has the same value for each number of poles.

Windings for Combined Pole-changing and Cascading. Four synchronous speeds may be obtained by operating two similar motors, each with a single pole-changing winding, in cascade and parallel. For general railway service the four speeds should be in the ratio of either 1 : 1·33 : 2 : 2·66 or 1 : 1·5 : 2 : 3.

Hence the numbers of poles must be in the ratio of 1 : 1·33 (e.g. 6 and 8) or 1 : 1·5 (e.g. 8 and 12), and both stator and rotor windings must be suitable for parallel and cascade operation of the motors.

Several schemes for such windings have been worked out, but in all cases a large number of leads have to be brought out from the winding (to a pole-changing switch) if the number of phases is to remain the same for both numbers of poles. In some cases the number of external connexions may be reduced considerably by changing the number of phases with the number of poles, e.g. 8 poles and three phases to 6 poles and two phases.

For symmetrical windings the mutual displacement (in electrical degrees or radians) of the coil groups of the several phases must be equal to the phase difference between the e.m.f's. of the supply system, e.g. 120 degrees or $\frac{2}{3}\pi$ radians for three phases, 90 degrees or $\frac{1}{2}\pi$ radians for two phases, 60 degrees or $\frac{1}{3}\pi$ radians for six phases.

The principles involved in the design of a pole-changing winding can be best explained by considering a winding to give either 8 or 6 poles with a three-phase supply.

Three-phase Pole-changing Winding for 8 and 6 Poles. The winding must be of the double-layer type, and the coil pitch may be 100 per cent for the larger number of poles (i.e. 75 per cent for the smaller number of poles). For symmetrical windings the number of slots per phase must be divisible by both pole numbers, and therefore the smallest total number of slots which can be employed is $3 \times 24 = 72$.

Now a double-layer, three-phase, winding for this number of slots requires 72 coils, there being 24 coils in each phase. For eight poles these coils must be arranged in eight groups, each group consisting of three coils, and for six poles they must be arranged in six groups, each group consisting of four coils. Diagrams showing the connexions between the groups of coils for the 8-pole and 6-pole combinations of the winding are given in Figs. 55, 56, which, for clearness, have been drawn with only one turn per coil.

The connexions shown in these diagrams are representative of those which would be adopted when pole-changing is not required. To obtain a pole-changing winding it will be necessary to bring out leads from a large number of coils, so that the connexions of either Fig. 55 or Fig. 56 may be obtained as desired. An examination of these diagrams will show that 48 leads are necessary to obtain eight poles, and 24 additional leads are necessary to obtain six poles. Hence ($48 + 24 =$) 72 leads will be required for the pole-changing winding, the disposition of the leads being shown in Fig. 57. If these leads are to be inter-connected exactly in the manner shown in Figs. 55, 56, then, obviously, 72 external cables will be required between the motor and the pole-changing switch. But by adopting a different scheme for the inter-connexions, the number of external cables between motor and pole-changing switch may be reduced to 36. It will be of interest to show how this result is obtained.*

First it is necessary to determine, from Figs. 55, 56, the direction of current in each coil which gives the correct polarities. We select, therefore, a given instant in the cycle, ascertain the directions of the line currents, and mark these directions on the slot portion of the conductors. In Fig. 57 the upper row of arrow-heads shows the directions of the currents to give eight poles,

* The scheme of interconnexions given here has been worked out by the author and was first published in his *Electric Motors and Control Systems*,

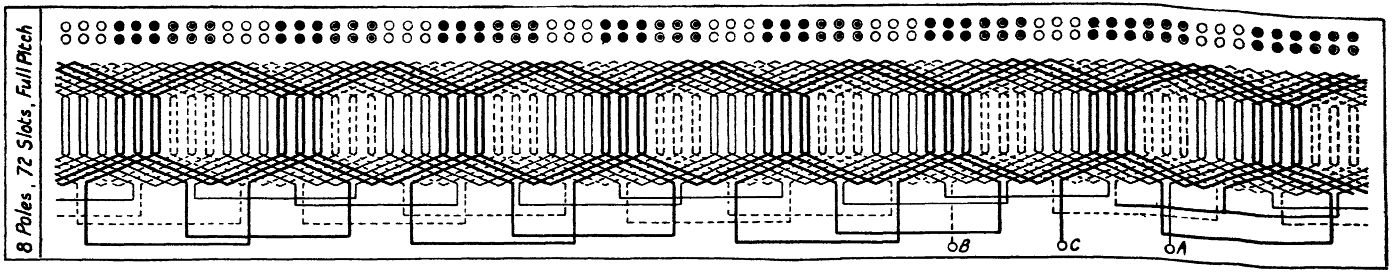


FIG. 55. CONNEXIONS AND DEVELOPMENT OF THREE-PHASE, WHOLE-COILED WINDING FOR 8 POLES

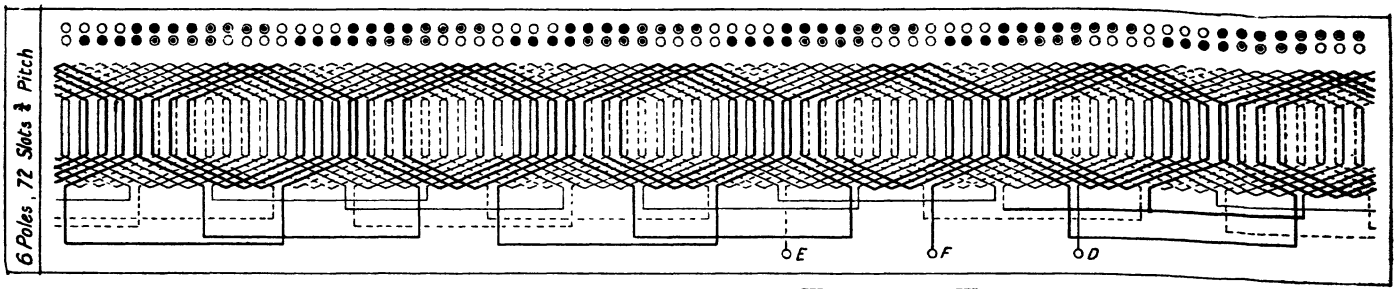


FIG. 56. CONNEXIONS AND DEVELOPMENT OF THREE-PHASE, WHOLE-COILED WINDING FOR 6 POLES

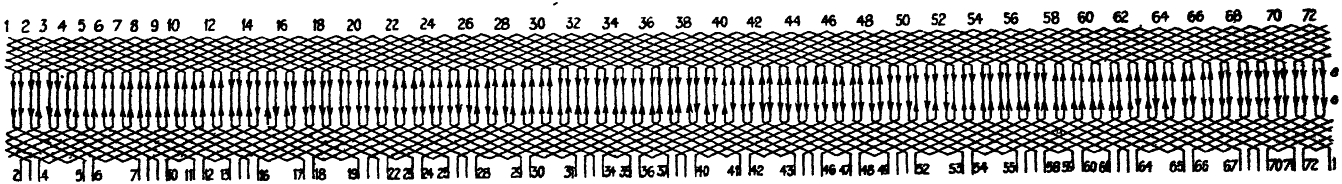


FIG. 57. DISPOSITION OF THE COIL LEADS TO OBTAIN A POLE-CHANGING WINDING FOR 8 AND 6 POLES

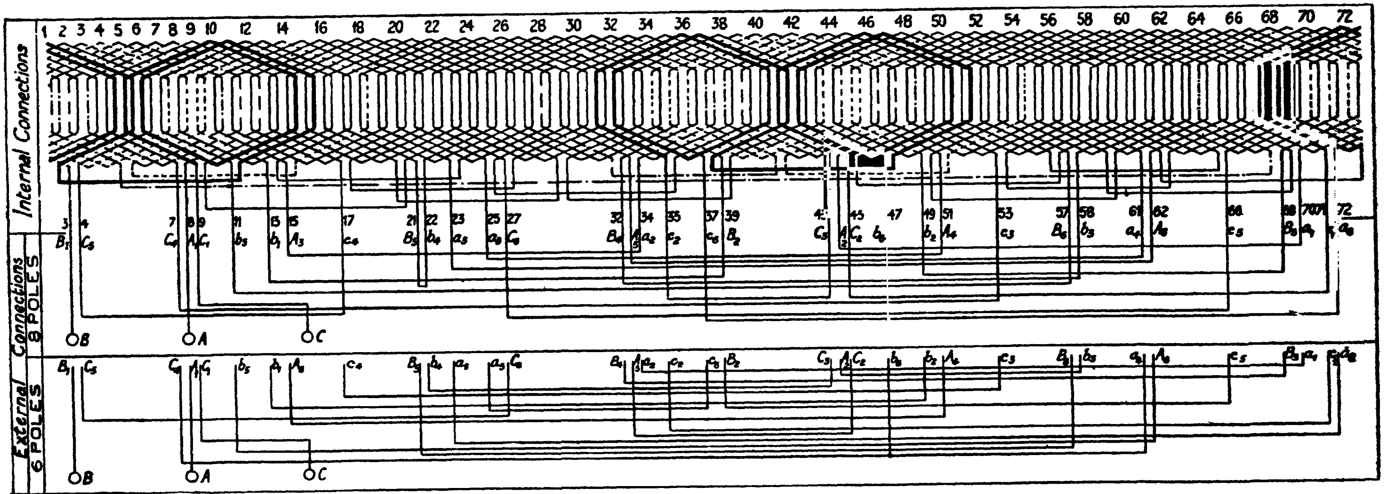


FIG. 58. CONNEXIONS AND DEVELOPMENT OF THREE-PHASE, WHOLE-COILED, POLE-CHANGING WINDING FOR 8 AND 6 POLES

while the lower row of arrow-heads shows the directions to give six poles, the instant selected corresponding to the currents in phases *A* and *C* having a positive direction (which is assumed to be *towards* the neutral point of the motor circuit) and the current in phase *B* having a negative direction.

Next prepare a table giving, for each set of poles, the phase positions of all the coils and the directions of current in the coils, thus—

Number of Coil (see Fig. 57)	Designation of Coil Leads	Phase Position		Direction of Current in Coil*		Number of Coil (see Fig. 57)	Designation of Coil Leads	Phase Position		Direction of Current in Coil*	
		8 Poles	6 Poles	8 Poles	6 Poles			8 Poles	6 Poles	8 Poles	6 Poles
2	1, 4	*C*	*B*	CCW	CCW	*38*	37, 40	*C*	*B*	CCW	CW
3, 4, 5	3, 6	*B*	*B*	CCW	CCW	*39, 40, 41*	39, 42	*B*	*B*	CCW	CW
6, 7, 8	5, 8	*A*	*A*	CCW	CCW	*42, 43, 44*	41, 44	*A*	*A*	CCW	CW
9	7, 10	*C*	*A*	CW	CCW	*45*	43, 46	*C*	*A*	CW	CW
10, 11	9, 12	*C*	*C*	CW	CW	*46, 47*	45, 48	*C*	*C*	CW	CCW
12, 13	11, 14	*B*	*C*	CW	CW	*48, 49*	47, 50	*B*	*C*	CW	CCW
14	13, 16	*B*	*B*	CW	CW	*50*	49, 52	*B*	*B*	CW	CCW
15, 16, 17	15, 18	*A*	*B*	CW	CW	*51, 52, 53*	51, 54	*A*	*B*	CW	CCW
18, 19, 20	17, 20	*C*	*A*	CCW	CW	*54, 55, 56*	53, 56	*C*	*A*	CCW	CCW
21	19, 22	*B*	*A*	CCW	CW	*57*	55, 58	*B*	*A*	CCW	CCW
22, 23	21, 24	*B*	*C*	CCW	CCW	*58, 59*	57, 60	*B*	*C*	CCW	CW
24, 25	23, 26	*A*	*C*	CCW	CCW	*60, 61*	59, 62	*A*	*C*	CCW	CW
26	25, 28	*A*	*B*	CCW	CCW	*62*	61, 64	*A*	*B*	CCW	CW
27, 28, 29	27, 30	*C*	*B*	CW	CCW	*63, 64, 65*	63, 66	*C*	*B*	CW	CW
30, 31, 32	29, 32	*B*	*A*	CW	CCW	*66, 67, 68*	65, 68	*B*	*A*	CW	CW
33	31, 34	*A*	*A*	CW	CCW	*69*	67, 70	*A*	*A*	CW	CW
34, 35	33, 36	*A*	*C*	CW	CW	*70, 71*	69, 72	*A*	*C*	CW	CCW
36, 37	35, 38	*C*	*C*	CCW	CW	*72, 1*	71, 2	*C*	*C*	CCW	CCW

An analysis of this table will show that certain coils are common to a given phase for each combination of the winding, while other coils have to be changed from one phase to another when the number of poles is changed. Moreover, with certain coils, the direction of current is the same for both sets of poles, but, with other coils, the direction of current must be reversed when the number of poles is changed. The analysis may be summarized thus—

(*a*) Coils which are common to a given phase, and in which the direction of current is the same for both sets of poles—

6, 7, 8, 69 (phase *A*); 3, 4, 5, 14 (phase *B*); 10, 11, 72, 1 (phase *C*).

(*b*) Coils which are common to a given phase, and in which the direction of current must be reversed when the number of poles is changed—

33, 42, 43, 44 (phase *A*); 39, 40, 41, 50 (phase *B*); 36, 37, 46, 47 (phase *C*).

(*c*) Coils which have to be changed in phase *without* reversal of current—

15, 16, 17, 26 (*A* to *B*); 24, 25, 34, 35 (*A* to *C*);
57, 66, 67, 68 (*B* to *A*); 12, 13, 22, 23 (*B* to *C*);
45, 54, 55, 56 (*C* to *A*); 2, 63, 64, 65 (*C* to *B*).

* CCW denotes counter-clockwise; CW denotes clockwise.

(*d*) Coils which have to be changed in phase *with* reversal of current—

51, 52, 53, 62 (A to B); 60, 61, 70, 71 (A to C);
21, 30, 31, 32 (B to A); 48, 49, 58, 59 (B to C);
9, 18, 19, 20 (C to A); 27, 28, 29, 38 (C to B).

The four coils in each of above sets may, therefore, be connected permanently in series. But each set of (four) coils consists of two groups, of which one group contains either a single coil or two adjacent coils connected in series, and the other group contains either three or two coils connected in series. Now each group of one, two, or three coils corresponds to two coil leads (*see* Fig. 57); hence, after the above series connexions have been made, we shall have only 36 leads remaining. The interconnexions between these leads must be made by the pole-changing switch.

The *complete connexions of the winding* for both sets of poles are shown in Fig. 58. In this diagram the 36 external leads have been marked according to the phase position occupied by the coils, to which the leads belong, when the winding is connected for eight poles. For example, the external leads, Nos. 8 and 70 (Figs. 57, 58), belonging to the set of (four) coils Nos. 6, 7, 8, 69, are marked A_1, a_1; the leads Nos. 3 and 16 belonging to the set of (four) coils Nos. 3, 4, 5, 14 are marked B_1, b_1; the leads Nos. 9 and 71 belonging to the set of coils Nos. 10, 11, 72, 1 are marked C_1, c_1. The leads of the other sets of coils have been marked in like manner. The leads from the six groups of coils forming one phase will, therefore, be marked as follows—A_1, a_1; A_2, a_2; A_3, a_3; A_4, a_4; A_5, a_5; A_6, a_6; and the leads for the B and C phases will be marked similarly.

To obtain eight poles, connexions are made between the leads as follows—

Phase A.—a_1 A_2; a_2 A_3; a_3 A_4; a_4 A_5; a_5 A_6.
Phase B.—b_1–B_2; b_2 B_3; b_3 B_4; b_4 B_5; b_5 B_6.
Phase C.—c_1–C_2; c_2 C_3; c_3 C_4; c_4–C_5; c_5 C_6.
Neutral point, a_6 b_6 c_6. Lines on A_1, B_1, C_1.

To obtain six poles certain groups of coils must be interchanged between phases and reversed in accordance with the above scheme. The connexions between the external leads are as follows —

Phase A. a_1–a_2; A_2 b_3; B_3 B_1; b_4 C_3; c_3–c_4.
Phase B.—b_1–b_2; B_2–c_5; C_5 C_6; c_6–a_3; A_3–A_4.
Phase C.—c_1 c_2; C_2–A_5; a_5 a_6; A_6 b_5; B_5–B_6.
Neutral point, a_4–b_6 C_4. Lines on A_1, B_1, C_1.

The *rotor winding* may be of the same type as the stator winding and may be connected in the same manner, but the pole-changing switch must be arranged internally so that only three slip-rings are required.

Pole-changing Winding for 8 and 6 Poles, Three and Two Phases. The simplest winding is of the *single-layer* type, designed for three-phases and 8 poles with a coil pitch of 100 per cent, the number of coil groups per phase being equal to the number of pairs of poles. To obtain 6 poles the coil groups must be re-connected for a *two-phase supply*, which involves bringing out leads from each of the 12 groups of coils (i.e. a total of 24 leads).

The necessity for changing the number of phases, when changing the number of poles, will be apparent from an examination of a diagram of a three-phase, 8-pole, half-coiled, single-layer, spiral winding with full-pitch coils. It

will be found that although a 6-pole, *three-phase* winding is impossible, a symmetrical 6-pole, *two-phase* (75 per cent pitch) winding is quite practicable and involves only a change in the connexions between the groups of coils. Thus, for a 24-slot wave winding, the connexions for eight poles, three phases, are

Phase I	1‾4_7‾10_13‾16_19‾22
Phase II	3‾6_9‾12_15‾18_21‾24
Phase III	5‾8_11‾14_17‾20_23‾2

and the connexions for six poles, two phases, are—

Phase I . . .	3‾6_10‾7_11‾14_18‾15_19‾22_2‾23
Phase II . . .	1‾4_8‾5_9‾12_16‾13_17‾20_24‾21

Diagrams showing the connexions of this winding—which is suitable for the stator—are given in Fig. 59. It will be observed that the winding is *half-coiled* when connected for three phases, and *whole-coiled* when connected for

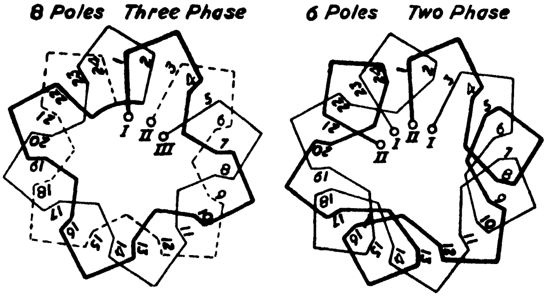


FIG. 59. CONNEXIONS OF SINGLE-LAYER POLE-CHANGING WINDING TO GIVE EIGHT POLES, THREE PHASE, AND SIX POLES, TWO PHASE

two phases. The two-phase winding may be supplied from the three-phase system by means of two "T-connected" auto-transformers.*

Rotor Winding for 8 and 6 Poles. Although each of the above 8/6-pole windings is suitable for either stator or rotor, their application to the rotor would necessitate an internal pole-changing switch. Rotor windings, however, may be devised for which no pole-changing switches are required. The simplest of these windings—originally developed by the Società Italiana Westinghouse for 4-speed locomotives—is derived from a quadruplex three-phase winding having isolated neutral points. These neutral points are connected to four slip-rings, and the common ends of the phases are connected to three other slip-rings.

A diagram showing the application of this principle to a 24-slot single-layer winding is given in Fig. 60, and a schematic diagram of the circuits is given in Fig. 61. The winding consists of twelve coils, hence each phase of the four three-phase circuits contains one coil. Now the angular spacing of the coils in the diagram is ($\frac{1}{12} \times 2\pi =$) $\frac{1}{6}\pi$ radians, or 30°, which corresponds to a phase difference of ($\frac{1}{12} \times 8\pi$) $= \frac{2}{3}\pi$ radians, or 120°, in an 8-pole field, and

* In certain cases it may be possible so to arrange the number of turns that the two phases of the winding may be T-connected and supplied direct from the three-phase system. For an example of such a winding *see Journ. I.E.E.* Vol. 69, p. 325.

($\frac{1}{12} \times 6\pi =$) $\frac{1}{2}\pi$ radians, or 90°, in a 6-pole field. Hence coils in the diagram which have an angular spacing of 120° will have a phase difference of ($4 \times \frac{2}{3}\pi$ $=$) $2\frac{2}{3}\pi$ radians—equivalent to 120 electrical degrees—in an 8-pole field and ($4 \times \frac{1}{2}\pi =$) 2π radians—equivalent to zero electrical degrees—in a 6-pole field.

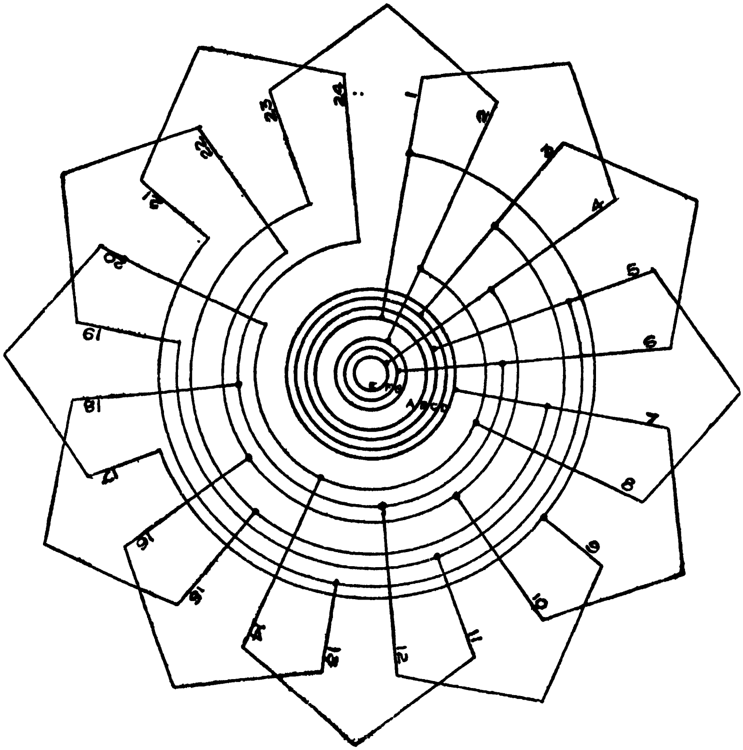


FIG. 60. CONNEXIONS OF ROTOR WINDING SUITABLE FOR 8 AND 6 POLES

Therefore, a three-phase winding for an 8-pole field may be formed from any three coils for which the angular spacing is 120°: and, as there are 12 coils, four similar three-phase windings will be obtained. Obviously these windings may be permanently connected in parallel.

The scheme of connexions for a winding having 24 single-turn coils is shown in Fig. 60, and the circuits of the four three-phase windings are as follows—

Phase A_1 ...	Ring E— 4- 1	Ring A	Phase A_2 ...	Ring E—10- 7	Ring D
Phase B_1 ...	Ring F—12- 9		Phase B_2 ...	Ring F—18-15	
Phase C_1 ...	Ring G—20-17		Phase C_2 ...	Ring G— 3-23	
Phase A_3 ...	Ring E—16-13	Ring B	Phase A_4 ...	Ring E—22-19	Ring C
Phase B_3 ...	Ring F—24-21		Phase B_4 ...	Ring F— 6- 3	
Phase C_3 ...	Ring G— 8- 5		Phase C_4 ...	Ring G—14-11	

Now if these windings are placed in a 6-pole field, the e.m.f. induced in the three phases of any given winding will be in phase, while the e.m.f's. induced in corresponding phases of the four circuits will have a phase difference of 90°. For example, the phase difference of the e.m.f's. induced in phases A_1 and A_2

is $(3 \times \frac{1}{4}\pi =)\ 1\frac{1}{2}\pi$ radians—equivalent to 270°—while the values for the remaining circuits are—

Phases A_1 and A_3 $(6 \times \frac{1}{2}\pi =)$ 3π radians = 180 degrees;
Phases A_1 and A_4—$(9 \times \frac{1}{2}\pi =)$ $4\frac{1}{2}\pi$ radians = 90 degrees;
Phases A_2 and A_4—$(6 \times \frac{1}{2}\pi =)$ 3π radians = 180 degrees.

Hence the corresponding phases of the four three-phase, 8-pole, windings form the (four) phases of a star-connected four-phase winding, so that, with a

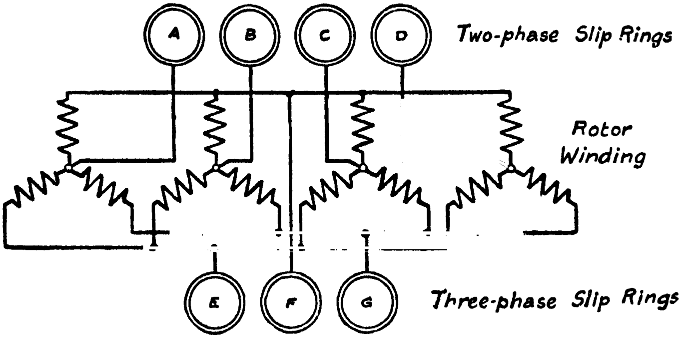


FIG. 61. DIAGRAM OF CIRCUITS FOR THE WINDING SHOWN IN FIG. 60

6-pole field, the rotor winding is equivalent to three four-phase star-connected windings in parallel. The circuits (*see* Fig. 60) are as follows—

Phase W_1....Ring A— 1 4
Phase X_1....Ring C—19–22
Phase Y_1....Ring B—13–16
Phase Z_1....Ring D – 7 10
} Ring E.

Phase W_2....Ring A— 9 12
Phase X_2....Ring C – 3 6
Phase Y_2....Ring B –21 24
Phase Z_2....Ring D—15–18
} Ring F.

Phase W_3....Ring A 17–2
Phase X_3....Ring C—11–14
Phase Y_3....Ring B— 5–8
Phase Z_3....Ring D 23 2
} Ring G.

The three neutral points of these windings, however, cannot be interconnected, as this would prevent the winding being used in the 8-pole field.

Four e.m.f's., or currents, differing 90° in phase, may be obtained from the following pairs of slip-rings: A–C, C–B, B–D, D–A, while two e.m.f's., or currents, differing 90° in phase, may be obtained from slip-rings, A–B, C–D.

Thus, in a 6-pole field, the rotor can supply either two-phase or four-phase current, and, in an 8-pole field, it can supply three-phase current.

The voltage relations will now be investigated. Let $3N$ denote the total number of turns in the stator winding, and Φ_3, Φ_2 denote respectively the fluxes corresponding to a terminal voltage V in each case. Then the turns in series per phase will be N for the three-phase 8-pole connexion and $3N/2$ for the two-phase 6-pole connexion. If the breadth coefficients for the 8-pole and the 6-pole windings be assumed as 0·96 and 0·9 respectively, then we must have

$$V/\sqrt{3} = 4{\cdot}44 \times 0{\cdot}96\Phi_3 Nf \times 10^{-2},$$

and $$V = 4{\cdot}44 \times 0{\cdot}9 \times 0{\cdot}92\Phi_2 f 3N/2 \times 10^{-2},$$

whence $$\Phi_2 = 1{\cdot}34\Phi_3.$$

For equal fluxes in two-phase and three-phase working the terminal voltage for the 6-pole winding must therefore be 0·75 of the normal three-phase line voltage. This voltage can readily be obtained from the auto-transformer.

The relationship between the open-circuit rotor voltages is obtained as follows.

Let N' denote the total number of turns in the rotor winding; then there are $\frac{1}{12}N'$ turns in series per phase for three-phase working, and $\frac{1}{6}N'$ turns in series per phase for two-phase working. Hence, with equal fluxes (Φ), and assuming the same breadth coefficients as above, the voltage (V_3) between the three-phase slip-rings is

$$V_3 = \sqrt{3} \times 4{\cdot}44 \times 0{\cdot}96 \times \Phi \times \tfrac{1}{12}N' \times f \times 10^{-2},$$

and the voltage (V_2) between the two-phase slip-rings is

$$V_2 = 4{\cdot}44 \times 0{\cdot}9 \times 0{\cdot}92 \times \Phi \times \tfrac{1}{6}N' \times f \times 10^{-2},$$

whence $V_2 = V_3$. Thus cascade working will be practicable for both sets of poles.

Cascade Operation of 8/6-pole Motors. This involves the consideration of (i) the number of phases in the windings, (ii) the rotor voltages at the two cascade synchronous speeds, (iii) the practicability of adapting the stator winding of the secondary motor to these voltages.

With high-voltage (3,300 V) motors having stator and rotor windings with the same number of phases for each number of poles, re-grouping of the coils of the stator winding of the secondary motor will be necessary if the connexions of Fig. 52 are employed because it is undesirable to wind the rotors for a high voltage. If such re-grouping is inconvenient or impracticable cascading is effected by connecting the two rotors, the starting rheostats being connected to the stator winding of the secondary motor. This method of cascading must obviously be employed if the rotor winding of Fig. 60 is used with the stator winding of Fig. 58.

General Considerations Relating to Three-phase Traction Motors

The selection of the *number of poles* involves considerations of power factor and operating speeds. To obtain a high power factor the smallest practicable number of poles should be chosen consistent with low leakage reactances for the windings. For example, with large, 1,000 h.p., motors six poles is the minimum, and with a low frequency ($16\frac{2}{3}$ c/s) a gearless drive is practicable. But with industrial frequencies (40 to 50 c/s) and six poles a geared drive is necessary owing to the relatively high synchronous speed of the motor. The use of gearing in this case is preferable to a gearless drive and the increase of the minimum number of poles to 12 or 16, as the latter would reduce seriously the power factor, and would result in exceptionally low power factors when lower speeds were obtained by pole-changing and cascading. The manner in which the number of poles affects the power factor of large motors is shown by the curves of Fig. 62.

The *leakage factor* is an important feature in the design, as it affects both the maximum power factor and the overload capacity. This factor (σ) is usually defined as the ratio of the magnetizing current to the ideal short-circuit current

(at normal voltage). Its relationship to the maximum power factor and overload capacity can be determined from the circle diagram, and is given by

$$\text{Maximum power factor} = (1 - \sigma)/(1 + \sigma)$$
$$\text{Overload capacity} = (1 + \sigma)/2\sqrt{\sigma}$$

Practical requirements, therefore, necessitate a low value for σ, i.e. a relatively small magnetizing current and a large ideal short-circuit current.*

The *air-gap* of a three-phase traction motor is generally larger than that of a stationary motor of similar size. To obtain a high power factor under these conditions a low frequency of supply is essential, together with few poles, nearly

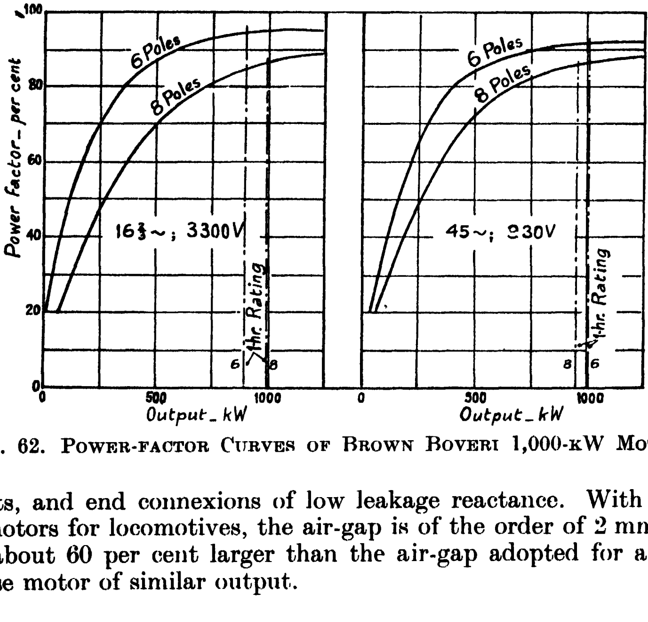


FIG. 62. POWER-FACTOR CURVES OF BROWN BOVERI 1,000-KW MOTORS

closed slots, and end connexions of low leakage reactance. With large Continental motors for locomotives, the air-gap is of the order of 2 mm (0·08 in.), which is about 60 per cent larger than the air-gap adopted for a stationary three-phase motor of similar output.

Losses and Efficiency. The losses comprise—stator and rotor I^2R, stator core loss, friction and windage. The rotor core loss at normal slip is negligible.

If P_M is the mechanical output we have, from equations (22), (23), rotor $I^2R = sP_M/(1 - s)$. Hence when the slip is small, the rotor I^2R loss is approximately equal to — slip × output.

With large motors the natural slip at full load is about $1\frac{1}{4}$ per cent, which, therefore, represents, to a close approximation, the rotor I^2R loss as a percentage of the output. Hence as the full load efficiency is of the order of 96 per cent the stator I^2R and core loss together are about $2\frac{1}{2}$ per cent of the output.

The total losses are, therefore, much lower than those in the corresponding d.c. and single-phase motors. Hence natural ventilation—which may be exceptionally good due to the open construction of the rotor spider—will normally be sufficient to dissipate the losses, especially as the largest losses occur in the stator, which has a large radiating surface.

* For the calculation of these quantities, see *Papers on Alternating-Current Machinery*, Hawkins, Smith, and Neville (Pitman).

EXAMPLES OF THREE-PHASE TRACTION MOTORS

Three-phase traction motors are usually built for locomotive service and are frame-mounted machines of large output, the power being transmitted from the motors to a number of coupled driving wheels by cranks and connecting rods. Owing to the constant-speed characteristic of the motors, a distributed, or individual-axle drive—as employed with direct-current and single-phase locomotives—is undesirable, as slight differences in the diameters of the driving wheels would cause unequal loading of the motors.

The largest application is in Italy. At one period over 700 locomotives (3,300 V, 16$\frac{2}{3}$ c/s) were in service, but many were destroyed during the 1939–45 war and have not been replaced. Two-speed locomotives for freight service

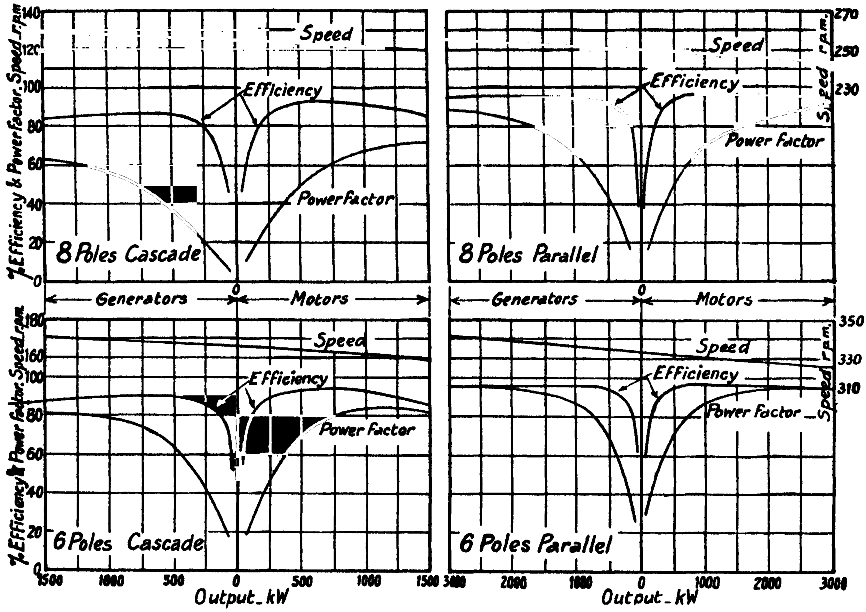


FIG. 63. PERFORMANCE CURVES OF BROWN BOVERI 3-PHASE, 3,300-VOLT, 16$\frac{2}{3}$-C/S, 4-SPEED LOCOMOTIVE MOTORS

on mountain grades have equipments with two 1,300 h.p., 8-pole gearless motors (collective drive), the two speeds being obtained by cascade and parallel operation. Four-speed locomotives for general service have two motors with pole-changing windings, the four speeds being obtained by pole-changing and cascading. Some motors are wound for 8 and 6 poles according to the schemes of Figs. 58 and 59, and are cascaded with both sets of poles. Other motors have a pole-changing winding to give 6, 8 and 12 poles and are cascaded with only 8 poles.

Fig. 64 shows details of a typical motor designed for 3,300 V, 8/6 poles, and rated at 1,300 h.p. when connected for 8 poles. The pole-changing switch for the rotor winding is located in the interior of the rotor, and is operated pneumatically.

Fig. 63 shows the performance curves for a pair of these motors (i.e. the power equipment of one locomotive) for cascade and parallel operation with

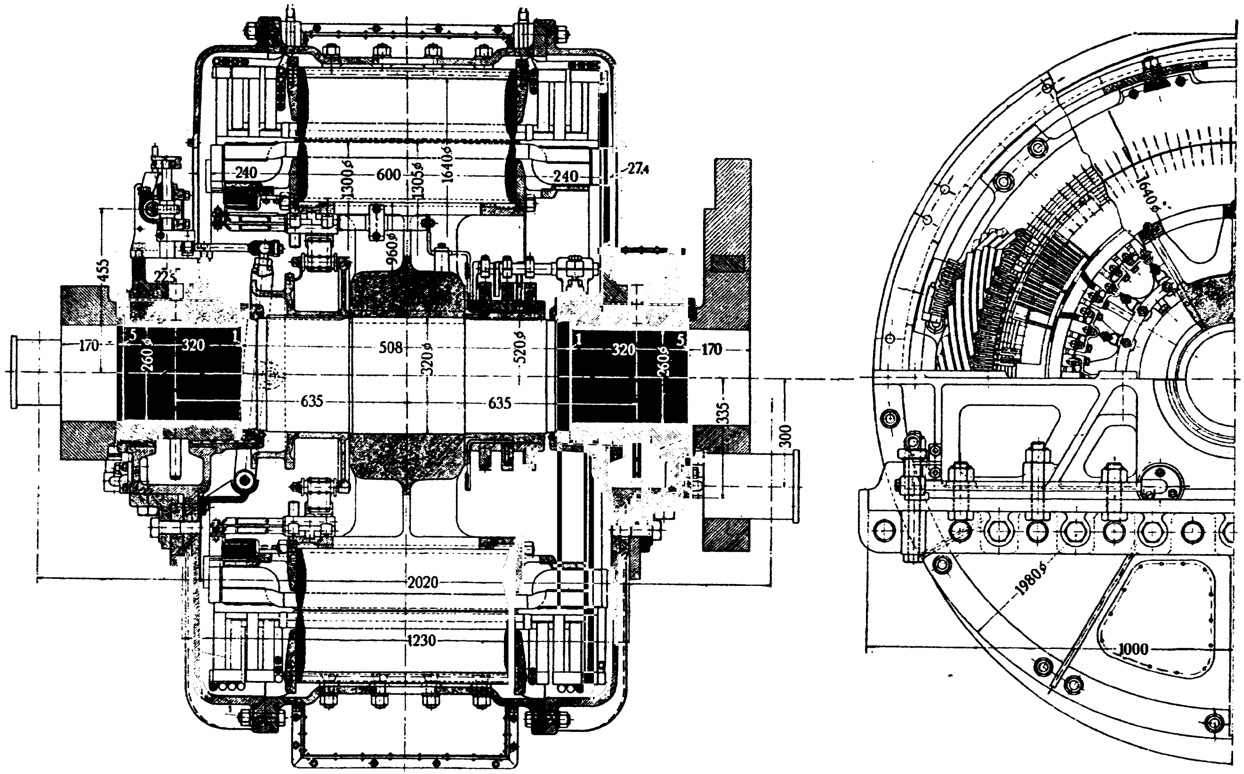


Fig. 64. Longitudinal and Cross-sections of Gearless Locomotive Motor (Brown Boveri)

both sets of poles. Curves are also given showing the performance of these machines when operating as induction generators—i.e. when used for regenera-

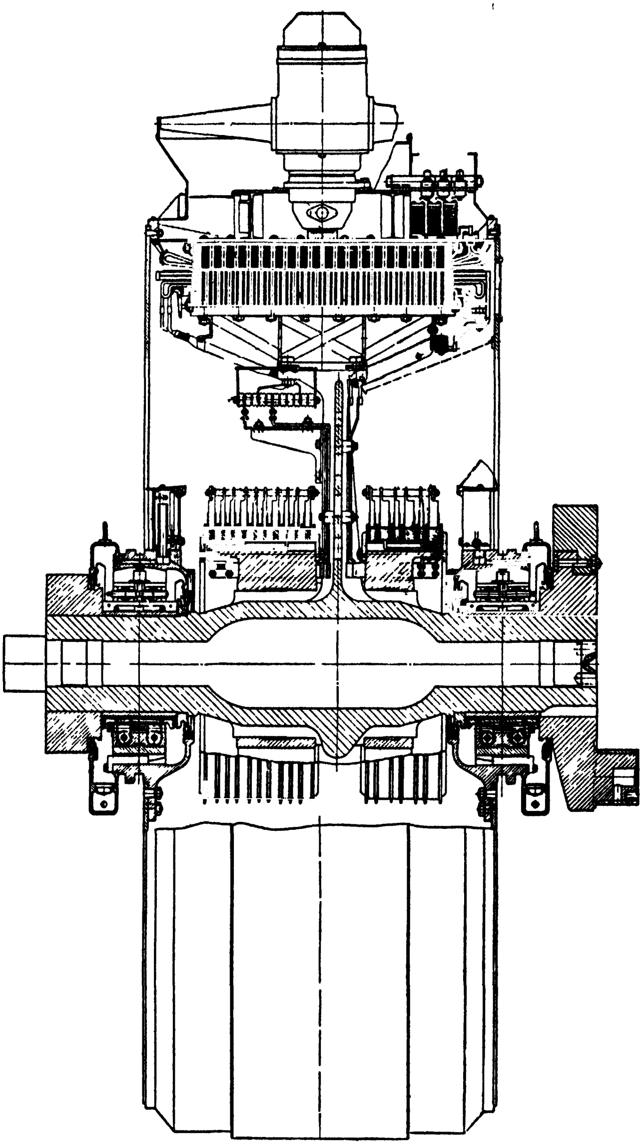


FIG. 65. SECTIONAL VIEW OF 2,500-H.P., 1,000-VOLT, 50-C/S MOTOR FOR 93-TON SPLIT-PHASE LOCOMOTIVE (METROPOLITAN-VICKERS)

tive braking. The high efficiency for both motor and generator operation should be noted.

Fig. 65 shows a sectional view of a British-built motor rated at 2,500 h.p.

(1 hour), 1,000 V, 50 c/s, of which 30 were built for the equipment of 93-ton split-phase locomotives (Kando converter system) for the Hungarian railways.

The motor is of interest not only for its large size but also for the scheme of pole-changing. Four speeds—corresponding to 18, 24, 36 and 72 poles—are obtained by two pole-changing windings on the rotor (which is the "primary" part of the motor) and a *single* winding, with 48 tappings, on the stator, the terminals being connected to the 48 electrodes of a liquid rheostat.

One rotor winding, with six slip-rings, is designed for 72 and 36 poles with a three-phase supply in each case, the scheme of pole-changing being similar to that of Fig. 54. The other winding, with ten slip rings, is designed to give 24 poles with a six-phase supply and 18 poles with a four-phase supply. The various supplies are obtained from the phase converter.

CHAPTER VII

THE TESTING OF TRACTION MOTORS

Introduction. Tests on traction motors may be divided into two classes, (i) factory tests, which include (*a*) commercial tests run on standard machines, (*b*) special tests applied to machines of a new design; and (ii) tests in service.

PART I

Factory Tests on Direct-current Traction Motors

Ratings. Traction motors are rated on an arbitrary basis, and two ratings (one-hour and continuous) are now adopted internationally. Both ratings are determined by load tests with the motors mounted on a testing stand (called stand tests) and the British Standard ratings are defined as follows—

One-hour Rating. The output at the motor shaft for one hour at normal voltage which produces temperature rises not exceeding the limits specified in the Appendix (i.e. for class B insulation* 120°C for the armature winding, 130°C for the field winding—both based upon resistance measurements—and 90°C for the commutator).

Continuous Rating. The output at the motor shaft for an unlimited period at normal voltage which produces temperature rises not exceeding those, given above, for the one-hour rating.

In each test the ventilation is to be the same as in service. The tests may be taken either with or without gearing.

Commercial Tests. To meet British Standards Institution's† requirements, commercial tests must include the following, of which brief details are given in the Appendix—

(i) A heat-run of one-hour's duration at the rated load.
(ii) A speed test at full load, in each direction of rotation, to check the position of the brushes and to ascertain if the speed of the motor is within the permissible limits. (The speeds in each direction of rotation must be within $\pm$ 3 per cent of the rated speed.)
(iii) An overspeed test.
(iv) Commutation tests (including circuit interruptions).
(v) An insulation test, consisting of a brief application of high voltage.

Tests (ii)–(v) are carried out with the machine hot.

With standard machines it is customary to test two similar machines together, operating one machine as a motor during the first half of the heat-run, and the other machine as a motor during the second half of the heat-run. The resistances of the armature and field windings of each machine are obtained

* As all modern traction motors have class B insulation, only the temperature limits for this class of insulation will be considered.

† The relevant Standard for traction motors is No. 173, and is published by the British Standards Institution.

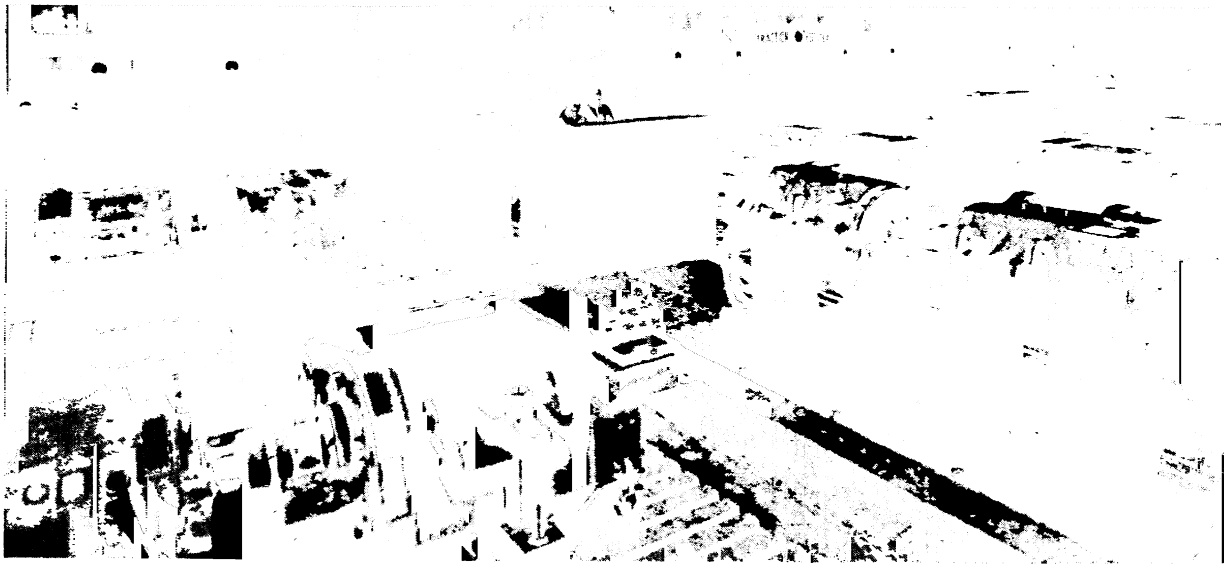


FIG. 66. TESTING STANDS FOR TRACTION MOTORS IN WORKS OF ENGLISH ELECTRIC COMPANY

at the start and finish of the run, in addition to the temperatures by thermometer.

Special Tests. The tests include (*a*) an efficiency and speed test over the operating range of the motor, (*b*) a number of heat-runs at various loads for the purpose of determining the thermal characteristics, (*c*) a core-loss and saturation test.

Production Tests. When a particular type of motor is manufactured in large quantities and the manufacturing processes are thoroughly stabilized—as will be evidenced by the uniformity of the results of commercial tests—the test procedure may be modified by restricting the full commercial tests to a few machines selected at random from the production line. The remaining machines are subjected to a short series of tests (called "production" tests) which may take the form of—(i) a no-load magnetization characteristic (saturation curve) with the main poles excited and the unloaded armature driven at constant speed; (ii) a no-load magnetization characteristic for the commutating poles, with the latter only excited and the armature driven at constant speed, the brushes being removed and a pair of pencil points being employed to enable the e.m.f. between the segments to be measured;* (iii) a test for checking the position of the brushes, the armature being driven at normal full-load speed with full-load current in the armature and commutating pole winding; (iv) an overspeed and high-voltage-per-segment (45 V) test at no load; (v) an insulation test at high voltage; (vi) measurements of the resistances of all windings.

Testing Stands. Load tests are usually run on a testing stand, which is arranged to accommodate two similar motors. When tests with gearing are necessary the two machines are mounted on opposite sides of a horizontal shaft and are geared together through standard gearing. But for ordinary routine and efficiency tests the two machines are tested without gears, the armature shafts being arranged in line and coupled together with a flexible coupling. Typical testing stands are shown in Fig. 66.

In cases where a single machine is to be tested the armature shaft is coupled to a brake, which should be either a Froude water-dynamometer or an electric regenerative dynamometer.

Load Tests on Two Similar Machines. When two similar machines are to be tested, one is operated as a motor and is loaded by the other machine acting as a separately excited generator, the field winding being connected in series with the motor. The generator may be loaded either on rheostats or by feeding back into the supply. In the latter case a booster is required to make up the difference between the supply voltage and that of the generator armature, while in both cases a booster will usually be required to maintain a constant voltage at the terminals of the motor.

The set is started with the generator loaded on rheostats, the rheostat load being cut out as soon as the generator is paralleled with the supply. The load is then regulated by adjusting the excitation of the "load" booster, while

* Details are given in a paper on "Quick and Accurate Production Testing of Large d.c. Apparatus," by M. J. Baldwin and H. D. Barnhart. *Trans. A.I.E.E.*, Vol. 69, p. 136.

normal voltage is maintained across the motor by regulating the excitation of the "line" booster. The *efficiency* of the machine operating as a motor is determined in the following manner—

Let V_1 = voltage across motor terminals,
V = voltage across motor and generator field (called the "total" voltage),
V_2 = voltage across generator armature,
I_1 = motor current in amperes (input),
I_2 = generator current in amperes (output).

Then the total losses are given by $VI_1 - V_2I_2$.

Now, since the generator field is connected in series with the motor and each machine is running at the same speed, the field I^2R, core, friction, and gear losses may be assumed as equal in each machine, while the armature I^2R

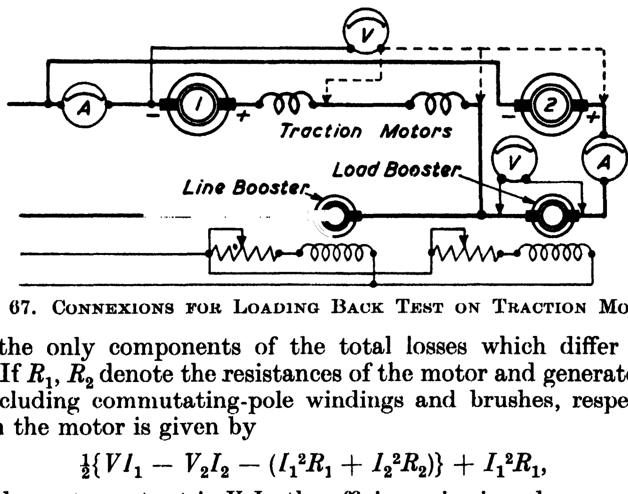


FIG. 67. CONNEXIONS FOR LOADING BACK TEST ON TRACTION MOTORS

losses are the only components of the total losses which differ in the two machines. If R_1, R_2 denote the resistances of the motor and generator armature circuits (including commutating-pole windings and brushes, respectively, the total loss in the motor is given by

$$\tfrac{1}{2}\{VI_1 - V_2I_2 - (I_1^2R_1 + I_2^2R_2)\} + I_1^2R_1,$$

and, since the motor output is V_1I_1, the efficiency is given by

$$\eta = [V_1I_1 - \tfrac{1}{2}\{VI_1 - V_2I_2 + (I_1^2R_1 - I_2^2R_2)\}]/V_1I_1$$

or
$$\eta = 1 - (0{\cdot}5/V_1)\{(V + I_1R_1) - (I_2/I_1)(V_2 + I_2R_2)\} \qquad . \qquad . \; (27)$$

a form which is convenient for calculation.

When the loading-back method is employed the efficiency is best determined from direct measurement of the motor input and the total losses, rather than from the motor input and generator output. Thus, if I and V_3 denote the current input from the supply system and the voltage at the terminals of the "load" booster armature, respectively, we have

$$\begin{aligned}\text{Total losses} &= VI_1 - V_2I_2 = VI_1 - (V - V_3)I_2 \\ &= V(I_2 - I_2) + V_3I_2 = VI + V_3I_2\end{aligned}$$

Hence the efficiency of the motor is given by

$$\begin{aligned}\eta &= [V_1I_1 - \tfrac{1}{2}\{(VI + V_3I_2) - (I_1^2R_1 + I_2^2R_2)\} + I_1^2R_1]/V_1I_1 \\ &= 1 - 0{\cdot}5(VI + V_3I_2 + I_1^2R_1 - I_2^2R_2)/V_1I_1 \qquad . \qquad . \qquad . \qquad (27a)\end{aligned}$$

Therefore the readings to be taken when running an efficiency test by the loading-back method are: motor current and voltage, generator armature current, line or supply current, "total" voltage, "load" booster armature voltage.

Calculation of Characteristic Curves from Test Results. In working out standard characteristic curves the test readings are corrected for a copper temperature of 110°C, and for a gear friction loss, at the rated load, equivalent to 2 per cent of the input (*see* Appendix).

The *speed is corrected* in the following manner—

Let R denote the resistance of the armature and field windings at a temperature of 110°C, R_1 the resistances during the test, n_1 the test speed, and n the corrected speed, both corresponding to a current I. If the terminal voltage during the test has been held at its normal value V_1, then

$$n_1/n = (V_1 - IR_1)/(V_1 - IR)$$

whence

$$n = n_1(V_1 - IR)/(V_1 - IR_1) \quad . \quad . \quad . \quad . \quad (28)$$

If the speed n_1 has been determined at a voltage (V'_1) other than normal, then

$$n = n_1(V_1 - IR)/(V'_1 - IR_1)$$

The *speed curve* is usually plotted in miles per hour, corresponding to the diameter of wheel (D in.) and gear ratio (γ) to be used. Thus the speed, S (in m.p.h.), corresponding to an armature speed n (r.p.m.), is

$$S = \frac{nD}{\gamma} \times \frac{60\pi}{12 \times 5{,}280} = 0{\cdot}00297\,\frac{nD}{\gamma} \quad . \quad . \quad . \quad (29)$$

The *torque curve* is obtained from the efficiency and speed curves. It is generally plotted in terms of the tractive-effort (expressed in lb) at the driving wheels. Thus, for a current of I_1 and (normal) voltage V_1, the tractive-effort (F) in lb is

$$F = \frac{V_1 I_1 \eta}{S} \times \frac{33{,}000}{100 \times 746} \times \frac{60}{5{,}280} = \frac{0{\cdot}005\,V_1 I_1 \eta}{S} \quad . \quad . \quad (30)$$

η and S denoting the percentage efficiency and speed (m.p.h.) respectively, which are obtained from the appropriate curves.

When it is desired to *separate the mechanical losses* (i.e. friction, windage, and gear loss, if any) from the electrical and magnetic losses (i.e. I^2R losses, "load" loss, and core loss) the set is run light—by supplying power to one machine at low voltage—and the input is measured over the range of speeds corresponding to the efficiency test. This input will then give the mechanical losses. The accuracy of the method, however, is not very high owing to the difficulty in obtaining steady readings of the input. A greater accuracy is possible by driving the set by a small motor and measuring the input to this machine.

Core Loss. The core loss (i.e. the iron loss in the armature core and the eddy-current losses (if any) in the armature conductors, armature flanges, etc.) may be determined by two methods; (i) by driving the traction motor by a smaller shunt motor and measuring the input to the latter when the former is (a) unexcited, (b) excited with various field currents, the speed being held

constant throughout; or (ii) by running the traction motor light, with separately excited field, and measuring the input to the armature at various excitations, the speed for each excitation being obtained from the speed-curve of the motor.

The second method, although not giving such accurate results as the first method, has the advantage that the tests can be carried out in a relatively short time. This method is employed when the conventional efficiency is to be calculated from running-light tests and resistance measurements (*see* Appendix).

The procedure is as follows—

The exciting field winding of the traction motor is separately excited and the armature is run light from a variable voltage supply. Readings are taken of the input to the armature at various values of the exciting current and speed (the speed for a given excitation being adjusted to that corresponding to this current on the speed curve). The input so obtained is equal to the core, friction, and windage losses.

To separate the core loss from the friction and windage loss the machine is run light—as a series motor—on a low voltage circuit, and the input to the armature is observed for speeds corresponding to those in the previous test. Since the excitation will be very low, the input to the armature may be taken as equivalent to the friction loss. Provided that the speeds have been correctly adjusted, the difference between the two tests will give the core loss without further calculations.

A typical set of readings and the core loss deduced therefrom is given in Table II, and the results are plotted in Fig. 68.

TABLE II

TYPICAL SET OF READINGS FOR CORE LOSS BY RUNNING-LIGHT METHOD
(Battery vehicle motor. Rated voltage, 60 V)

Running as a Separately-excited Motor					Running as a Series Motor				Core Loss (Watts)
Speed r.p.m.	Field Amperes	Armature Amperes	Armature Volts	Watts Input	Speed r.p.m.	Terminal Volts	Amperes Input	Watts Input	
1,240	200	20·8	48	1,000	1,240	6·5	9·6	62·4	937
1,330	175	20·1	49	984	1,330	7·1	9·6	68·2	916
1,440	150	19·0	50	950	1,440	7·7	9·6	74	876
1,630	125	16·6	52·5	874	1,630	8·7	9·6	83·6	790
1,890	100	12·85	55	708	1,890	9·9	9·65	95·6	613
2,000	92	11·5	55·5	638	2,000	10·6	9·7	103	535

The core loss so obtained corresponds to "no-load" conditions (i.e. undistorted main flux). When the motor is loaded the effects of armature reaction cause distortion of the main flux and result in higher core losses in the armature teeth, together with higher eddy-current losses in the armature conductors. These additional losses are called "stray" losses, and are of considerable importance in machines working with tapped fields in consequence of the field distortion being greater in these machines than in machines working with full

field. This matter has been investigated by Dr. F. W. Carter and is discussed at length in a paper on "Rating and Service Capacity of Traction Motors" (*Journ. I.E.E.*, Vol. 65, p. 994).

Thermal Characteristic. The thermal characteristic of a traction motor is a curve showing the time that the motor will carry various loads, at normal voltage, for a temperature rise of 95°C, the machine being at atmospheric temperature at the start. The characteristic is determined by carrying out heat runs at different loads and normal voltage.

The thermal characteristic is useful in showing how the temperature of a motor is affected by steady loads of definite duration. But the continuous rating of the motor obtained from this characteristic will differ from the

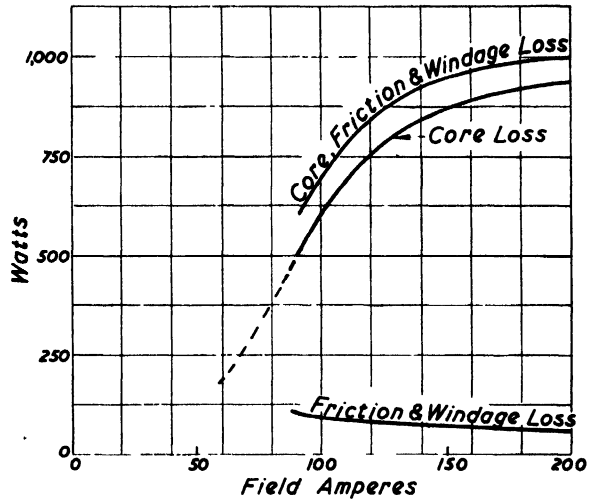


FIG. 68. RESULTS OF NO-LOAD CORE-LOSS TEST

continuous rating of the machine when operating under service conditions, on account of the distribution of losses not being the same in each case.

When the service on which a given motor has to operate is known, the approximate temperature rise of the machine can be obtained from a heat-run, on the testing stand, in which the losses in the motor are equivalent to, and distributed in the same ratio as, the average losses in service, the ventilation being, as far as practicable, the same as in service. The heat-run is continued until a constant temperature is reached. The temperature rise obtained by this method, however, is usually slightly higher than that obtained in service, on account of the better ventilation in the latter case.

The method of obtaining the voltage and current at which this test must be conducted is as follows. From the service speed-time curves the corresponding curves of the voltage and current for each motor are obtained. The mean voltage and r.m.s. current are then calculated over the whole period in which the motor is in service, and these values adopted for the heat-run. It is apparent that the losses in the motor during this test will have the same value as the average losses in service, and, moreover, the ratio of the distribution will be the same in each case.

When this test is run on a self-ventilated machine, the speed of the armature during the test must be equal to that corresponding to the average speed in service. The voltage corresponding to this speed may differ from the mean voltage in service, and in this case the test must be run with the total armature loss and the field loss equivalent to the average values obtained above, although the ratio of the armature I^2R loss to the core loss may not be the same in the two cases.

PART II

Factory Tests on Single-phase Traction Motors

The *commercial test* applied to a single-phase motor is similar to that applied to a direct-current motor. The motor is run for one hour at its rated load with normal voltage and frequency, the resistances and temperatures being

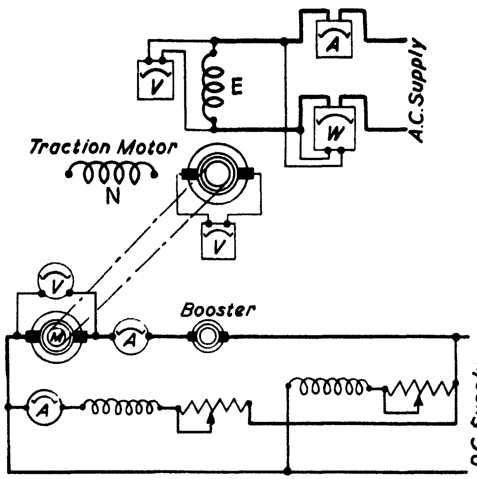


Fig. 69. Connexions for Core-loss Test on Compensated-series Motor

obtained in the usual manner. This test is followed by a speed test in each direction of rotation, a commutation test at various loads, including starting, and an insulation test. In addition, the impedance of the motor is measured at normal frequency with the armature stationary.

The loading-back method cannot be applied conveniently to single-phase motors, and in consequence the load must take the form either of a d.c. generator or a mechanical brake (e.g. the Froude water dynamometer).

In *efficiency tests* readings are obtained of the input (by wattmeter and ammeter), torque and speed over a range of loads at constant voltage and frequency. The readings are then corrected to a definite copper temperature and gear loss (if any) in a manner similar to that adopted for d.c. motors, and the efficiency and power factor calculated, while the torque and armature speed are converted into tractive-effort and train speed, to correspond to the operating conditions. The results are then plotted against input amperes (as abscissae), thus giving the characteristic curves of the motor.

Core Loss. In single-phase motors core losses occur in the field structure (or stator core) as well as in the armature core. The loss in the latter consists of two components, one due to the alternating flux and the other due to the rotation of the armature. The core loss in the stator (and the component of the armature core loss which is due to the alternating flux) is supplied by the exciting current, and can therefore be measured by a wattmeter in this circuit. The other component of the armature core loss (which is due to the rotation of the armature) is determined by measuring the power required to drive the armature. In these tests the brushes must be raised from the commutator, as otherwise the copper losses in the field and armature windings (resulting from the circulating currents in the coils short-circuited by the brushes) will be included with the core losses. A pair of pilot brushes with pointed contacts is fitted for the purpose of measuring the voltage induced in the armature and thereby enabling the flux to be calculated.

The *method of procedure* is as follows. The exciting winding is separately excited from a variable voltage supply of the correct frequency, and a wattmeter, an ammeter, and a voltmeter are connected in the circuit (Fig. 69).

The armature is driven at various speeds from a small shunt motor, the armature of which is connected to a variable voltage supply, while the field is separately excited at a constant current. An ammeter and a voltmeter are connected in the armature circuit of the driving motor.*

The brushes of the motor under test are removed, and a series of readings are then taken over a range of exciting currents (from zero to about 50 per cent overload) at four or five different speeds,† the speed being maintained constant for each set of readings. In this manner both components of the rotor core loss, together with the stator core loss, are determined at the same time. Provided that there are no circulating currents in the armature due to transformer action,‡ the wattmeter in the field circuit will measure the core loss in the stator and armature (due to the alternating flux) + the I^2R loss in the field winding. Similarly, the input to the driving motor, when corrected for the I^2R loss in its armature, will, in the absence of circulating currents, represent the core loss (due to the rotation of the armature) + the friction and constant losses in the set. The latter are, of course, given by the corrected readings corresponding to zero excitation on the machine under test.

In order to determine the losses (due to circulating currents) in the armature coils which are short-circuited by the brushes, the above tests are repeated with all the brushes in position. The wattmeter reading in this case includes the losses (due to transformer action) in these coils.

*Generally, the rating of the driving motor should be about 10 to 15 per cent of that of the machine under test. To obtain good results the following conditions should be fulfilled—

(i) Maximum current input to driving motor should not exceed 60 per cent of the full load current, (ii) input current when driving the machine under test unexcited should not exceed 20 to 25 per cent of the full load current.

† Or for each value of the exciting current the corresponding speed is obtained from the speed-curve and readings taken at this speed with and without excitation. In this manner the points on the characteristic core-loss curve are determined directly, but the results will be affected to a much greater extent by inaccuracies in individual readings than when the test is conducted in the manner detailed above. Moreover, the results only give the characteristic core-loss curve for one operating voltage.

‡ Circulating currents may be produced in multiple-circuit armature windings if they are unbalanced magnetically or electrically.

PART III

Service Tests

Service tests can be grouped into two classes, (i) those which are conducted under *actual* service conditions, involving runs of various lengths (corresponding to the distances between the stations) at a given mean schedule speed; and (ii) those which are conducted under *equivalent*, or average, service conditions, on a level test track, for the purpose of obtaining data of the motors.

The tests in class (i) are of the order of "official" tests, and are usually run to ascertain if a given equipment fulfils the manufacturers' guarantees; while those in class (ii) are of the nature of experimental tests, the object of which is the determination of data from which the "service-capacity" curves of the motors can be obtained. These curves show the mean schedule speeds at which a motor is capable of operating various services with a given temperature rise; the nature of the service being expressed by (*a*) the number of stops per mile, (*b*) the train weight per motor.

Now the heating due to the I^2R and core losses depends upon the magnitude and distribution of these losses, and will vary with the class of the service. Thus, in suburban service, the greater portion of the I^2R losses occurs during acceleration and speed-curve running; while the core loss has its greatest value at the end of the accelerating period, and will probably exceed the armature I^2R loss during the free-running period. On account of the large thermal capacity of the motor, the temperature of the various parts will not follow appreciably the fluctuations in the losses, and a steady temperature will be attained when the average rate of generation of heat is balanced by the rate at which the heat can be dissipated in radiation, convection, etc.

To *determine the service capacity curves of a motor* it must be operated under uniform service conditions (corresponding to a particular service) until the temperature has attained a constant value. Having decided the class of service (i.e. the schedule speed, stops per mile, and train weight per motor) an appropriate speed-time curve is drawn, from which, in combination with the characteristic curves of the motor, the accelerating current, the time during which power is "on," the coasting time, braking time, and duration of stop are obtained.

A car equipped with motors and loaded to the required weight per motor is then operated on level track to this speed-time curve, the runs being continued until the temperature of the motors becomes steady. Additional tests are made for other service conditions, the motors being at the atmospheric temperature at the start of each test. The line voltage in all tests is maintained, as far as practicable, at the normal value. Provided that the service conditions under which the tests are run have been selected to give the same limiting temperature rise in each case, the service capacity can be obtained directly from the tests.

The determination of a number of service-capacity curves in this manner would consume much time. But as service-capacity curves can be calculated from the service thermal characteristic of the motor, it is necessary to perform only sufficient tests to determine the latter.

The method by which the *service thermal characteristic* of the motor is determined is as follows. Records are obtained of the current and voltage by means of suitable graphic-recording instruments, while the temperatures of the field and frame are observed at frequent intervals, and the final temperatures of all parts are ascertained at the completion of each test.

The resistances of the field and armature windings are taken, if possible, during the lay-over periods.

The I^2R losses in the armature, brush contacts, and field windings are calculated.* The core loss corresponding to the various excitations and speeds is obtained from the core-loss curves, and the "stray" loss and other indeterminate losses causing heating are obtained from the segregated losses deduced from the stand efficiency test. In this manner the total losses in the motor

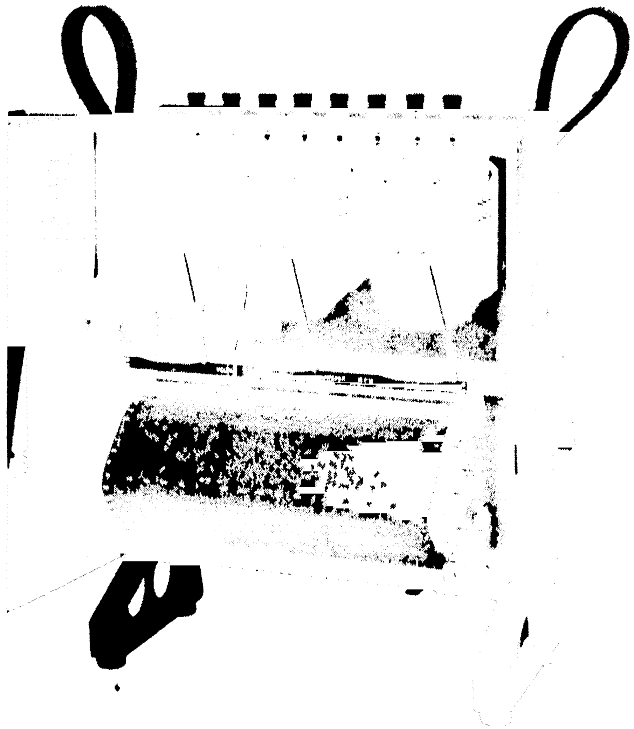


FIG. 70 GRAPHIC RECORDING INSTRUMENT (EVERSHED AND VIGNOLES)

are determined for a series of runs. From the average losses and the highest observable temperature rise the watts dissipated per degree rise of temperature is computed for each run and is plotted against the mean running speed. Hence from this curve (which usually approximates very closely to a straight line) the temperature rise for any service can be predetermined when the corresponding average losses and the mean running speed are known.

On "*official*" *service tests* the energy consumption, temperature rise of the motors, and schedule speed have to be determined. With d.c. equipments it

* In some cases the I^2R loss in the field coils has been determined directly, by means of a watt-hour meter connected in the field circuit. The average loss is then obtained by dividing the watt-hours registered during the test by the duration of the test (in hours). If the average resistance of the field coils is also determined, the square of the average current during the test can be readily obtained, which, when multiplied by the average armature resistance, will give the average armature I^2R loss.

is the practice to record the line voltage and the current input to one motor. With a.c. equipments, however, the current and power input, together with the line voltage, must be determined on the high-voltage side of the main transformer.

Graphic recording instruments for train testing must be designed to withstand the large amount of vibration incidental to train operation, and are usually distinguished from the commercial forms of recording instruments by the high torque and large damping of the moving element.

A typical instrument is illustrated in Fig. 70. This instrument gives records of five different quantities—current, voltage, speed, duration of brake applications, duration of stops—on one chart 12 in. wide. The speed record is obtained from the voltage of a magneto-generator coupled to one of the wheels, an adjustable resistor in the circuit providing the means whereby the chart record is given directly in miles per hour. The duration of the brake applications is recorded by a pen operated by an electromagnet, which is controlled by contacts on the brake cylinder. The duration of the stops is recorded by a second electromagnetic pen controlled by a push button operated by an observer.

CHAPTER VIII

THE CONTROL OF TRAMWAY, TROLLEYBUS AND BATTERY VEHICLE MOTORS

General Principles

Duty-cycle. In selecting a suitable method of control for an electric motor, or a group of motors, a knowledge of the duty-cycle is necessary. With tramway and suburban railway services the duty-cycle consists of (i) a starting period at high current input—of from 10 to 20 seconds' duration—(ii) a speed-curve-running period during which the acceleration and the current input diminish gradually, (iii) a coasting period, (iv) a braking period, (v) a brief period (from 10 to 20 seconds) of rest. A typical duty-cycle corresponding to suburban service is shown in Fig. 26.

The frequency varies from 15 to 20 duty-cycles per hour for suburban service up to about 60 duty-cycles per hour for tramway service.

As the average current input to each motor during the starting period is usually of the order of the rated current, a considerable loss of energy would occur if the motors were started with a series rheostat. But, since the operating conditions on tramways and suburban railways necessitate the use of multi-motor equipments—tramcars being equipped with two, and in certain cases four, similar motors, and motor-coach suburban trains being equipped with four or more similar motors—alternative methods of control are available by grouping the motors in series and parallel. These combinations may be utilized during starting because the torque of a series motor depends only upon the current input to the motor, and therefore with a multi-motor equipment two or more similar motors, when *connected in series*, will produce the same total torque as when they are connected in parallel, provided that the current per motor is the same in both cases.

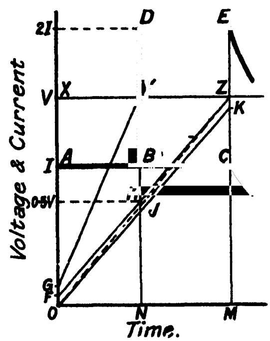
The series combination of a multi-motor equipment, however, can only be employed during the initial portion of the starting period, as the parallel combination is necessary to obtain full speed.

Therefore, with a two-motor equipment, the motors may be connected in series during the first portion of the starting period and in parallel for the remainder of the period. With a four-motor equipment three combinations of the motors are possible, (i) series, (ii) series-parallel, (iii) parallel. These methods of starting are called "series-parallel" and "double-series-parallel" control, respectively. They not only result in considerable saving in energy compared with the rheostatic method of starting, but also provide two and three efficient running speeds.

Energy Loss at Starting. First consider two similar motors to be started by the series-parallel method. Let the line voltage be constant and the motor current be maintained at a constant value, I, throughout the starting period.

The current-time and voltage-time diagram for the starting period is shown in Fig. 71, in which the motor current is represented by the horizontal line ABC drawn at ordinate I, the line current by the stepped line $ABDE$, and the line voltage by the horizontal line XYZ drawn at ordinate V. The voltage drop in one motor, due to the current I, is represented by OF, and the voltage drop

in the two motors when connected in series by OG ($= 2OF$). The starting period is represented by OM, of which the portions ON, NM correspond to the series and parallel connexions respectively of the motors. The point N is determined by the condition that the connexions must be changed from series to parallel when the voltage per motor is equal to one-half of the line voltage. Hence, if a horizontal line be drawn through the ordinate corresponding to half line voltage, the point of intersection, H, with the line ZF gives the abscissa required.



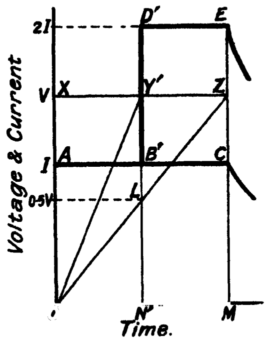


FIG. 71 FIG. 72

STARTING CONDITIONS FOR SERIES-PARALLEL CONTROL

The point, N, and the *durations of the series and parallel portions of the starting period*, are easily determined analytically. Thus, if T denotes the duration of the starting period, t_s, t_p the durations of the series and parallel portions respectively, V the line voltage, and v the voltage drop in each motor corresponding to the current I, then, from the similar triangles OJN, OKM, we have

$$\frac{t_s}{T} = \frac{JN}{KM} = \frac{\frac{1}{2}V - v}{V - v}$$

whence $$t_s = \tfrac{1}{2}T(V - 2v)/(V - v)$$

and $$t_p = T[1 - \tfrac{1}{2}(V - 2v)/(V - v)]$$

The variation of voltage at the terminals of the two motors during the series portion of the starting period is represented by GY, and the variation of voltage at the terminals of *each* motor throughout the whole starting period is represented by FZ. Hence the variation of the counter-e.m.f. of each motor throughout the starting period is represented by OK, ZK being equal to OF (which represents the voltage drop in one motor due to the current I).

Hence ordinates between XY and GY represent the voltage drop in the starting rheostat when the motors are connected in series. Similarly ordinates between YZ and HZ represent voltage drop in the starting rheostat when the motors are connected in parallel and the line current is $2I$.

Therefore the energy loss in the starting rheostats during series running is represented by the product of the *motor* current and the triangular area XGY. Similarly the energy loss in the rheostats during parallel running is represented by the product of the *line* current and the triangular area YHZ.

For the *ideal case*, when the voltage drop in the motors is negligible, the current-time and voltage-time diagram takes the form of Fig. 72, in which the counter-e.m.f. and terminal voltage per motor are both represented by the line OZ. The duration of the series portion (ON') of the starting period is now

equal to that of the parallel portion ($N'M$). The energy loss in the starting rheostats during series running is proportional to the triangular area XOY', and that during parallel running is proportional to twice the area $Y'LZ$, i.e. to the area XOY' (since $Y'Z = XY'$, and $Y'L = \frac{1}{2}OX$). Thus the energy loss during starting is divided equally between the series and parallel periods. The total energy loss during starting is proportional to the area of the rectangle $OXY'N'$.

Now the energy output from one motor during the whole starting period is represented by the product of the motor current and the triangular area OZM: it is, therefore, proportional to the area OZM. Hence the output from the two motors during the starting periods is proportional to the area of the rectangle $OXZM$. Therefore *the total loss of energy in the starting rheostats is equal to one-half of the energy output from the motors during the starting period.*

Energy Saving due to Series-parallel Control. Since the energy taken from the supply system = energy output from motors + losses in rheostats, therefore, with series-parallel control, the energy taken from supply system $= 1{\cdot}5 \times$ energy output from motors, and the overall efficiency during starting $= 1/1{\cdot}5 = 2/3$, or $66\frac{2}{3}$ per cent.

With rheostatic control throughout the whole of the starting period (i.e. with the motors connected permanently in parallel) the energy loss in the rheostats is proportional to twice the area OXZ, i.e. to the area of the rectangle $OXZM$, Fig. 72. Thus in this case the energy loss in the rheostats is equal to the energy output from the motors, and the overall efficiency is 50 per cent.

Series-parallel control, therefore, results in a saving of energy equal (in the ideal case) to one-half of the energy output of the (two) motors during the starting period.

Other important advantages of series-parallel control are—two speeds are available in the ratio of 1 : 2, approximately; the starting rheostats are smaller, lighter, and cheaper than those necessary for ordinary rheostatic control under similar conditions.

Energy Saving due to Double Series-parallel Control. When four similar motors are available, three combinations of the motors are possible, series, series-parallel, and parallel. Hence with ideal starting conditions, i.e. negligible voltage drop in the motors, constant current per motor, and constant line voltage, the durations of the series and series-parallel portions of the starting period are each equal to one-fourth of the whole starting period. It is easy to show that under these circumstances the energy loss in the starting rheostats for the series and series-parallel periods is equal to one-half of that for the parallel period, and that the energy loss during the parallel period is equal to one-fourth of the energy output from the motors during the whole starting period. Thus the overall efficiency $= 1/(1 + \frac{1}{4} + \frac{1}{8}) = \frac{8}{11}$, or $72{\cdot}73$ per cent.

When six similar motors are available and each motor is wound for one-half of the line voltage—as is the practice with 3,000-volt equipments—the three motor combinations are: (i) series, (ii) first series-parallel with two groups each consisting of three motors connected in series, (iii) second series-parallel with three groups each consisting of two motors connected in series. In this case, with ideal starting conditions, the durations of the three portions of the starting period are all equal, and the energy loss in the starting rheostats is equal to one-third of the energy output during the whole of the starting period. Therefore the overall efficiency in this case is $1/(1 + \frac{1}{3}) = \frac{3}{4}$, or 75 per cent.

Comparison of Rheostatic, Series-parallel, and Double Series-parallel Control. The comparison for ideal starting conditions is shown in the accompanying table, in which is also given the approximate ratio of speeds for the various combinations of the motors when the rheostats are cut out and the motor current has a given value.

System of Control	No. of Speeds Available	Approx. Ratio of Speeds	Rheostatic Losses as Percentage of Energy Output (Ideal Conditions)	Overall Efficiency during Starting (Ideal) Conditions Per cent
Rheostatic Parallel	1	—	100	50
Series-Parallel	2	1 : 2	50	66·6
Double series-parallel (4 motors)	3	1 : 2 : 4	37·5	72·7
Double series-parallel (6 motors)	3	1 : 2 : 3	33·3	75

Applications of Double Series-parallel Control. This system is employed on heavy locomotives which require a large number of rheostatic steps in order to obtain smooth starting. With suburban trains, however, the slight saving in energy of double series-parallel control compared with series-parallel control would be entirely offset by the additional cost, weight, and maintenance of the double series-parallel equipment.

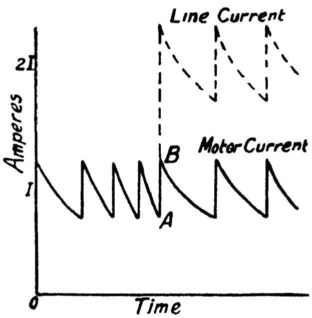


FIG. 73. VARIATION OF CURRENT DURING STARTING

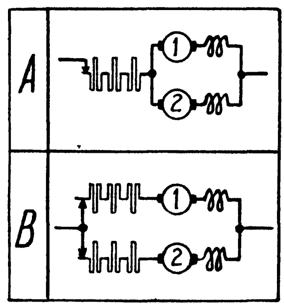


FIG. 74. ALTERNATIVE METHODS OF CONNECTING MOTORS AND RHEOSTATS

Practical Requirements. In the practical application of series-parallel and double series-parallel control the starting must be effected with a limited number of steps in the starting rheostats, in order to avoid excessive cost and complication of the controller. Therefore the motor current during starting cannot be maintained at an absolutely constant value, as hitherto assumed, but varies between definite limits as the sections of the rheostat are cut out. By suitable design of the rheostats and correct manipulation of the controller a given average value of the motor current can be maintained throughout the starting period, as indicated in Fig. 73. To obtain this result with series-parallel control, it is necessary that the change of motor combinations from series to parallel be made when all the rheostat has been cut out and the motor current has decreased to its lower limit—i.e. at point A, Fig. 73.

The change of motor combinations from series to parallel necessitates the re-insertion of a portion of the starting rheostat in order to limit the motor current to the prescribed value. The rheostats may be re-inserted according to either of the alternative methods shown in Fig. 74, both of which are extensively employed in practice. If the limits of the motor current during parallel running are to be the same as those during series running the value of resistance cut into the motor circuit when the motors are first connected in parallel must be such that the upper limit of current (i.e. point *B*, Fig. 73) is obtained. The methods of calculating the requisite values of resistance to satisfy these conditions are discussed later.

The starting conditions represented in Fig. 73 in which uniform variation of motor current occurs during the whole of the starting period, may be considered as ideal for the limited number of steps which can be employed in practice. These ideal conditions necessitate that the transition be effected without interrupting the current in either motor—a result which may be

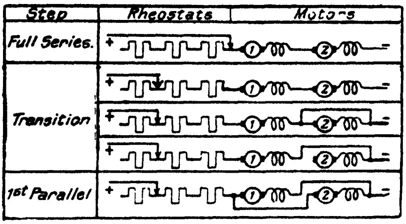


FIG. 75. CONNEXIONS FOR SHUNT TRANSITION

accomplished by the "bridge" method. In many cases, however, the complication entailed in the controller with this method of transition is not warranted, and a simpler method, called "shunt" transition—in which one motor only is supplied with current during transition—is employed. These methods will now be discussed in detail.

Shunt Transition. Three transition steps are necessary, the connexions being shown in Fig. 75. At the first step a portion of the starting rheostat is re-inserted, the motors being still connected in series. The value of this resistance should be such that the current input on the second transition step (when one of the motors is short-circuited) is approximately equal to the upper limit of current during the starting period (i.e. point *B*, Fig. 73). At the second step, one motor (No. 2) is short-circuited by connecting the negative terminal of motor No. 1 directly to the negative supply main. At the succeeding step the series connexion between the motors is opened, so that motor No. 2 is now ready to be connected in parallel with motor No. 1, thus completing the transition.

Bridge Transition. This is effected in a single step without opening either of the motor circuits. The starting rheostats, however, must be divided into two separate portions, so that when the motors are operating in parallel each motor may have a rheostat connected in series with it, as shown at *B*, Fig. 74. When the motors are operating in series the two portions of the starting rheostat must be connected *between* the motors in the manner shown at *A*,

Fig. 76, in which the numbered arrow-heads indicate the order of cutting out the sections of the rheostats. At the full-series combination of the motors, *B*, a connexion *X* is established directly between the motors, and the original series connexion is opened, together with the short-circuiting contacts across the sections of the rheostat. At the transition step, *C*, the free ends of the rheostats are connected to the supply, so that the motors, rheostats, and connexion *X* form a Wheatstone bridge network, as shown in the conventional diagram of Fig. 77. The connexion *X* now forms an equalizing connexion

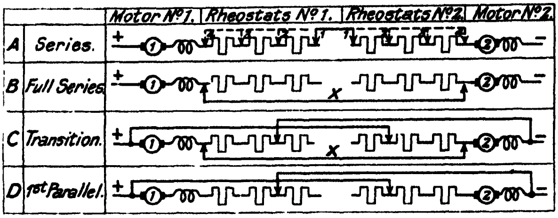


FIG. 76. CONNEXIONS FOR BRIDGE TRANSITION

between the two parallel paths—motor No. 1 and rheostat group No. 1, motor No. 2 and rheostat group No. 2—and the opening of this connexion gives the normal parallel combination, of the motors, as is shown at *D*, Fig. 76. If, when transition is effected, the values of the rheostats are such that the motor current increases to its upper limit, then the transition will have been effected under the ideal conditions shown in Fig. 73.

Bridge transition, therefore, enables the normal accelerating torque to be available from both motors throughout the whole of the starting period.

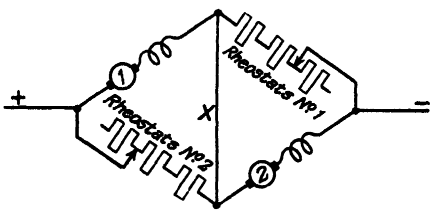


FIG. 77. WHEATSTONE NETWORK DIAGRAM TO ILLUSTRATE BRIDGE TRANSITION

Alternative Methods of Reducing Energy Loss at Starting. The rheostatic losses at starting may be eliminated by *variable voltage control,* the motors being supplied from a variable (adjustable) voltage generator or converter (metadyne or grid-controlled mercury-arc rectifier), or by means of a booster connected between the current collector and the motors.

The variable-voltage-generator method is employed on vehicles carrying their own power or converting plant, and its application is discussed in Chapter XVII. The metadyne converter and booster methods have been supplied to motor-coaches and locomotives, and have the advantage they they are readily adaptable to regenerative braking: their principles and applications are discussed in Chapter XII.

With a *single-motor equipment* (which is usually employed on trolleybuses) the rheostatic losses at starting may be reduced by shortening the rheostatic accelerating period and employing a high initial acceleration.

For example, in a particular trolleybus equipment an acceleration of 2·8 m.p.h.p.s. is obtained with a given armature current and field strength, the speed corresponding to normal voltage being 12·5 m.p.h. and the duration of the rheostatic acceleration being 4·46 sec. By an increase of field strength an acceleration of 3 m.p.h.p.s. can be obtained with the same armature current, the speed corresponding to normal voltage now being 11·25 m.p.h. and the rheostatic accelerating period 3·75 sec. The rheostatic losses are approximately 234 kW-sec and 197 kW-sec respectively, and the reduction due to the higher acceleration is 16 per cent.

SERIES-PARALLEL DRUM-TYPE CONTROLLERS

Before discussing actual tramcar controllers—the connexions of which are rather complicated—it may be desirable to determine the simplest form of drum controller which will fulfil the requirements, which are (i) to control the speed and direction of motion of the car, and the retardation during electric braking, (ii) to provide means for cutting-out, if necessary, a defective motor and operating the car with the remaining motor. In some cases provision must also be made for preventing a car running backwards when stopped on a steep rising gradient.

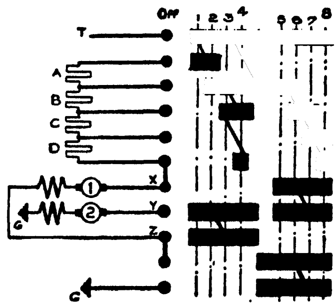


FIG. 78. CONNEXIONS FOR SERIES-PARALLEL CONTROLLER WITH SHUNT TRANSITION

Simple Series-parallel Controller. A drum-type controller consists essentially of a contact drum, or cylinder, carrying insulated and interconnected *segments*, which, when the drum is moved through certain angles, make contact with appropriate fixed contacts (called *fingers*) to which the motors, rheostats and supply mains are connected.

The connexions of the simplest form of series-parallel controller for two non-reversing motors and shunt transition are shown in Fig. 78, in which the contact drum is represented by its flat development.

The fingers are represented by the vertical row of large dots, and the segments by the black rectangles. The operating positions, called "notches," are indicated by the vertical chain-dotted lines (numbered 1 to 8) drawn through the segments. These lines coincide with the centre-line of the fingers in the operating positions. Thus on the first notch the top and second fingers are connected together, and the eighth and ninth fingers (from the top) are also connected. The motors are, therefore, in series and all the starting rheostats are in circuit. Sections of the rheostat are cut out on the succeeding notches, until, on the last series notch (No. 4) the motors are connected in series across the supply. Notch 4 is therefore called a *running position*. Similarly notch 8 is a running position. The other notches, 1, 2, 3, 5, 6, 7, are called *rheostatic positions*.

Reversing Drum. Although in practice provision must be made for operating the motors of a car in either direction of rotation from the same controller, operation in the reverse or backward direction of the car is not required under normal running conditions. Separate contact drums and operating handles are therefore employed for (*a*) starting and speed regulation, (*b*) controlling the direction of motion. These (separate) drums are called the "power drum" and

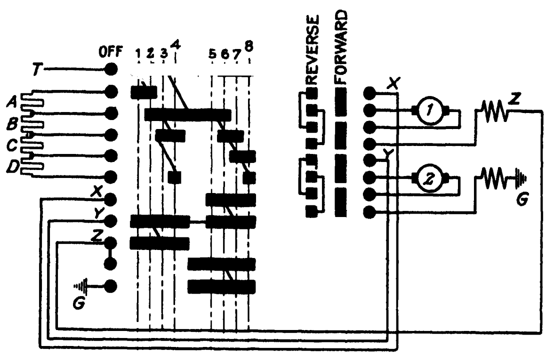


FIG. 79. CONNEXIONS FOR SERIES-PARALLEL CONTROLLER WITH REVERSING DRUM

the "reversing drum" respectively. The reversing drum consists of a small contact drum and fingers for reversing the armature connexions. This drum must be mechanically interlocked with the power drum so that it cannot be operated unless the latter is in the "off" position. Four fingers and two sets of interconnected contact segments are required per motor, the connexions being shown in Fig. 79.

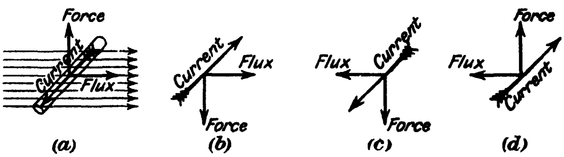


FIG. 80. DIAGRAMS SHOWING DIRECTION OF FORCE ON CURRENT-CARRYING CONDUCTOR IN MAGNETIC FIELD

Suppression of Arcing at Controller Contacts : Magnetic Blow-out. Provision must be made in all traction controllers for the suppression of arcing at the contacts, otherwise excessive burning will occur at the fingers and segments at which circuits are opened. In all modern controllers arcing is suppressed by means of a "magnetic blow-out." A powerful magnetic field is provided at the contacts where arcing occurs, and the conditions are so arranged that the arcs are rapidly extinguished or "blown out."

The operation of the magnetic blow-out depends upon the fundamental principles of electro-magnetism that a current-carrying conductor situated in a magnetic field—the direction of which is perpendicular to the axis of the conductor—is acted upon by a force tending to move the conductor out of the field, i.e. the direction of the force is perpendicular to both the axis of the conductor and the direction of the magnetic lines. Elementary diagrams showing the relationship between these quantities are given in Fig. 80.

The magnitude of the force is proportional to the product of current and magnetic flux density.

The magnetic field is produced by an electromagnet excited by the main current, and is directed by pole-pieces to the spaces in which arcing occurs.

With a *transverse* distribution, the flux is perpendicular to the plane containing the contacts and the arcs are blown in an *axial*, or longitudinal, direction.

With an *axial* (longitudinal) blow-out field, arcs forming between fingers and segments will be blown in a direction perpendicular to the axis of the contact drum, according to the directions of current and flux. Obviously, only the radially-outward direction is permissible.

In order to prevent the arcs striking adjacent fingers, fireproof barriers, called "arc deflectors," are inserted horizontally between the fingers and segments.

Electric Braking. Since dynamo-electric machines are reversible in their functions, the motors on a car can be operated as self-excited electric generators —provided that certain conditions are satisfied—when driven by the momentum of the car. When thus driven as generators, the machines are loaded by rheostats, and the energy which is dissipated in the latter, as well as that necessary to supply the losses in the generators, is derived from the kinetic energy of the moving car. In some cases the exciting coils of magnetic track brakes are connected in series with the loading rheostats, to provide additional braking effort

Conditions Governing Electric Braking. To operate the series-wound car-motors as self-excited series generators when driven by the momentum of the car, two conditions must be satisfied—

1. The armature and field connexions must be reversed relatively to each other.
2. A rheostatic load, the maximum resistance of which does not exceed a pre-determined value, must be connected across the terminals of the machines.

The reversal of connexions is necessary because, under normal braking conditions, the direction of rotation is the same as that when the car is running under power. Hence, due to the series excitation of the field winding, a reversal of the armature relatively to the field winding, or vice versa, is required in order that the machine may build-up as a generator when it is connected to a load.

The second condition, in which the maximum value of the loading resistance is specified, arises from an inherent characteristic of series generators, i.e. that a generator of this type will not excite if the resistance of the load connected to its terminals exceeds a critical value.

Connexions of Motors for Electric Braking. With two-motor equipments, the motors are connected in parallel for electric braking as the series connexion of the machines would produce an excessive voltage across the loading rheostats, thereby necessitating not only special insulation but rheostats of considerably higher resistance than those required for starting purposes. With the parallel connexion of the motors, however, the starting rheostats, with one or two additional sections, may be utilized as loading rheostats. But in this case precautions have to be taken to ensure stable operation of the generators.

Stability in the operation of two parallel-connected series generators is

obtained either by equalizing the exciting currents, e.g. by connecting the field windings in parallel, as in Fig. 81 (*a*), or by cross excitation, as in Fig. 81 (*b*).

These schemes, however, possess an important difference under abnormal conditions. For instance, if the direction of rotation of the generators is

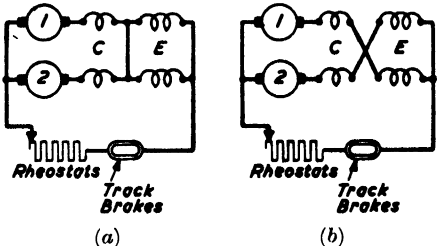


FIG. 81. METHODS OF CONNECTING MOTORS FOR ELECTRIC BRAKING

reversed (due to, say, a run-back) and the armature and field connexions are the same as for normal braking conditions, no braking effect can be obtained with the connexions of Fig. 81 (*a*), as the machines will fail to excite. On the other hand, an emergency braking effect will be obtained with the connexions

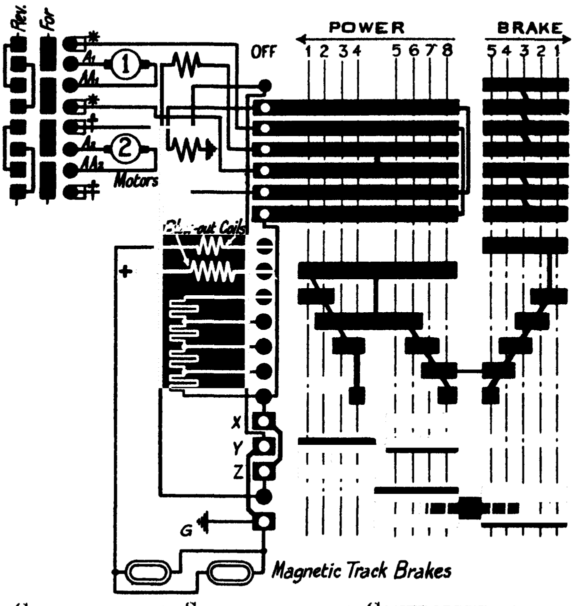


FIG. 82. CONNEXIONS OF SERIES-PARALLEL CONTROLLER, ARRANGED FOR ELECTRIC BRAKING, WITH RUN-BACK PREVENTER AND MOTOR CUT-OUTS

To cut out motor No. 1, raise fingers marked *
To cut out motor No. 2, raise fingers marked †.

of Fig. 81 (*b*), as the machines will build-up in series and will be short-circuited upon themselves. Another difficulty when using the equalized-field connexion is the possibility, with incorrect settings of the brushes or commutating poles, of the machines building up in series on their commutating-pole excitation, in which case very little current may pass in the external circuit although the armature currents may be large.

Series-parallel Controller Arranged for Electric Braking. In controllers of this type, the control of the motors during starting and braking is effected by a single operating handle; starting and speed regulation being effected by moving the handle to one side of the "off" position (in, say, a clockwise direction), and electric braking by moving the handle to the other side of the "off" position (in, say, a counter-clockwise direction). A separate drum and handle is provided for controlling the direction of motion of the car.

Fig. 82 shows how the controller of Fig. 79 is modified to obtain braking with the equalized-field connexion. The reversal of connexions between

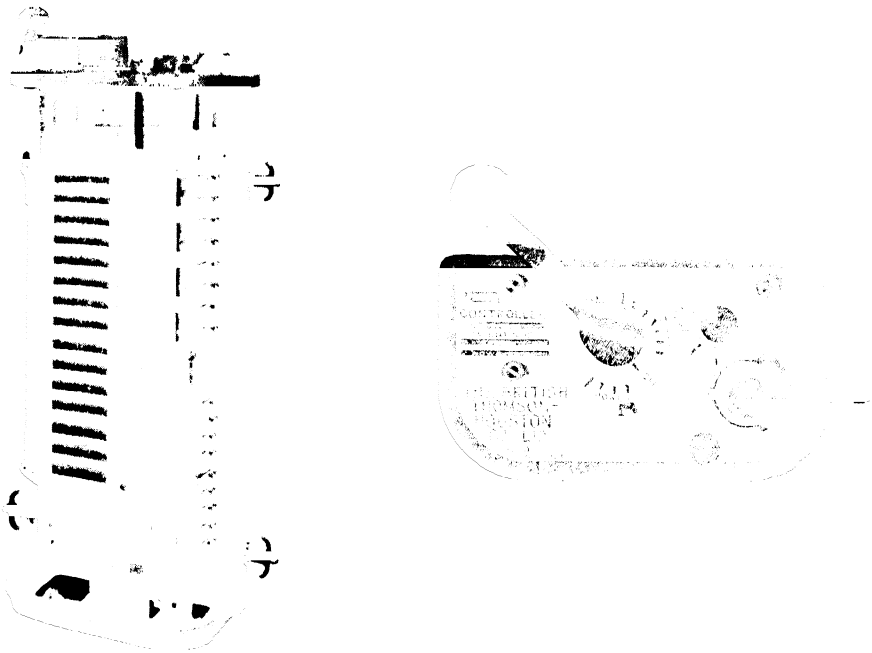


FIG. 83. B.T.-H. DRUM-TYPE (*B* 510) TRAMCAR CONTROLLER
Maley and Taunton air-brake valve mounted under the cap-plate.

"power" and "brake" is effected by an auxiliary drum and additional fingers. This diagram also shows the simplest device for cutting-out a defective motor (four fingers with locking-off cams), which, however, is not now used.

Run-back Preventer. A car stopped on a steep rising gradient may be prevented from running backwards by short-circuiting one motor without reversal of connexions. Any backward motion therefore causes the machine to excite as a short-circuited generator and so check the car's progress.*

Fig. 82 shows one method of adapting the controller for short-circuiting both motors in the "off" position. The "power" segments of the auxiliary drum are extended and additional segments are provided on the main drum.

* An electric brake, depending for its action on the motion of the car, cannot "hold" the car on an incline, although it can prevent the car from running away. Mechanical brakes are necessary for holding a car on an incline.

In cases where the motors are cross-connected for braking, as in Fig. 81(*b*), the braking positions of the controller are effective for both directions of motion of the car, and may, therefore, be used for preventing a run-back.

Example of Hand-operated Tramcar Controller

B 510 Controller of British Thomson-Houston Co. This controller is designed for motors having individual ratings up to 45 h.p. at 600 volts. Fig. 83 shows a controller with cover removed and arc deflectors closed; it is fitted with an air-brake valve which is operated by a cam when the driving handle is moved to the braking positions.

The fingers, Fig. 84, are of the pivoted type with renewable tips. The contact pressure is obtained from spiral compression springs which are adjustable, and flexible copper shunts connect each finger to its terminal base, so that no current is carried by either the hinge or the compression spring.

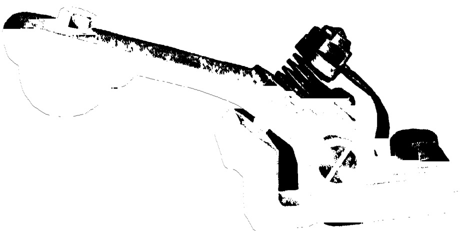


Fig. 84. B.T.-H. Controller Finger

The segments are fitted to split "body castings," which are clamped to a mica-insulated square shaft. Renewable tips are fitted to those segments subjected to arcing.

The magnetic blow-out is of the "individual-finger" type, a blow-out coil being fitted to the finger bases of each of the main fingers. Iron plates are moulded in the arc deflectors to direct the flux to the contact tips of the fingers. This type of blow-out closely resembles that employed with contactor-type controllers.

The cut-out switch is of the drum type: it is located immediately below the reversing drum, and its operating spindle extends through the cap-plate as shown in Fig. 83. The cut-out spindle is operated by the reversing handle, which must be removed from the spindle of the reversing drum for this purpose. The latter operation is possible only when the reversing drum is in the "off" position (i.e. when the main drum is "off," as an interlocking device between the drums ensures that the reversing drum can be moved only when the main drum is "off").

The cut-out switch has three positions, namely, both motors in circuit, No. 1 motor cut out, No. 2 motor cut out. The connexions of its nine fingers and three groups of segments are shown in the simplified diagram in Fig. 85. When this switch is in either of the "cut-out" positions an interlocking device prevents the main drum being moved beyond the "full series" position.

A simplified diagram of connexions is given in Fig. 85, in which the fingers and segments of the reversing drum are omitted, but the positions of the fingers in the armature circuits are indicated. Transition is by the shunt method (Fig. 75), and the motors are cross-connected for braking (Fig. 81 (*b*).) When

either motor is cut out, electric braking can be obtained with the remaining motor, but in this case the position of the reversing handle must always correspond to the actual direction of motion of the car.

Tramcar Remote Control (power-operated) Systems and Control Gear

Individual Contactor Control System. The necessity for high acceleration in modern tramways, together with the provision of ample passenger accommodation in the cars, requires four-motor equipments with ratings aggregating

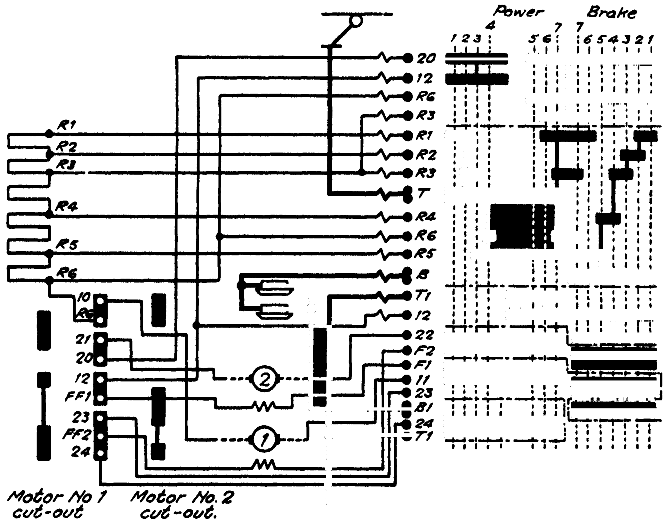


Fig. 85. Connexions of B.T.-H. Drum-type Tramcar Controller
Dotted lines represent positions of fingers of reversing drum.

180 to 260 h.p. For the control of such equipments power-operated contactors are preferable to hand-operated drum-type controllers, and systems employing both electro-pneumatic and electromagnetic types of contactors have been developed. These systems are simplified forms of multiple-unit equipments for motor-coach trains which are considered in detail in Chapter IX.

Essentially the control equipment consists of (i) a group of about 9 contactors for the main circuits and starting rheostats, (ii) a reverser, (iii) a change-over switch for "power" and "brake," (iv) a master controller. Items (i), (ii), (iii) may be either assembled on a framework, as shown in Fig. 86, for installing in the car body, or arranged in one or two cases for mounting on the underframe of the car.

Electro-pneumatic Contactor. This consists of a fixed contact, *A*, Fig. 87A, with a blow-out coil, *B*, attached; and a moving contact, *C*, actuated by a single-acting air cylinder, *D*, the air supply to which is controlled by an electromagnetic valve, *E*, shown in detail in Fig. 87B.

Admission of air to the cylinder causes the contacts to close, and a spring above the piston to be compressed: release of air causes the contacts to open.

Electro-magnetic Reverser. This is of the drum type and is actuated by two single-acting opposed air cylinders, a rack on the common piston rod engaging a pinion on the shaft carrying the contact drum. Each cylinder has its own control valve, and admission of air to either cylinder sets the contact drum in one position or the other.

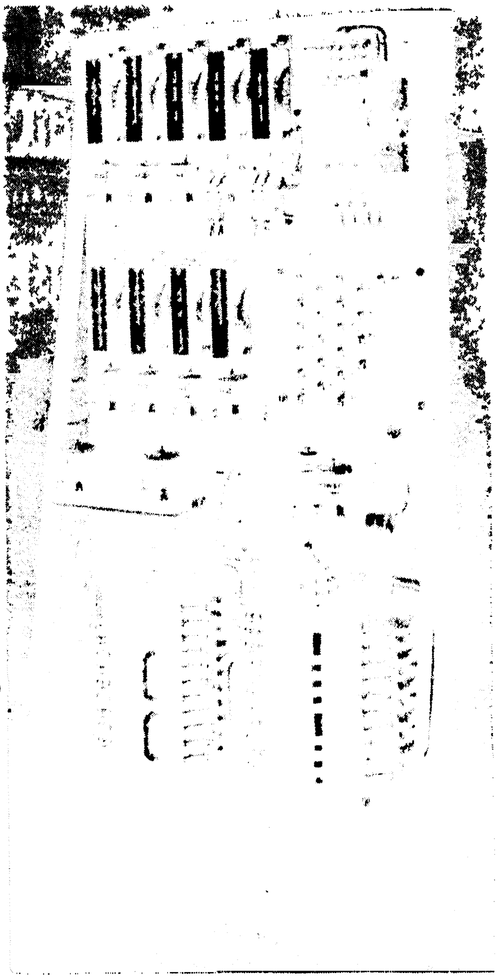


FIG. 86. METROPOLITAN-VICKERS CAB-MOUNTED ELECTRO PNEUMATIC CONTROL GEAR FOR TRAMCAR

(a) Contactors, (b) Reverser, (c) Power/brake change-over switch.

The power and brake change-over switch is of similar construction.

Master Controller. As the solenoids of the valve magnets require only a few watts for their operation a low voltage (obtained from a storage battery) is employed for the control circuit. The master controller therefore is of simple

construction as only small currents have to be controlled and no magnetic blow-out is necessary.

Main-circuit Connexions. These are shown in Fig. 88, which refers to a

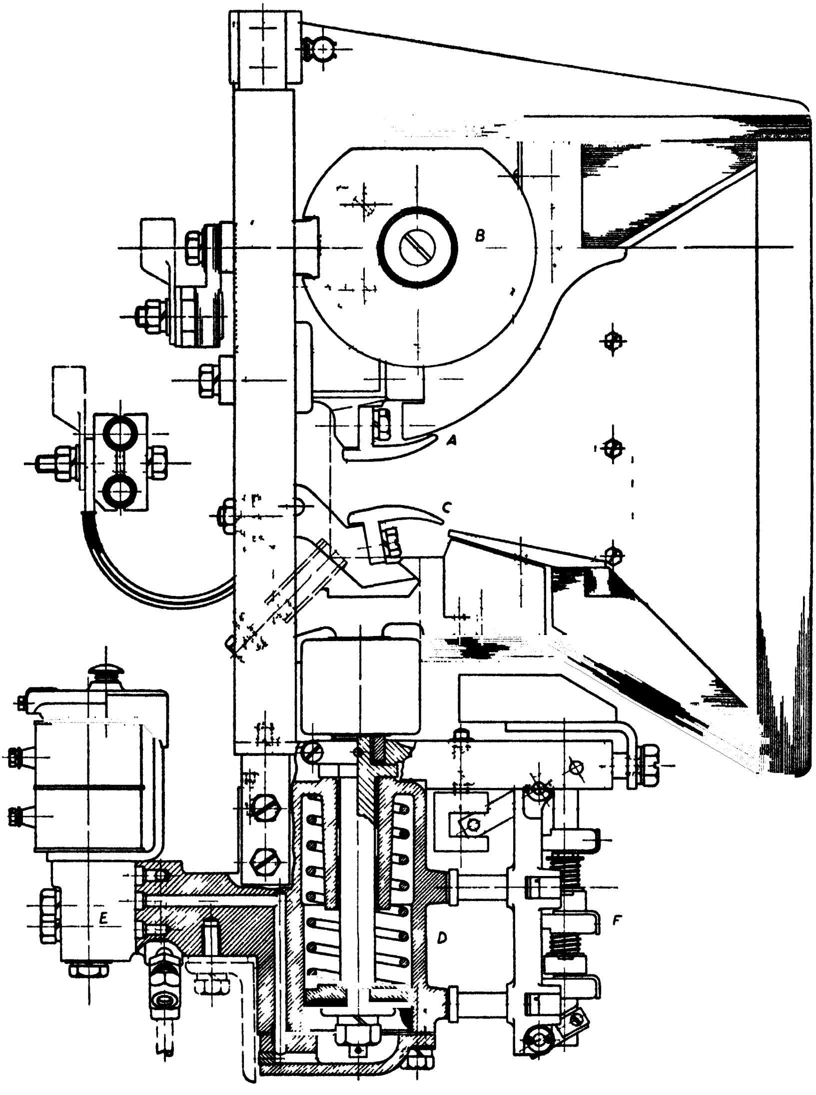


FIG. 87A B T H. ELECTRO PNEUMATIC CONTACTOR FOR RAILWAY SERVICE

four-motor equipment with magnetic track brakes. Provision is made—by additional operating positions of the power/brake change-over switch—for operation with either pair of motors cut out.

The sequence chart shows the contactors which are closed on each operating position of the master controller, and the schematic diagrams show the general

scheme of the circuit connexions for both power and brake operation of the motors.

An alternative system of control provides for automatic acceleration with bridge transition, the scheme being similar to that employed on motor-coach trains and described in Chapter IX.

Multi-notch Control System. This system was originally developed in America for the Presidents' Conference Committee (P.C.C.) type of street railway car, the object being to obtain high and smooth acceleration and rheostatic braking within the limits of adhesion. These requirements called for a practically "stepless" control system. Two American designs were evolved, one having almost 100 rheostatic steps, whilst the other has considerably more.

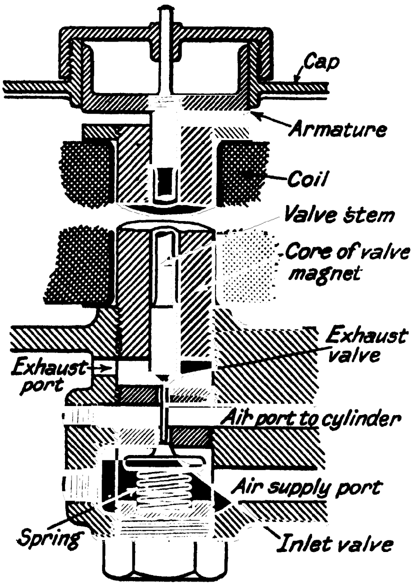


FIG. 87B. DETAIL OF ELECTROMAGNETIC VALVE

In this country an equipment fulfilling the same requirements and following similar principles has been developed by Messrs. Allen West & Co. in conjunction with Messrs. Crompton Parkinson.*

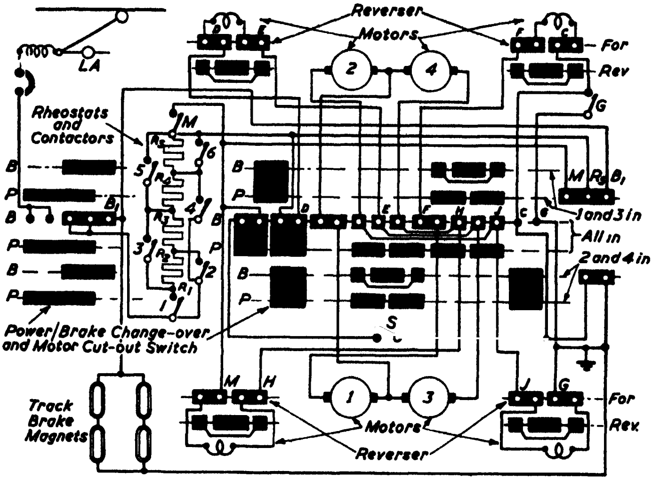
Contactors. In this design the large number of rheostatic steps is obtained by means of simple, coil-less, magnetic-operated, contactor elements arranged in a circle with the operating mechanism (two diametrically opposite permanent magnets) in the centre. Fig. 89 shows the complete group of rheostatic contactor elements combined with the rheostat, the whole assembly being called the "accelerator." The continuous sections of the strip-type rheostat, *A*, are clamped to the supporting steel pins which project radially from the moving-contact portion of each contactor element, and which thus constitute tappings. The circular assembly of contactor elements with their associated rheostat are arranged electrically in two halves, the moving contacts making contact with a common circular bus-ring. The current enters one half of the rheostat and passes to the other half by a diametrically opposite pair of contactor elements making contact on the common bus-ring.

Each moving contact, *B*, is actuated by a pivoted lever carrying an iron armature *C*. The moving contact is mounted on a leaf spring which biases both the moving contact itself and its actuating lever to the "open" position relative to the common bus-ring.

The two permanent magnets, *D*, are fixed at opposite ends of arms fitted to a sleeve which is mounted on a central shaft, the sleeve being driven through gearing by a small pilot motor, *M*. The rotation of the magnets closes in sequence diametrically opposite contactor elements through the attraction of

* This equipment is designated by the trade name "Vambac," which is an abbreviation for "Variable automatic multi-notch braking and acceleration control."

the armatures on their actuating levers. The latter, however, do not touch the magnets and do not impose any frictional drag on the pilot motor. The speed of the pilot motor is controlled automatically so as to maintain the current in the motors approximately constant at the value to give the required acceleration and braking, corresponding to the position of the master controller.



Sequence of Contactors										
	Step	S	M	G	1	2	3	4	5	6
Series	1	●			●					
	2	●			●	●				
	3	●			●	●	●			
	4	●				●	●	●		
	5	●				●	●		●	
	6	●				●	●	●	●	
	7	●			●	●	●	●	●	●
	T	●		●		●			●	
	T		●	●		●			●	
Parallel	8		●	●		●			●	
	9		●	●		●	●		●	
	10		●	●				●	●	
	11		●	●		●	●	●	●	
	12		●	●	●	●	●	●	●	●
Brake	1		●	●	●	●				
	2		●	●	●	●	●			
	3		●	●		●	●	●		
	4		●	●		●	●		●	
	5		●	●			●	●	●	
	6		●	●		●	●	●	●	
	7		●	●	●	●	●	●	●	

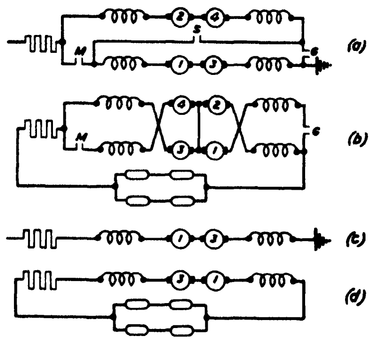


FIG. 88. MAIN-CIRCUIT CONNEXIONS FOR 4-MOTOR EQUIPMENT (250/275-V MOTORS) (METROPOLITAN-VICKERS)

The schematic diagrams show the circuits for (a) power, all motors; (b) braking, all motors; (c) power, two motors only; (d) brake, two motors only.

Additional contactors of the electro-magnetic type, mounted on a panel, are employed for the main power and rheostatic braking circuits, for field shunting, and for cutting out additional rheostats (which provide for slow running or "crawling").

Main Circuit Connexions. Fig. 90 shows the scheme of connexions. Two motors are shown for simplicity, but in practice two pairs of 275-V motors are used, the motors of each pair being permanently connected in series. Series-parallel control is not employed. Slow running speeds are obtained with only contactors L and M_1 closed, the accelerator, A, remaining in its initial

position. Automatic acceleration is obtained with contactors, L, M_1, C_1, C_2 closed, the accelerator moving at a controlled rate to its "full speed" position,

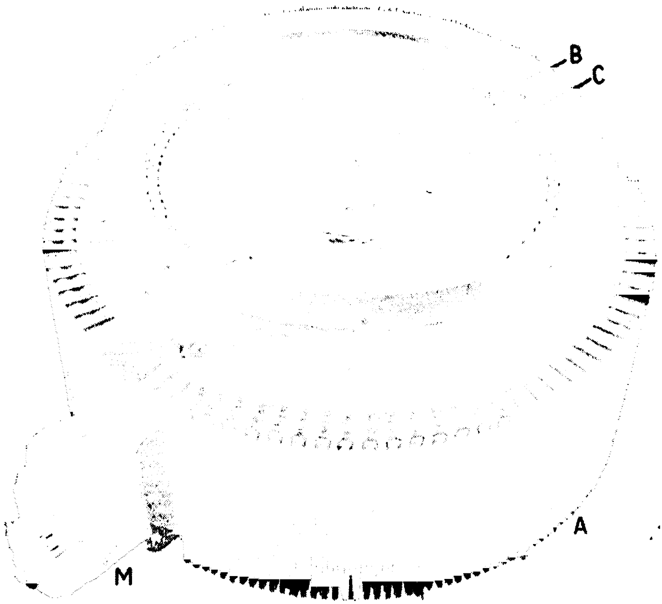


FIG. 89. COMPLETE GROUP OF RHEOSTATIC ELEMENTS AND CONTACTORS (ALLEN-WEST)

at which point M_2 closes to short circuit the accelerator, and F_1 and F_2 close to weaken the motor fields.

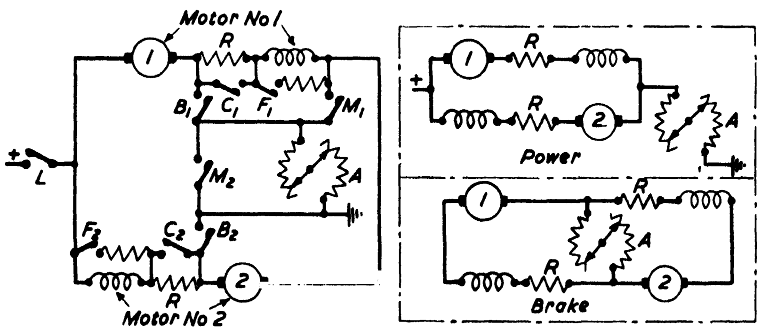


FIG. 90. SCHEME OF CONNEXIONS FOR "VAMBAC" EQUIPMENT (ALLEN-WEST)

Rheostatic braking is obtained with the crossed-field connexion by closing contactors B_1 and B_2. By arranging the circuits in the manner shown in Fig. 90 no reversal of the motor connexions is necessary when changing from "power" to "brake."

The braking torque is held controlled by the movement of the accelerator which gradually cuts the rheostat out of circuit.

Pilot Motor. The field of this motor is separately excited from a low-voltage (36-V) battery whilst the armature is supplied at variable voltage by means of a reversing potentiometer rheostat also connected to the battery. The drive from the pilot motor, with its gear box and the rotating magnets of the accelerator, is taken up by an electromagnetic clutch, the coil of which is controlled by auxiliary contacts on the short-circuiting contactor M_2, such that once the accelerator has cut out all the rheostat and is short-circuited by

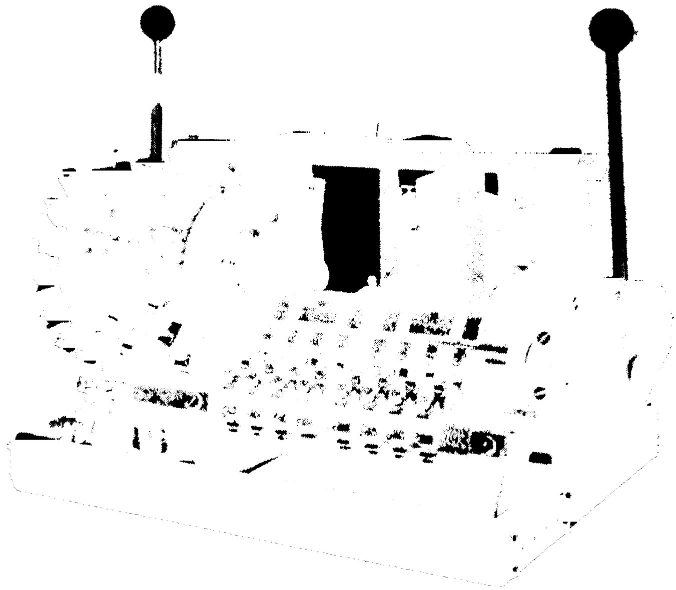


FIG. 91. MASTER CONTROLLER (COVER REMOVED) FOR "VAMBAC" CONTROL (ALLEN WEST)

the closing of contactor, M_2, the clutch is de-energized and the rotating magnet-arm is spring-returned to its initial position. By this means the accelerator, having fully accelerated the car, is always ready for a rheostatic brake application, and similarly, having brought the car to standstill by rheostatic braking, there is no delay if an immediate re-application of power is required.

Master Controller. This is usually combined with the reverser, as shown in Fig. 91, and arranged for location under the driver's seat.

The operating handle, which has a central "off" position, moves forward for acceleration and backward for rheostatic braking. The degree of movement of the handle from the "off" position, besides closing the appropriate contactors for "power" or "brake," determines the acceleration and the rate of braking. This is effected by setting, through spring tension, the position of the contact arm of the pilot-motor potentiometer (which is a miniature permanent-magnet-operated rheostat). The force exerted by the spring is opposed by the pull due to a control solenoid which carries a current proportional to that in the main motors. When the accelerator advances from its initial position the current

in the main motors increases, and proportionately increases the pull of the solenoid until a state of balance exists on the potentiometer-arm between the spring tension due to the position of the master controller handle and the pull due to the control solenoid, thus holding the speed of the accelerator to give the acceleration or rate of braking desired.

Trolleybus Control Systems and Controllers

Control Requirements. Trolleybuses are usually equipped with a single motor (series or compound) which is started on the simple rheostatic system. To reduce the losses in the rheostats they are cut out at a low vehicle speed (about 12 m.p.h.), and therefore field weakening is necessary to obtain normal service speeds.

Electric braking (rheostatic) is usually employed in order to reduce the wear of the brake shoes and drums, the final retardation being effected by mechanical brakes, operated usually by compressed air. Regenerative braking, with compound-motor equipments, is now confined to such hilly routes on which the advantages of recuperation outweigh the difficulties in the operation of the sub-stations and distribution system.

The *control system* must therefore provide for—(i) rheostatic starting, (ii) field weakening, (iii) electric braking, (iv) mechanical braking. These operations must be effected by the driver's feet (as his hands are required for steering and applications of the hand brake) and, following the practice in mechanically-propelled vehicles with foot control, two pedals are provided, one for accelerating (i.e. starting and speed control), and the other for braking (electric and mechanical). The pedals must require only a light pressure for actuating the controls, and therefore a remote-control system is necessary, with power-operated contactors for the main (heavy-current) circuits and pedal-operated master controllers (power and brake) for the control (light-current circuits). As the vehicle is always driven in one direction (reversing being only required for manoeuvring) the reversing switch is of the hand-operated main-current type.

In addition provision is usually made, on vehicles for service in Great Britain, for *emergency operation* from the lighting battery, the ampere-hour capacity of which is increased for this purpose. The battery is arranged in two sections (each either 24 or 36 V) which are normally connected in parallel (for lighting and charging) but may be connected in series when emergency propulsion is required. A change-over switch is therefore necessary together with a contactor (controlled by a foot-operated switch) for connecting the battery directly to the motor (which operates on its series field winding without starting rheostats).

Control Schemes. The control system for a *non-regenerative* compound motor provides for a total of ten steps for motor operation [i.e. 6 starting (rheostatic), and 4 running (including 3 weak field steps)] and two steps of rheostatic braking.

For motor operation the shunt field winding is cut out on the first weak-field step and diverter resistors are connected across the series field winding on the succeeding steps.

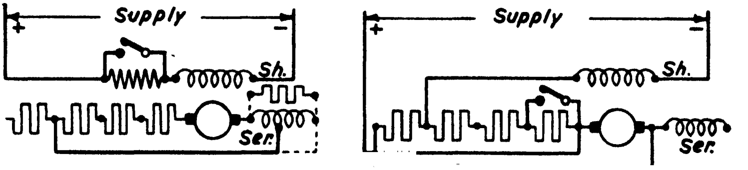
For rheostatic braking either of the schemes shown in Fig. 92 are employed, each of which gives a similar braking (speed/tractive-effort) characteristic

with a definite limiting torque (*see* Fig. 32). In both cases the shunt-field winding is separately excited from the supply system and the armature is loaded on rheostats. De-compounding in one case is obtained by including a portion of the series field winding in the load circuit, and in the other case by including in the shunt field circuit a portion of the voltage drop in the load rheostats in such a direction as to oppose the supply voltage.

For a *regenerative* (compound) motor—in which the shunt field ampere-turns are much greater than those in a non-regenerative motor—the control system would provide for four weak-field steps (3 with weakened shunt field and one with the series field alone).

As regenerative braking ceases at a speed of about 13 m.p.h. two steps of rheostatic braking are provided, the machine operating as a differentially-compounded generator.

The control system for a *series* motor would provide for 5 starting (rheostatic)



(*a*) (*b*)

FIG. 92. CONNEXIONS FOR RHEOSTATIC BRAKING

(*a*) Contra-field, or differentially compounded, method; (*b*) stabilized shunt-field method. Two stages of braking are obtained with the contactor (i) open, and (ii) closed. The dotted connexions in (*a*) show an alternative (shunted-field) method to the tapped series field winding.

steps and 4 running steps (3 with weakened field). Rheostatic braking is not usually employed as misuse of the electric brake may cause over-stressing and damage to the mechanical transmission gear.

Equipment for Starting and Speed Control. The contactors are of the electromagnetic type, the operating coils being usually wound for the line voltage (525 V), but in some cases low-voltage operation from the lighting battery is employed. A typical contactor is shown in Fig. 93. The hinged-armature method of actuating the moving contact is employed as this form of construction is simpler than that with a plunger.

The *master controller* has two sets of contacts (one for accelerating and one for rheostatic braking) which are of the silver-tipped butt type. The moving contacts are mounted on rocking levers which are linked to the respective pedals. A typical controller is shown in Fig. 94.

The *reverser* is of the drum type and is hand operated: it is located in the same case as the master controller. When provision is to be made for battery operation additional contacts and operating positions (forward and reverse) are necessary. These additional contacts may also include those for changing the connexions of the battery (*see* Fig. 96), or alternatively a separate change-over switch (parallel-series) may be employed.

The master controller shown in Fig. 94 is intended for location under the driver's seat and the contactors are mounted on a panel for location either in the cab or on the chassis. A more compact arrangement is shown in Fig. 95, which enables all the control-circuit wiring to be completed at the factory.

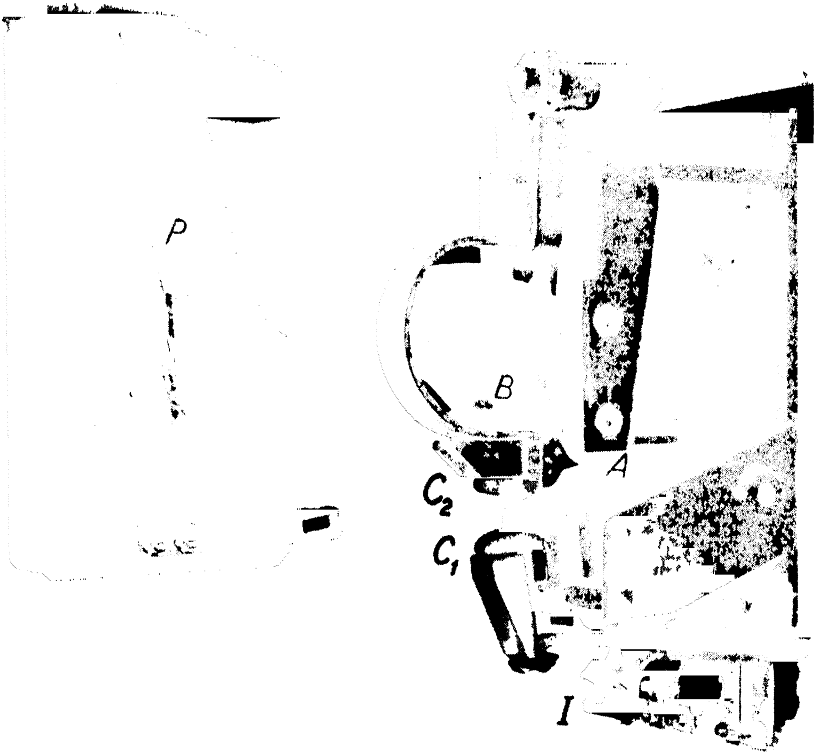


Fig. 93. Trolleybus Contactor (English Electric)

Arc chute detached to show contacts etc. (A) hinged armature. (B) blow-out coil. (C_1, C_2) contacts. (P) pole-piece. (I) interlocking contacts.

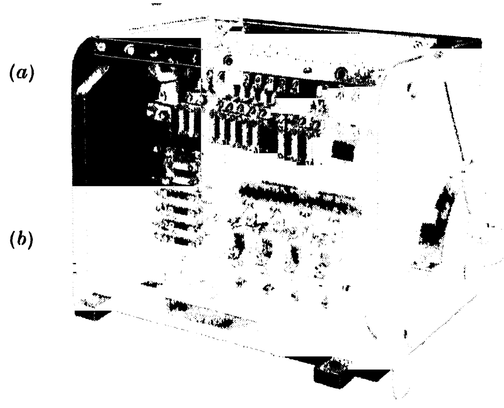


Fig. 94. Trolleybus Master Controller (B.T.-H.)

(a) Contacts for power contactors, (b) contacts for "brake" contactors, (c) reversing drum.

The reversing handle is operated by the driver's left hand (for a right-hand-drive vehicle) and the linkwork from the pedals is connected to the levers at the bottom of the master controller.

Connexions. Fig. 96 shows the scheme of connexions of the main and control circuits for a compound-motor non-regenerative equipment with stabilized rheostatic braking and battery manoeuvring. The master controller is of the type shown in Fig. 94, and the contactors are of the electromagnetic type with the operating coils wound for the line voltage.

The auxiliary circuits (in addition to the control circuit) supplied at line voltage include (i) the visual dewirement indicator, (ii) the motor driving the

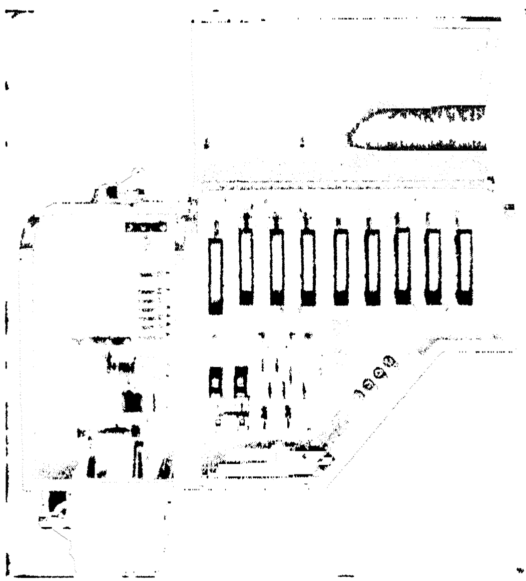
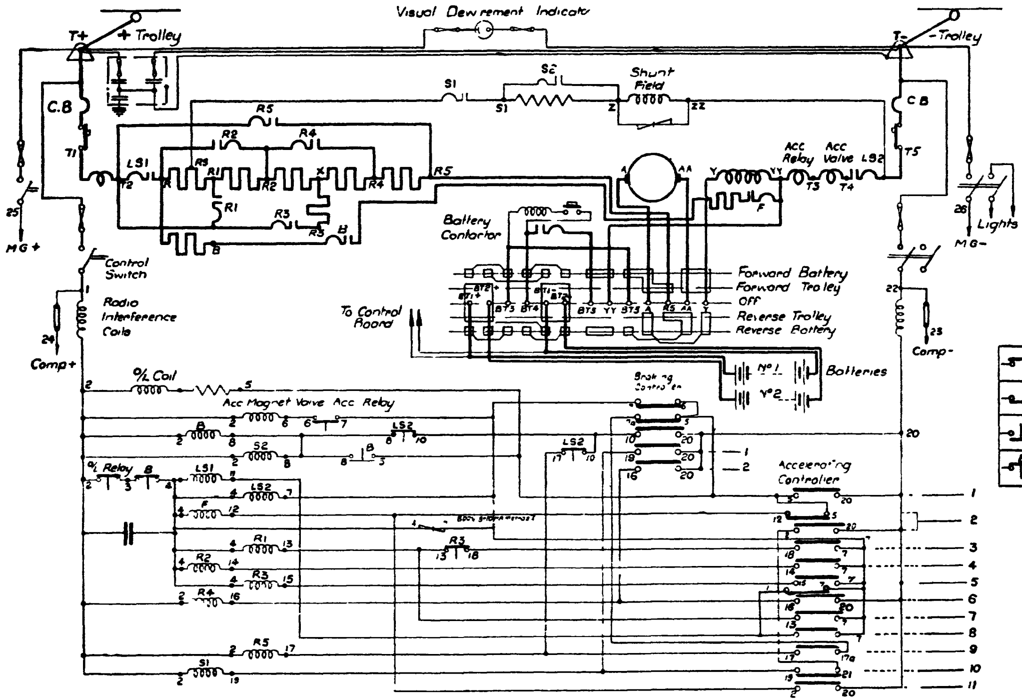


FIG. 95. CAB-MOUNTED CONTROL UNIT (B.T. H.)

air compressor for the air brake, (iii) the motor of the motor-generator set for low-voltage lighting (*see* Chapter XIII) and battery charging.

Radio interference due to arcs at the contacts of the master controller is suppressed by coils connected in the supply leads to the controller, and radio interference due to sparking at the trolley collectors is suppressed by a combination of capacitors (condensers).

Automatic (current-limit) Control of Accelerating Period. In Great Britain and many other countries the driving technique of a trolleybus is similar (except for gear changing) to that of an internal-combustion-engined bus, i.e. the speed and acceleration depend on the amount of depression of the "accelerating" pedal. With a trolleybus a rapid movement of this pedal would cause overloading of the motor and the opening of the circuit breaker. Such mal-operation may be neutralized by arranging that the rate at which the starting rheostat is cut out, and the field is weakened, is dependent on the current input to the motor. This "current-limit" method of controlling the acceleration has a



SEQUENCE OF SWITCHES	Notch	LS1	LS2	R1	R2	R3	R4	R5	F	B	S1	S2
Resistance Notches	1	●	●						●			
	2	●	●								●	
	3	●	●	●							●	
	4	●	●	●	●						●	
	5		●		●	●					●	
	6		●		●	●	●				●	
	7		●	●	●	●	●				●	
	8	●	●	●	●	●	●				●	
Full Field	9	●	●	●	●	●	●	●			●	
Inter Field	10	●	●	●	●	●	●	●				
Weak Field	11	●	●	●	●	●	●	●	●			
Rheostatic Braking	1							●		●	●	●
	2						●	●		●	●	●

	Indicates a normally open contact which closes when pedal is depressed
	Indicates a normally closed contact which opens when pedal is depressed
B	Indicates interlock contacts made when B Contactor is closed
B	Indicates interlock contacts made when B contactor is open

FIG. 96. CONNEXIONS FOR MAIN AND CONTROL CIRCUITS FOR COMPOUND-MOTOR, NON-REGENERATIVE EQUIPMENT (METROPOLITAN-VICKERS)

universal application to d.c. motor-coach trains and is described in detail in Chapter IX.

Trolleybuses, however, do not require such elaborate systems, and simpler schemes based on the same principle have been developed.* One scheme (incorporating both current and time-limits) is shown diagrammatically in Fig. 97A. A telescopic link is inserted between the accelerating pedal and the operating lever of the master controller (which is of the type illustrated in Fig. 94), and the movement of the latter is retarded by a piston, *P*, working in a cylinder filled with oil, the flow from which is controlled by electromagnetic valves, *B*, *C*.

When the vehicle is stationary and the accelerating pedal is depressed, the initial current closes valve *B*, and the oil flows through valves *A* and *C*, allowing the master controller to advance at a predetermined rate. If the load is such that the motor current increases above the permissible value the current-limit relay causes valve, *C*, to close and slow down the rate of movement of the master controller.

A quick-return valve, *D*, allows a quick return of the master controller to the "off" position.

If power is re-applied during coasting a quick advancement of the master controller is obtained (due to valves *B* and *C* being open) until the motor current rises to the value which causes valve *B* to close.

Fig. 97B shows a master controller in which a similar scheme of acceleration control is incorporated. The telescopic link is shown in the operating pull rod below the controller.

Battery Vehicle Control Systems

Control Requirements. Battery vehicles are employed chiefly for local deliveries of goods in large towns. The daily duty of a vehicle consists of many short runs, involving frequent starting and stopping. As the ampere-hour capacity of the battery is restricted by considerations of weight and cost, economy in discharge is important, and may be effected by (i) reducing the rheostatic losses at starting, (ii) providing two efficient running speeds (preferably in the ratio of approximately 1 : 2). Both requirements are fulfilled by arranging the battery in two halves, which may be grouped either in parallel or series.

Control Systems. For small vehicles simple rheostatic starting is employed and the control equipment consists of a drum-type controller and reverser.

For larger vehicles, handling loads of 12 cwt and above, a more economical control system is necessary, and the parallel-series battery grouping system is employed. The two halves of the battery are connected in parallel during the initial starting and for running at approximately half maximum speed, and are connected in series when maximum speed is required. The transition from the parallel to the series connexion of the sections of the battery is made by a closed-circuit bridge method and the sequence of the steps is shown diagrammatically in Fig. 98.

A remote control system is employed, with electromagnetic contactors (for the main circuits), a small pedal-operated master controller and a hand-operated reversing switch with an additional position for "charge." Fig. 99 shows the equipment mounted on a vehicle.

* See *Transactions A.I.E.E.*, Vol. 71, "A new control for trolley coaches," by N. H. Willby. Also *B.T.H. Activities*, Vol. 19, p. 439.

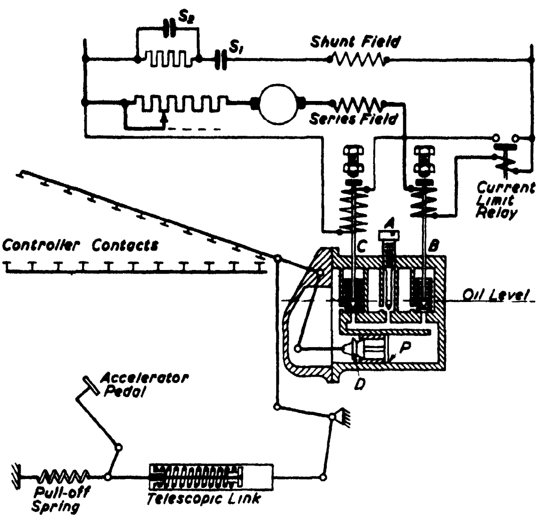


FIG. 97A. SIMPLIFIED SCHEME OF ACCELERATION CONTROL FOR TROLLEYBUSES (METROPOLITAN-VICKERS)

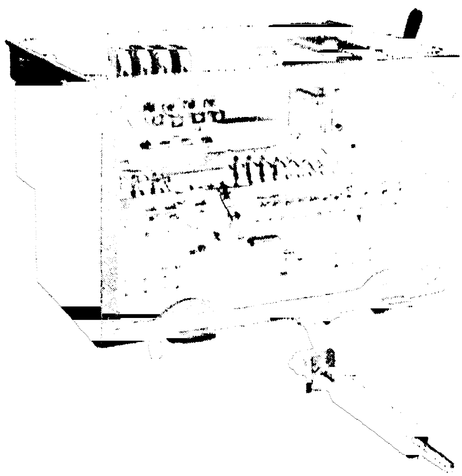


FIG. 97B. TROLLEYBUS MASTER CONTROLLER WITH AUTOMATIC ACCELERATION CONTROL (G.E.C.)

A typical *master controller* is shown in Fig. 100, and a *diagram of connexions* of a parallel-series control system, suitable for vehicles carrying loads up to 1½ tons, is given in Fig. 101.

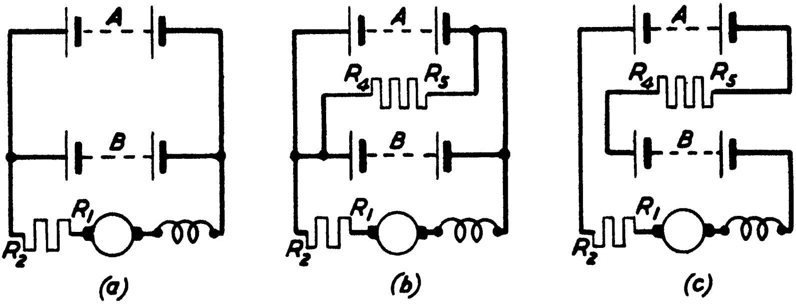


FIG. 98. DIAGRAM SHOWING TRANSITION FROM PARALLEL TO SERIES CONNEXION OF BATTERIES

(*a*) Batteries in parallel (*b*) transitional resistor ($R_4 - R_5$) inserted (*c*) batteries in series

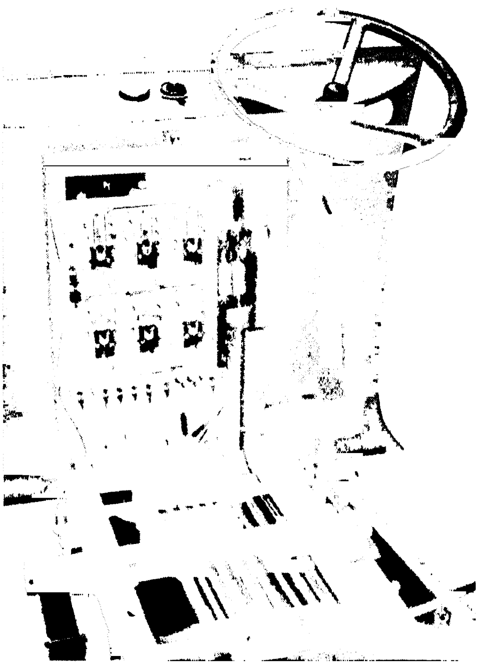


FIG. 99. CONTROL EQUIPMENT MOUNTED ON VEHICLE (B.T.-H.)

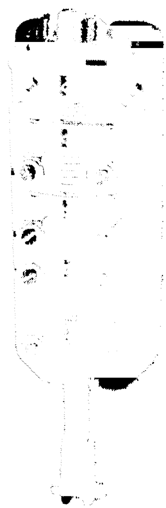
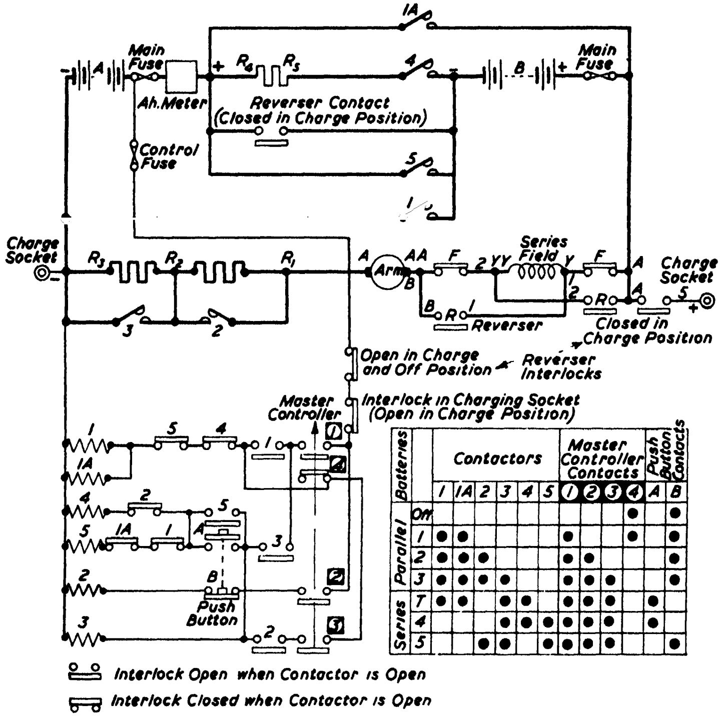


FIG. 100. MASTER CONTROLLER (B.T.-H.)

The master controller has three operating positions, giving two rheostatic steps and a running step with the sections of the battery in parallel. To obtain full speed a push-button on the steering column is pressed for a few seconds and released, the pedal of the master controller remaining depressed. The sequence

of operations is as follows—contactor No. 2 opens (by the opening of the contacts, *B*, of the push-button); No. 4 closes (due to the closing of the interlocks



Batteries		Contactors 1	1A	2	3	4	5	Master Controller Contacts (1)	(2)	(3)	(4)	Push Button Contacts A	B
Parallel	Off										●		●
	1	●	●					●			●		●
	2	●	●	●				●	●				●
	3	●	●	●	●			●	●	●			●
Series	T	●	●		●	●		●	●	●		●	
	4				●	●	●	●	●	●		●	
	5			●	●		●	●	●	●			●

FIG. 101. CONNEXIONS FOR PARALLEL-SERIES CONTROL SYSTEM (B.T.-H.)

on No. 2); Nos. 1, 1A open (due to the opening of the interlocks on No. 4); No. 5 closes (due to the closing of the interlocks on Nos. 1, 1A); No. 2 closes when the push-button is released and its interlocks cause No. 4 to open, so that only the essential contactors (Nos. 2, 3, 5) remain closed.

CHAPTER IX

THE CONTROL OF DIRECT-CURRENT RAILWAY MOTORS

CONTROL SYSTEMS FOR MOTOR-COACH TRAINS

ELECTRIC trains for suburban service are usually made up of a number of motor and trailer coaches, and the composition of the train is altered to suit the traffic. Thus for light traffic one motor-coach and one trailer may be sufficient, while for heavy traffic two or more motor-coaches and a number of trailers may be required. Each motor-coach may be equipped with two or four motors, but all of the motors throughout the train must be controlled simultaneously from one point.

Multiple-unit Control. To obtain the maximum flexibility in the make-up of trains an indirect or remote control system is required in which the various series-parallel controllers for the motors are power operated,* the operating currents of all these controllers passing through a small master controller manipulated by the driver, who has, therefore, complete control over the whole of the motors. Such a system is called *multiple-unit control,* because, for control purposes, each motor coach and its equipment is considered as *one unit*, and the control circuits of all the units are connected in parallel or multiple.

This system is now standard for all motor-coach trains and is also employed on locomotives in cases where two or more locomotives may require to be coupled for hauling heavy trains over gradients.

General Classification of Multiple-unit Systems. Multiple-unit systems are classified according to the system of operation and the type of motor-controller. Thus the motor-controller may be operated either entirely electrically (called "*all electric*" operation), or by pneumatic cylinders having electrically-operated control valves (called "*electro-pneumatic*" operation). Again, the motor-controller may consist either of a number of individual switches (called "*unit switches*" or "*contactors*") each having its own operating mechanism and being self-contained, or, alternatively, a group of switches or contactors may be operated by cams mounted on a common shaft, so that only a single operating mechanism is required for the switch group. Motor-controllers of the former type are called *unit-switch controllers*, or *individual-contactor controllers*; those of the latter type are called *cam-shaft controllers*.

Further, with a power-operated motor-controller two methods of notching-up, or progression from step to step, are possible, i.e. (i) synchronously with the movements of the master controller (and therefore under the direct control of the motorman just as if a hand-operated motor-controller were in use), (ii) automatically, the notching-up, when once started by the motorman, being completed automatically (by means of relays and auxiliary switches) without further effort or skill being necessary on the part of the motorman.

* The term "motor-controller" is employed subsequently to denote such power-operated series-parallel controllers, including also the reversers.

Application. The *non-automatic system* is obviously necessary for locomotives as the service duty may vary from day to day. The motor-controllers are usually of the electro-pneumatic individual-contactor type.

The *automatic system* was developed especially for motor-coach trains as the starting conditions, except for gradients, are practically constant, the passengers—even in a packed suburban train—contributing only about 15 to 25 per cent to the total weight. Moreover, the adhesive weight is always sufficient to permit the full accelerating tractive effort to be employed under all rail conditions.

Although all the above systems are in extensive use, the present tendency is towards electro-pneumatic operation, the motor-controllers being either of the

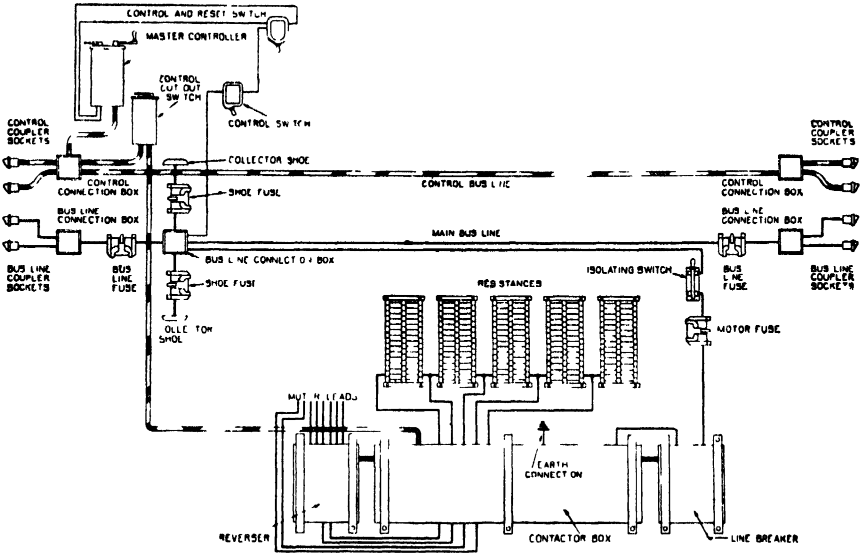


FIG. 102. LAYOUT OF CONTROL APPARATUS ON MOTOR-COACH OF MULTIPLE-UNIT TRAIN

Line voltage control supply, earth return

individual-contactor or cam-shaft type for surface railways, and the cam-shaft type for underground railways with restricted under-floor space.

The chief reasons for preferring electro-pneumatic, instead of electromagnetic, operation for motor-controllers of the individual-contactor type are—(i) the contactor group is more compact; (ii) each valve-magnet requires only a few watts for operation, whereas the solenoid of an electromagnetic contactor may require from 100 to 200 watts; (iii) a low voltage control supply can be employed, which simplifies the auxiliary and interlocking contacts on the contactors and their circuits, reduces the size of the master controller and gives greater reliability to the whole of the control apparatus and circuits. As electric trains are almost universally equipped with compressed-air brakes, the supply of air for electro-pneumatic operation is always available.

Control Equipment on Coaches. The equipment which is necessary on a *motor coach* for control purposes comprises—

Master controller, series-parallel motor-controller (with rheostats and reverser), control-circuit multi-core cable (control bus-line) with coupler sockets, accelerating relay; also a motor-generator, battery or tapped line-connected resistor (potentiometer) if the control circuit is to be supplied at low voltage.

That necessary on a *trailer coach* comprises—

Master controller (when required), control-circuit multi-core cable with coupler sockets.

To ensure protection of the motor equipment against excessive currents, and to isolate, when necessary, the control apparatus of any motor-coach from the control bus-line, each motor-coach is further equipped with—

Over-current relay with driver's control switch, isolating switch for master controller, cut-out switch and fuses for isolating the control circuit of the motor-controller from the control bus-line.

A diagram is given in Fig. 102 to show the layout of the apparatus and the method of connexions for a motor-coach equipped with two motors and intended for single-end operation. It will be observed that branch connexions from the control bus-line and also from the main bus-line (the object of which is to interconnect the collector shoes on the several motor-coaches of a train) are made by means of special connexion boxes.

Control Supply. The control circuits of *electromagnetic* systems are usually supplied at line voltage (600 V), but the operating coils of the contactors and reverser are usually wound for about 120 V, which necessitates a number of coils, or the equivalent resistors, being connected in series on each step of the motor-controller. The switching of the coils and resistors is effected by auxiliary contacts on the contactors.

In *electro-pneumatic* systems, however, due to the small power required by the valve magnets, a low voltage is desirable, and voltages between 50 V and 120 V are employed. The supply is obtained either from a motor-generator (with usually a storage battery floating across the generator) or from the earthed end of a potentiometer rheostat connected across the traction circuit.

A supply of *compressed air* is essential for electro-pneumatic systems. The air is stored in an auxiliary reservoir, which is supplied from a main reservoir, the air pressure in the latter being maintained within prescribed limits.

Individual-contactor Multiple-unit System

Contactors. Individual contactors for multiple-unit systems are more robust than those for tramcar and trolleybus equipments. The magnetic blow-out is more powerful and the arc chutes are larger in order to give high arc-rupturing capacity. A drawing of a typical 1,500-V contactor is shown in Fig. 87A.

Each contactor is fitted with auxiliary contacts, actuated by the moving contact and shown at *F*, Fig. 87A, for the purpose of interlocking and automatically switching the control circuits in automatic control systems as explained later.

Fig 103 Low height Master Controller with Contactor type Contacts (B T H)
Arc chutes to right of bevel gear are lowered to expose contacts

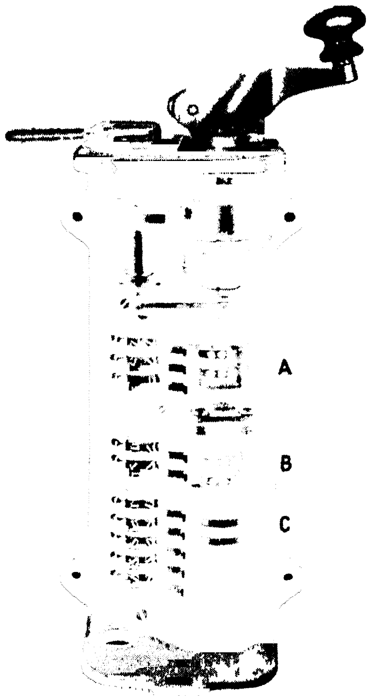


Fig 104 Master Controller with Drum-type Contacts (Metropolitan Vickers)

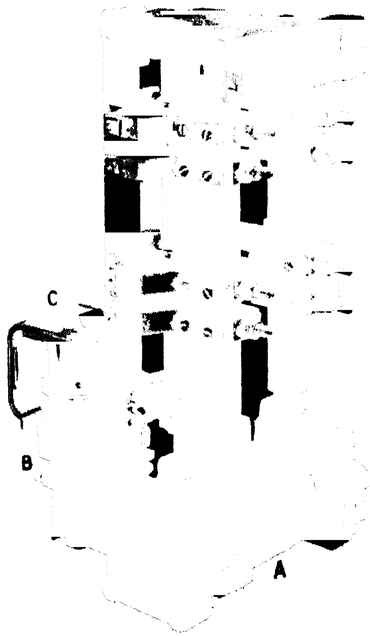


Fig 105 Electro pneumatic Reverser (Metropolitan Vickers)

From 12 to 20 contactors are necessary for series-parallel control with bridge transition, the smaller number for 600-V circuits and the larger number for 1,500-V and 3,000-V circuits.

Master Controllers. All master controllers for multiple-unit trains are provided with a "dead-man's" handle, to ensure that, in the event of the driver becoming incapacitated, the power supply to the motors will be interrupted and the brakes will be applied automatically.

Fig. 103 shows a modern controller designed with low height for operation by a seated driver. The contacts are of the contactor type, and those controlling the series and parallel switching of the contactors are actuated by a horizontal cam-shaft with a bevel drive from the driver's handle. The pair of contacts (called the "cut-off" or "dead-man's" contacts) to the left are connected directly in the supply circuit to the controller and are actuated by the driving handle through a spring latch. These contacts close when the upper (hinged) portion of the handle is depressed in the "off" position and moved to an operating position: they open automatically when the handle is released whatever its position. When the handle is released in an operating position the cut-out contacts cannot be reclosed until the handle is returned to the "off" position.

The hinged portion of the handle also controls a pilot valve connected to the train pipe of the air brake. This valve is closed when either (*a*) the reversing lever is in one of the running positions and the driving handle is depressed, or (*b*) the reversing lever is in the "off" position.

Hence the driving handle must normally be held depressed, and if any abnormal circumstances cause it to be released the power supply to the motors will be cut off and the brakes applied automatically.

Fig. 104 shows an alternative form of master controller with drum-type contacts incorporating the "dead-man's-handle" feature. The fingers are arranged in line, and the drum contacts are arranged axially, *A*, being the reversing contacts, *B*, the "dead-man's" contacts, and *C* the contacts controlling the contactors.

Reversers. These are usually of the drum type with two operating positions. In the electromagnetic type the operation is by two electromagnets, one for each position. In the electro-pneumatic type (Fig. 105) a pinion on the shaft of the contact drum engages a rack formed on the common piston rod of two single-acting opposed air cylinders, *A*, each with its own control valve, one being shown at *B*. Admission of air to either cylinder sets the drum to one position or the other. Auxiliary contacts, *C*, are fitted to both types for interlocking the control circuit of the reverser with that of the main contactors.

Automatic Control. Automatic closing of the rheostatic contactors at a given motor current may be obtained by means of a current-limit relay (called an "accelerating" or "notching" relay) and auxiliary contacts on each contactor. The relay consists essentially of a pair of contacts (connected in the control circuit) and an actuating electromagnet excited by the motor current. The contacts are closed when the current is below a prescribed value and open when the current is above this value.

The *general scheme of operation* is that, as each "rheostatic contactor" closes, it transfers (by means of auxiliary contacts) its operating coil from an

actuating circuit (which includes the contacts of the relay) to a *retaining circuit* (which is independent of the relay contacts), and connects the operating coil of the contactor for the succeeding step to the actuating circuit via the contacts of the relay.

Hence the contactors will automatically "notch-up," step by step, until the motor combination corresponding to the position of the master controller is reached, the rate at which the notching progresses being controlled indirectly by the motor current. The notching, however, may be arrested at any point, if desired, by opening the actuating circuit.

Since the resistance steps are cut out only when the motor current falls to the prescribed value, it is possible, by correctly grading the rheostats, to obtain the same variation of motor current on each notch, so that the average accelerating current, and therefore the *average acceleration, will be constant.*

Accelerating Relay. The simplest form of relay consists of an open type vertical solenoid with a plunger carrying a contact disc which bridges a pair of contacts when the plunger is in its lowest position. But the attracted armature type, of which one form is shown in Fig. 113, is employed in many modern equipments on account of its compactness and robustness.

Protective Relays. *Over-current protection* in modern equipments takes the form of a relay having a series operating coil and contacts, normally closed, connected in the control circuit of the "line" contactors (which are designed to rupture currents much in excess of normal). When the relay trips, the contacts are latched in the open position and must be reset by energizing a shunt coil, the circuit of which is usually taken through off-position contacts on the master controller. A shunt tripping coil, controlled by a push button, is also provided to enable emergency tripping to be effected by the driver.

Low-voltage protection is necessary on trains operating without a main bus-line to ensure that if the main supply to any motor-controller is interrupted (due to the current collectors crossing a gap) the control supply to that controller will also be interrupted. This is effected by a relay (called a "low-voltage," "no-volt," or "line," relay) having a shunt operating coil connected, with a suitable series resistor, across the traction supply and a pair of contacts (which are closed when the operating coil is excited) in series with the control circuit of the line contactors.

Combined low-voltage and low-current protection may be necessary when the motor of the motor-generator for the auxiliary supply has a speed-stabilizing (shunt) winding.

In this case a series winding (connected in the motor circuit) is added to the low-voltage relay. Arrangements are made (*see* Fig. 114) so that the initial closing of the contacts is effected by the voltage coil and the retention of these contacts in the closed position is dependent on the current in the series coil.

Connexions for Automatic Control (Electro-pneumatic System). Schematic diagrams of the main-circuit and control-circuit connexions are given in Figs. 106, 107. The control-circuit supply to the master controller is taken through the contacts of a pneumatic switch (called a "control governor") in addition to the usual isolating switch. The control circuit is therefore interlocked with the air supply, and the circuit to the master controller is closed only when the air pressure is above a definite value.

The sequence of operations is briefly as follows—

When the reversing handle is thrown to an operating position the re-set coils of the overload relays are energized via wire 7.

When the driving handle is moved to the first position, the circuit to the re-set coils is broken, and a circuit is established to wires 1 or 2 (according to the position of the reversing drum) and also to wire 3, in the circuit of which is included the contacts of the "dead man's" feature. (These contacts are also included in the circuits of wires 4, 5, 6, which are energized on subsequent

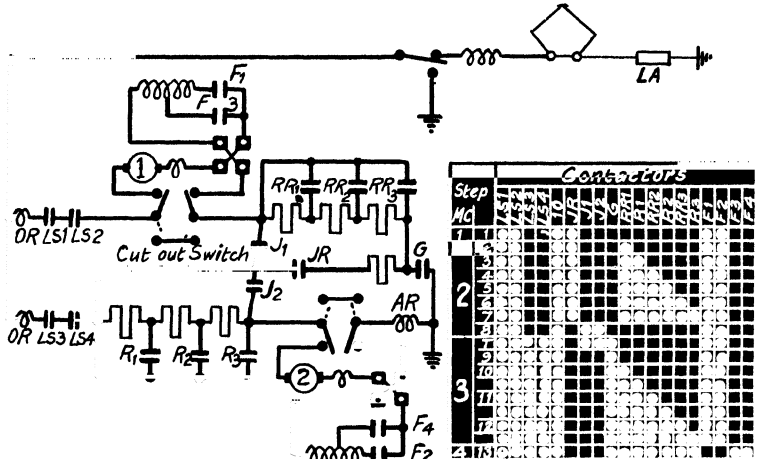


FIG. 106 MAIN-CIRCUIT CONNEXIONS FOR METROPOLITAN-VICKERS ELECTRO-PNEUMATIC 1,500-VOLT CONTROL SYSTEM

AR, *OR* denote the accelerating relay and the over-current relays respectively. Contactor No 10 (not shown in diagram) is connected between R_3 of the lower group of rheostats, and the cut-out for No 2 motor

steps.) The reverser is therefore set, and its interlocking contacts energize (via wires 1 or 2) the magnets of contactors LS_1, LS_2, 10. Observe that this circuit includes the contacts of the overload relays and the auxiliary contacts on the motor cut-out switches.

Wire 3 establishes the circuits to the "full-field" contactors, F_1, F_2, and the "series" contactor JR, the former being interlocked with the field-tapping contactors F_3, F_4, and the latter being interlocked with the line contactors LS_1, LS_3. Thus contactors LS_1, LS_2, 10, F_1, F, JR_2 are closed, and the motors are connected in series with each other and with the full resistance in the circuit.

When the driving handle is moved to the second position the "actuating" circuit is established, and contactor RR_1 closes, provided that the motor current has fallen to the prescribed value. Observe that the circuit to the accelerating relay includes the interlocking contacts on contactors LS_1, JR. The closing of contactor RR_1 transfers its magnet coil to the "retaining" circuit —which is supplied by wire 3—and connects the magnet coil of contactor R_1 to the "actuating" circuit. Automatic action then continues until the "bridge" contactors J_1, J_2 (two contactors being connected in series) close and the "full series" step is obtained. Observe that when J_1 closes, its interlock opens the circuit of the magnet coils of contactors JR, RR_1, R_1, RR_2, R_2, RR_3, R_3. Observe also that the opening of JR opens the "actuating" circuit.

The "transition-actuating" circuit is established, via wire 5, at the third position of the master controller, and, if the motor current is below the prescribed value, the "parallel" contactors—LS_3, LS_4, G—are closed and the bridge contactors are opened. Observe that the closing of LS_4 transfers the magnet

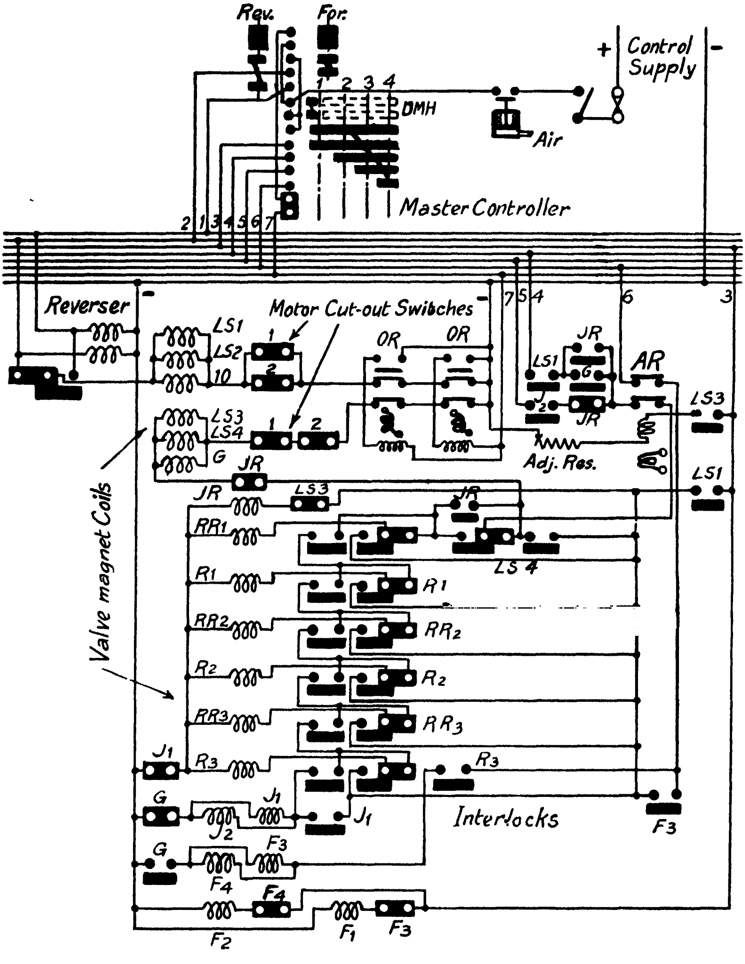


FIG. 107. CONTROL-CIRCUIT CONNEXIONS FOR METROPOLITAN-VICKERS ELECTRO-PNEUMATIC CONTROL SYSTEM

The interlocks are shown in the "open" position of the contactors.
AR, *OR* denote the accelerating relay and the overload relays respectively.

coils, LS_3, LS_4, G, to the "retaining" circuit supplied by wire 3, and that the closing of G opens the circuit of J_1, J_2. Observe also the interlocking of the "parallel" contactors with both the "series" contactor and the auxiliary contacts of the motor cut-out switches, the re-establishment of the "actuating" circuit (via wire 4) and the energizing of the auxiliary shunt coil of the accelerating relay.

On the last step of the controller the control circuit of the field-tapping contactors, F_3, F_4, is established via wire 6, the special contacts of the accelerating relay, and the interlocks on contactors R_3 and G. Hence tapped-field operation cannot be obtained until the motors are in "full" parallel and the current is below the prescribed value.

Connexions for Automatic Control (Electro-magnetic System). Schematic diagrams of the motor and control circuits are given in Figs. 108, 109.* The

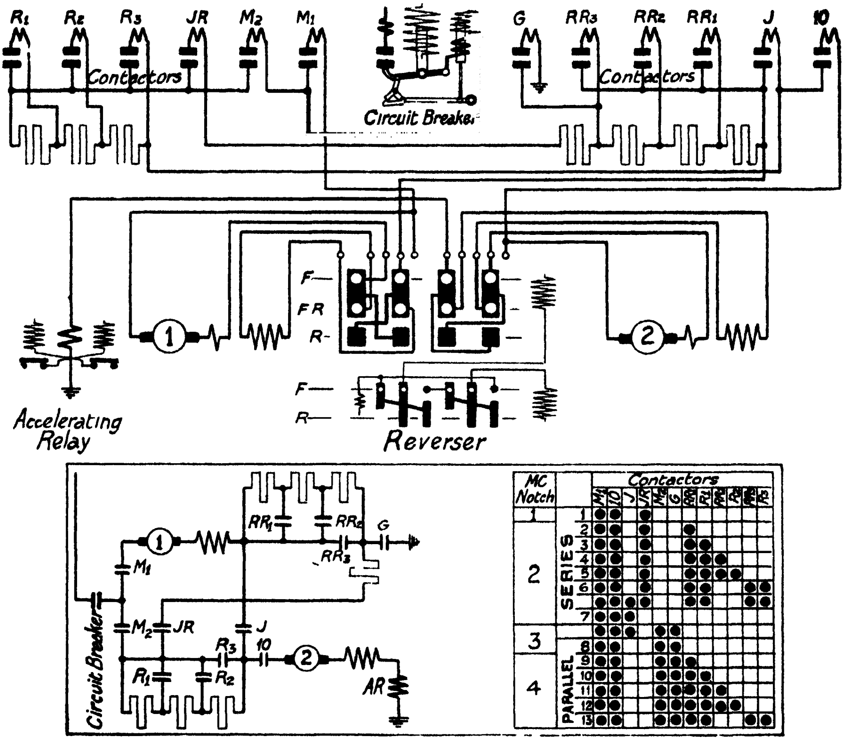


FIG. 108. MAIN-CIRCUIT CONNEXIONS FOR METROPOLITAN-VICKERS AUTOMATIC ALL-ELECTRIC CONTROL SYSTEM

control circuit is supplied directly from the traction circuit and, as the contactor operating coils are designed for about 100 volts, the operating circuits must always include six coils or their equivalent in resistance.

The control-circuit bus-line cable consists of seven wires, of which five are for the purpose of controlling the contactors and the remaining two are for the purpose of controlling the automatic circuit-breaker. Each of the wires constituting the control circuit proper has a definite function: thus No. 1 wire controls the "actuating" circuit of the contactors; No. 2 wire controls

* These connexions and the scheme of numbering the contactors are typical of the Metropolitan-Vickers control systems on some of the suburban motor-coach trains operating on the Southern Region of British Railways.

the "retaining" circuit of the contactors; No. 3 wire controls the transition into parallel; and wires Nos. 4 and 5 control respectively the "forward" and "reverse" operating coils of the reverser, as well as the "line" contactor and the contactor No. 10 in No. 2 motor circuit.

When the master controller is placed on the first (series) notch "forward," two circuits are established. One circuit is from No. 5 finger (Fig. 109), through the "forward" operating coil and interlocks of the reverser,* through the operating coils of contactors M_1 and 10, and thence to earth through a rheostat. The other circuit is from No. 2 finger, through the interlocks of contactors M_1, M_2, and thence through the operating coil of the "series" contactor (JR) to earth via the

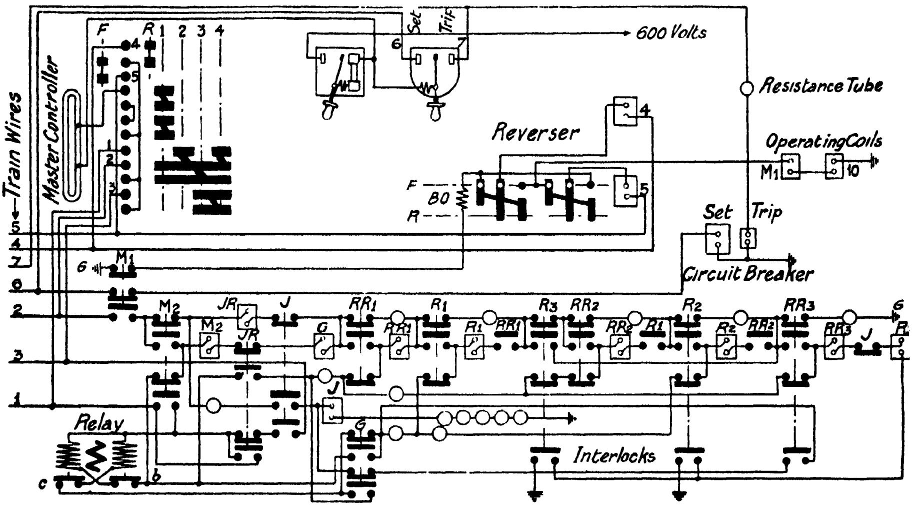


FIG. 109. CONTROL-CIRCUIT CONNEXIONS FOR METROPOLITAN-VICKERS AUTOMATIC ALL-ELECTRIC CONTROL SYSTEM

auxiliary contacts of contactors, M_1, M_2, J, RR_1, R_1, R_3, RR_2, R_2, RR_3, and a number of substitutional resistors. This circuit forms the "retaining" circuit for the contactors which are subsequently energized automatically.

If the master controller is moved to the second notch, another circuit (called the "actuating" circuit) is established by means of wire No. 1 and the current-limit or accelerating relay. This circuit, however, is under the control of the relay, and the contactors will be energized only provided that the relay contacts are short-circuited. As neither potential coil of the relay is energized on the first notch of the controller, the relay contacts will be short-circuited when the controller moves to the second notch, and, therefore, a circuit will be established through the operating coil of contactor RR_1 to earth, via the contacts (b) of the relay and the interlocks of contactors RR_1, JR, R_1, R_3, RR_2, R_2, RR_3. When contactor RR_1 closes, its operating coil is transferred automatically to the "retaining" circuit (energized from No. 2 wire) by means of the interlocks on this contactor, and, at the same time, the circuit of the potential coil (c) of the relay is opened. Before this change has taken place, however, the plunger

* If the reverser is in the "reverse" position, the "forward" coil is energized with the full line voltage provided that contactor M_1 is open.

controlling contacts (*c*) of the relay—from which the succeeding contactor, R_1, is energized—will have been lifted, and the time-limit device on the plunger will prevent the premature release of the latter when the circuit of the potential coil is opened. Therefore, the contacts (*c*) cannot close until the plunger is released by the series coil, which occurs when the motor current falls to the predetermined value at which the relay is set. Contactor R_1 will then be energized, via the contacts (*c*) of the relay and the interlocks of contactors G, R_1, RR_1, R_3, RR_2, R_2, RR_3.

Next, contactor RR_2 is energized via the contacts (*b*) of the relay and contactors JR, R_3, RR_2, R_1, R_2, RR_3.

In a similar manner contactor R_2, and contactors RR_3 and R_3, are energized sequentially on the next two notches. The transference of the earth connexion of the "retaining" circuit from contactor RR_3 to contactor R_3 should be noted, as well as the automatic opening of contactors RR_2 and R_2.

On the next notch—the last series notch—the "bridge" contactor J is energized, and when this contactor closes, its auxiliary contacts open the circuit of contactors JR, RR_1, R_1, RR_3, R_3. The contactors closed on the last series notch are, therefore, M_1, 10, J.

It should be noted that no more "notching-up" can take place unless No. 3 wire is energized. Moreover, during the above process of notching-up, the notching can be stopped at any desired point by placing the master controller on the first notch, which interrupts the "actuating" circuit.

If the master controller is moved to the last (or fourth) notch "forward," contactors M_2 and G will be energized from No. 3 wire via the contacts (*b*) of the relay and contactors J, JR, M_2, JR, RR_1, R_1, R_3, RR_2, R_2, RR_3. When contactors M_2 and G close, the control circuit is transferred to No. 2 wire via the interlocks of contactors, M_1, M_2, JR, RR_1, R_1, R_3, RR_2, R_2, RR_3. Thus the object of No. 3 wire is to energize the "parallel" contactors (M_2 and G), and these contactors cannot close until J has closed and JR has opened. At starting, the master controller may be placed directly on the last notch, and "notching-up" will take place through all the series notches, the transition notch, and the parallel notches.

When M_2 closes, J is opened automatically, and the potential coils of the relay are supplied through the auxiliary contacts of M_2. On the series notches the potential coils of the relay are supplied through the auxiliary contacts of JR. Thus the automatic relay can only control the "actuating" circuit, provided that *either* JR or M_2 is closed.

The notching-up to "full parallel" takes place in a manner similar to that for the series steps. Observe, however, that the operation of the contactors is now governed by the current in motor No. 1. On some of the parallel notches the closing of corresponding "rheostatic" contactors takes place successively instead of simultaneously. For example, a rheostat section in No. 1 motor circuit is not cut out until the contactor of the corresponding section in No. 2 motor circuit is fully closed. The next rheostat section in No. 2 motor circuit, however, cannot be cut out until the relay contacts close again.

Cam-shaft Control Systems

Advantages. Cam-shaft systems possess important advantages over systems employing individual contactors. For example, the cam-shaft system has fewer parts subjected to wear; it is less costly to maintain than an individual-contactor system; a definite sequence of automatic contactor operations is

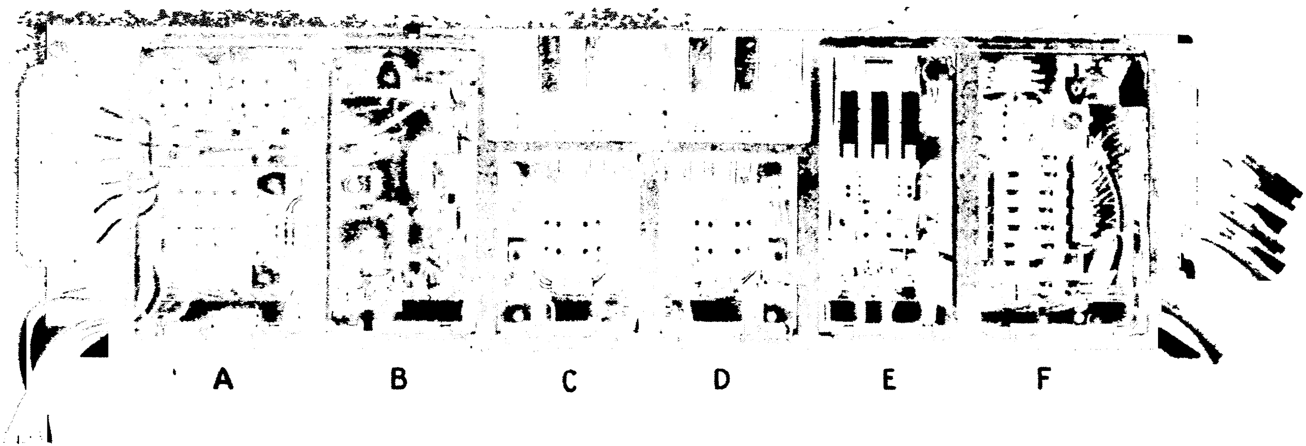


Fig. 110. B.T.-H. Type PCM Electro-pneumatic Cam-shaft Control Gear Mounted in Compartment-type Frame for Installing on Underframe of Motor-coach

The equipment contained in the several compartments is as follows—*A*, field-shunting contactors and reverser. *B*, protective and field-shunting relays *C*, *D*, line contactors with extended arc chutes *E*, series-parallel transfer contactors: *F*, cam-shaft contactors.

obtained with a very small number of auxiliary contacts; the control equipment is lighter, occupies less space, and can be inspected easier and quicker than a corresponding equipment with individual contactors. Moreover, the control-circuit wiring for automatic control is very much simpler than that for the corresponding system having individual contactors.

Electro-pneumatic System. In the system which has been developed in this country by the British Thomson-Houston Co., and in America by the General Electric Co.,* the cam-shaft actuates only the "rheostatic" contactors. A separate group of three cam-operated contactors (called a transfer switch) effects the series and parallel combinations of the motors, and individual contactors act as circuit breakers. The complete equipment assembled for underframe mounting is shown in Fig. 110.

A unique feature is that the cam-shaft rotates in one direction to cut out resistors in the series combination of the motors, and, when the full-series position is reached and transition has been effected, it returns through the same positions, but in the reverse order, to cut out resistors in the parallel combination of the motors. When the full-parallel position is reached the cam-shaft has returned to its original position. Hence when power is cut off no further movement of the cam-shaft is necessary.

This feature enables nine rheostatic steps to be obtained in each combination of the motors, and in consequence only a relatively small variation of current and tractive-effort occurs on each step.

Cam-shaft Contactors. The eleven "rheostatic" contactors are arranged in two groups on opposite sides of the cam-shaft as shown in Fig. 111. Each contactor is closed by a spring and opened by a cam. The cams, of moulded insulating material, are fixed to a square shaft, one end of which is fitted with a star-wheel and the other end with a pinion, which engages with a rack connecting two single-acting pistons as shown diagrammatically in Fig. 112. One piston, A, operates under oil pressure and the other, B, under air pressure, and the motion of the rack and cam-shaft is due to the *difference* of pressure on the pistons. The use of air and oil gives smoother operation than air alone.

Operation. In the initial position of the cam-shaft, and with the valve-magnet unexcited, air pressure is maintained in the "air" cylinder and atmospheric pressure exists in the oil reservoir. When the valve-magnet coil is excited, air is exhausted from the "air" cylinder and is admitted to the oil reservoir. The pressure on piston A is now greater than that on piston B, and the cam-shaft, if unrestricted, would rotate in the clockwise direction at a speed determined by the flow of oil through the valve C.

Step-by-step movement of the cam-shaft is obtained by a star-wheel, pawl, and an electromagnet (called the "stop" magnet), which, when excited, is capable of holding the pawl in any notch of the star-wheel. The operating coil of this magnet is in series with the contacts of the accelerating, or "notching," relay, which in this case are closed when the motor current is *above* a prescribed value and open when the current falls to, or is below, this value. Hence the

* This system has been adopted since 1938 for all deep-level ("tube") trains of London Transport on account of its suitability for underframe mounting with wheels of small diameter. It is also employed on other lines of London Transport and has an extensive application in America.

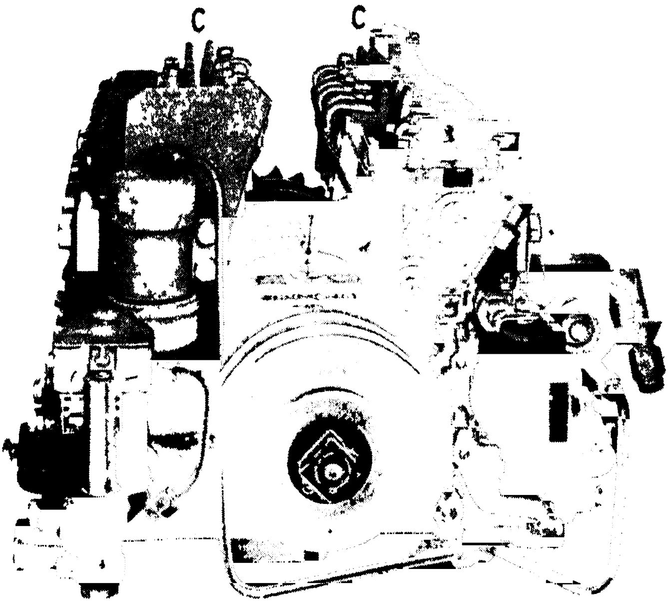


FIG. 111. GROUP OF CAM-SHAFT CONTACTORS WITH OPERATING MECHANISM (IN FOREGROUND) (B T. H.)

The contacts of the eleven contactors are shown at C

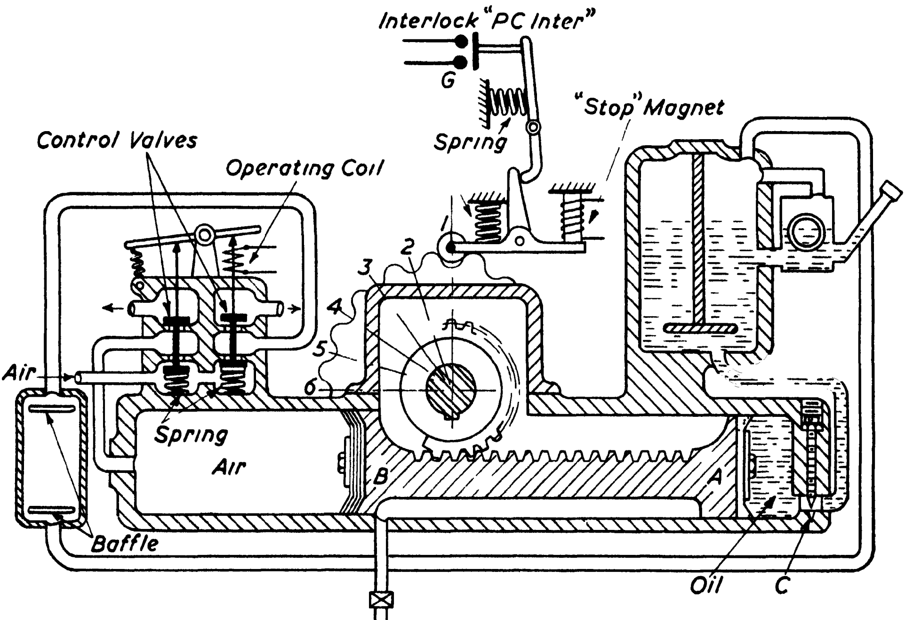


FIG. 112. OPERATING MECHANISM OF B.T.-H. TYPE PCM CAM-SHAFT CONTACTOR GROUP

star-wheel can move through a notch only when the motor current *falls to*, or is below, the prescribed value.

When the cam-shaft reaches the limit of its travel, interlocking contacts complete the circuit of the valve-magnet coil of the series-parallel transfer switch, and, if the master controller is in the position for parallel operation, transition is effected by the bridge method. The operation of the transfer switch opens the circuit of the cam-shaft valve-magnet, thereby exhausting air from the oil reservoir and admitting air to the "air" cylinder. Pressure is now applied to piston B, and the cam-shaft moves step-by-step in the reverse direction until the initial position is reached, corresponding to the full parallel combination of the motors.

Transfer Switch. The three contactors P, S, G, Fig. 114, are actuated by a cam mechanism which is operated by an air cylinder. With no air pressure in the cylinder, i.e. the valve-magnet is unexcited, S is closed and P,G, are open, and when air is admitted, due to the excitation of the valve-magnet, P, G, are closed and S opens.

Accelerating Relay. This is of the attracted-armature type and is shown in Fig. 113. The armature, A, which has a knife-edge pivot at its upper end, carries a contact at its lower end and is spring-biased to open the contacts when unexcited. The operating coils in the present case consist of two series coils—one in the circuit of each motor—and two shunt coils—one called the "lift" coil and the other the "rate" coil.

The "lift" coil, B, is energized momentarily while the star-wheel of the cam-shaft is passing from notch to notch, the circuit being controlled by the contacts G, Fig. 112 (labelled *PC Inter* in Fig. 114), actuated by the pawl: its function, therefore, is to attract the armature and close the relay contacts while the motor current is rising in the series coils.

The "rate" coil, C, is controlled by either a special switch in the driver's cab or additional contacts on the reverse drum of the master controller. When energized its ampere-turns oppose those of the series coils, so that the armature releases at a higher motor current than when the series coils act alone. The function of the rate coil, therefore, is to enable two values of accelerating current to be obtained with a given setting of the armature control spring. This feature is useful where trains have to operate both underground and on the surface, as it allows a higher acceleration to be employed on the underground lines where the rails are always in a good (dry) condition.

The function of the auxiliary electromagnet, D, (called the relay stop magnet), is to stop the action of the relay by holding the contacts closed. The operating coil is excited on the first (shunting) step of the master controller (*see* Fig. 114) and in this condition an extension on the armature engages the lower end of the relay armature. The "stop" magnet of the cam-shaft therefore remains excited and the cam-shaft is prevented from notching.

Connexions. Schematic diagrams of main (600 volts) and control (low-voltage) circuits for the equipment illustrated in Fig. 110 are shown in Fig. 114.

In position No. 1 of the master controller a supply is given to train wires Nos. 1, 2, 4, (the reversing lever being in the "forward" position) and the valve-magnets of the line contactors LB_1, LB_2 are energized provided that the contacts of the low-current-and-voltage relay and other interlocks are already

closed. The closing of the line contactors completes the main circuit, but although the valve-magnet of the cam-shaft is excited, notching cannot proceed because the contacts of the accelerating relay are closed and the star-wheel is held by the stop magnet.

When the master-controller is moved to positions Nos. 2 or 3 automatic notching to either full series or full parallel occurs as already explained. Observe

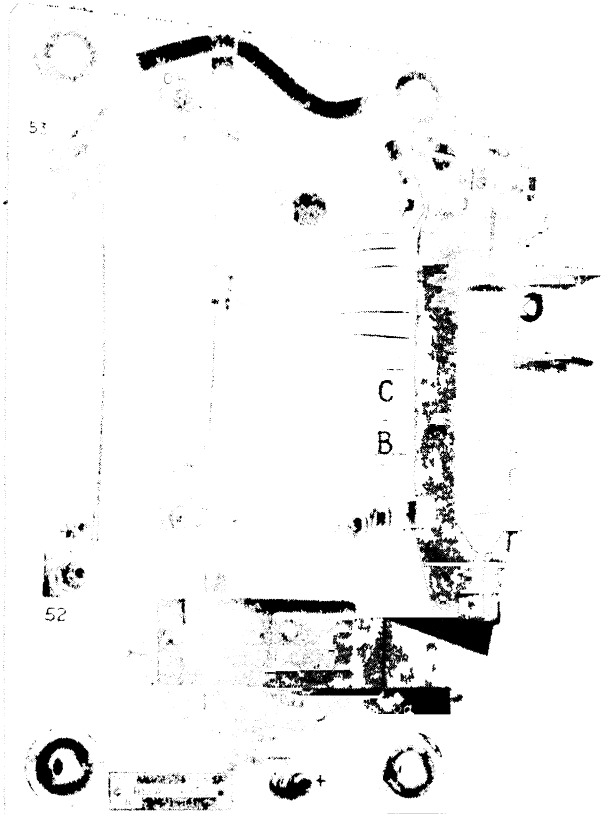


FIG. 113 ACCELERATING OR NOTCHING RELAY AS USED WITH TYPE PCM EQUIPMENT (B.T.-H.)

(i) that the operation of the transfer switch, *PSG*, depends upon the interlock *PC*10 being closed (assuming train wire No. 3 to be connected to the control supply), and (ii) that interlocks on *PSG* establish a retaining circuit for its valve-magnet and open the circuit of the cam-shaft valve-magnet. The cam-shaft therefore returns to its initial position, No. 1, which is now the full-parallel position.*

* Full parallel, full field, is obtained when the cam shaft reaches position No. 2. The final step enables one step of field weakening, if desired, to be obtained under the control of the accelerating relay.

In the equipments in service on the London Transport lines provision is made for two stages of field weakening. A special hand-operated switch is provided in the driver's cab together with additional contacts on the master controller and an extra wire in the control bus line. When this switch is closed

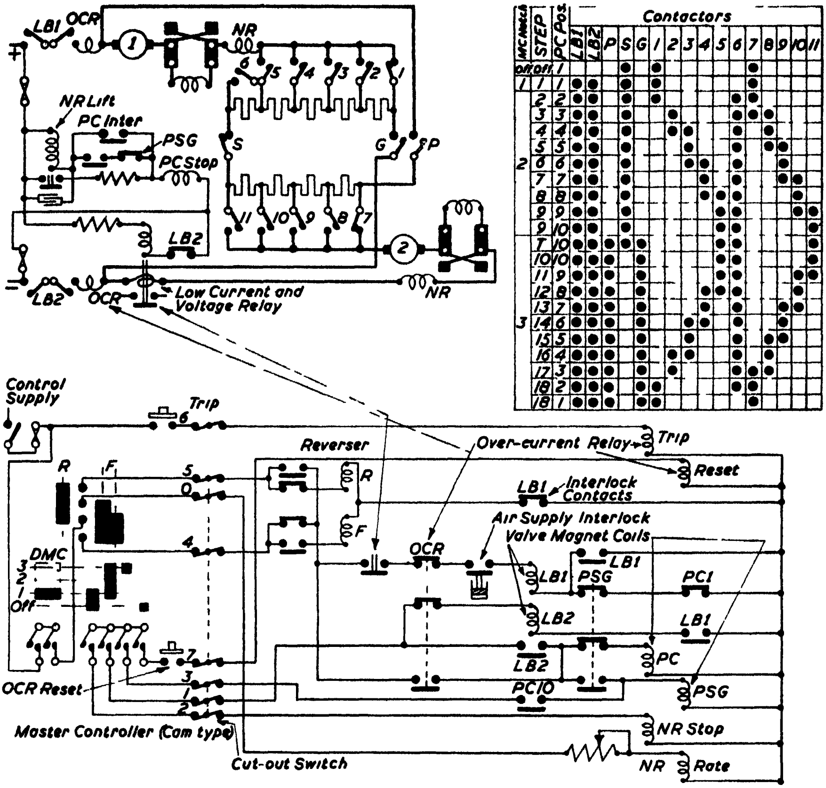


FIG. 114. SIMPLIFIED MAIN- AND CONTROL-CIRCUIT CONNEXIONS FOR TYPE PCM EQUIPMENT (B.T.-H.)

OCR and *NR* denote the over-current and notching (accelerating) relays respectively, *PSG* denotes the series-parallel transfer switch

and the master controller is in position No. 3 the first stage of field weakening is obtained with the cam-shaft in position No. 1, and the second stage is obtained under the control of a series relay.*

Electro-pneumatic Cam-shaft equipment for 1,500-volt Circuits. A typical equipment for compartment mounting on a motor-coach is shown in Fig. 115, and differs from the 600-V equipment only in (i) additional insulation, (ii) barriers between the cam-shaft contactors, (iii) individual contactors for transfer from series to parallel, (iv) three stages of field weakening.

* For connexions, *see* paper by E. Webster on "Railway Traction Control Equipment on Suburban London Transport," *Proc. I.E.E.* Vol. 96, Pt. II, p. 537.

All-electric (English Electric Co.'s) System. The essential difference between this and the electro-pneumatic system is the method of operation. The cam-shaft is operated by a small electric motor and the reverser is operated by electromagnets. The line, or circuit-breaking, contactors are of either the electromagnetic or cam types, the latter being operated by cams fitted to an extension of the main cam-shaft.

A typical 1,500-V motor-coach equipment is shown in Fig. 116.

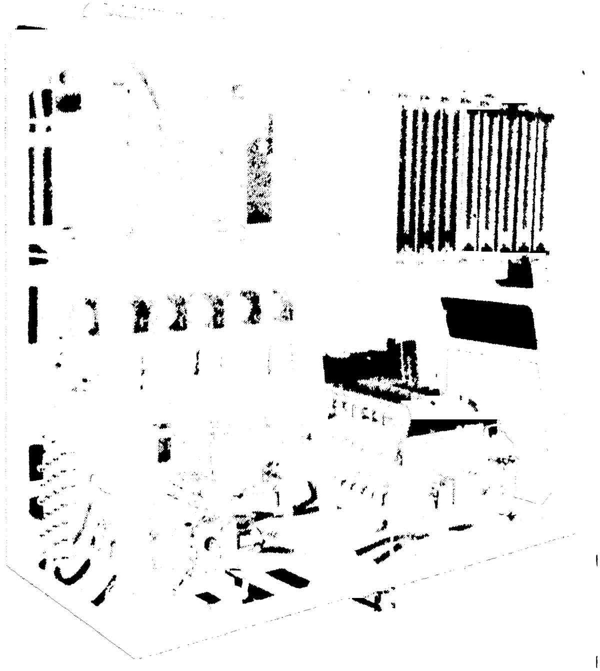


FIG. 115. B.T.-H. 1,500-V ELECTRO-PNEUMATIC CAM-SHAFT CONTROL EQUIPMENT
(*a*) Line and *PSG* contactors; (*b*) cam-shaft contactors; (*c*) over-current relays; (*d*) field-shunting contactors, (*e*) accelerating and field-shunting relays, and reverser.

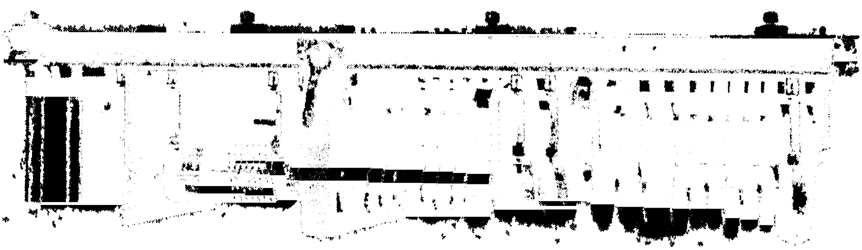
Principle of Operation. The cam-shaft is driven through gearing by a low-voltage separately-excited motor, the starting and stopping of which is controlled by a contactor (called the "cam-shaft-motor relay"). In automatic equipments this relay is controlled by the accelerating relay in conjunction with an interlocking drum (called the "position regulator") on the cam-shaft, the supply to which is controlled by two relays (called "position relays") energized from the master controller. In non-automatic equipments the cam-shaft-motor relay is controlled entirely by the position regulator, and several position relays are necessary.

The reversing switch for the cam-shaft motor is fitted to the line, or circuit-breaking, contactors, and is so arranged that the motor cannot start in the forward direction until these contactors have closed, nor start in the backward direction (for moving the cam-shaft to the "off" position) until these contactors have opened. Hence the ainm circuit is always broken by the line contactors.

Connexions of Automatic Equipment with "Advance" Feature. A simplified diagram of connexions is shown in Fig. 117.

The series and parallel positions of the cam-shaft are controlled by two relays, S and P, which are energized in positions Nos. 2 and 3, respectively, of the master controller. In position No. 1 of the master controller the cam-shaft can only move to position No. 1, as the supply to the cam-shaft-motor relay is then cut-off by the position regulator, due to relay S not being energized.

The *current-limit*, or accelerating, *relay* operates in conjunction with a special steel cam fitted to the end of the cam-shaft. The profile of this cam consists of a number of notches, each notch corresponding to an operating position of the cam-shaft. The pawl which engages with these notches carries one of the contacts of the relay, the other contact being attached to the pivoted armature. (These contacts control the cam-shaft motor relay.) The contacts, relay armature, and pawl are so arranged that when the pawl occupies a notch in the cam the contacts are closed if the relay armature is released, but are open



(a) (b) (c) (d) (e)

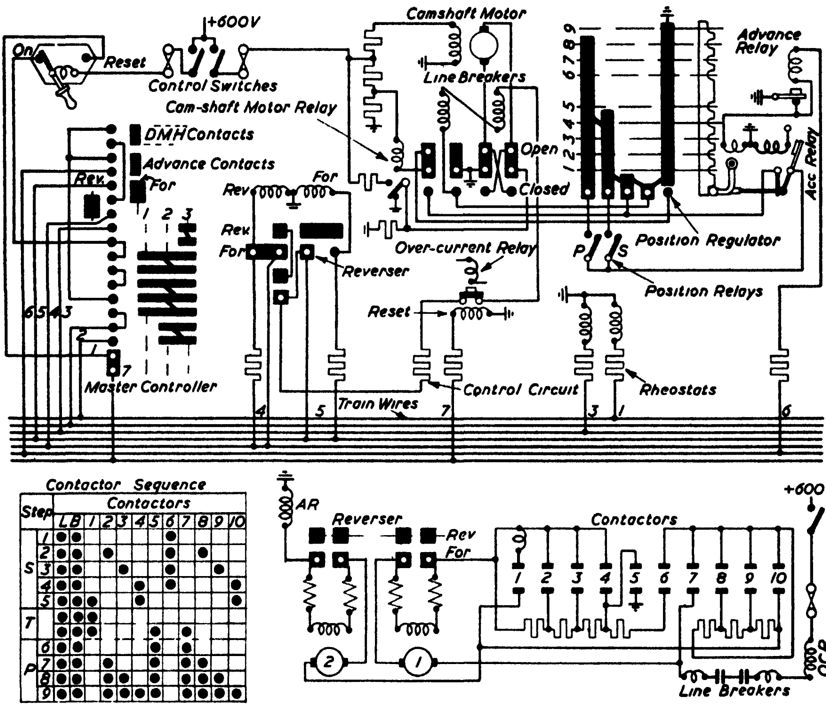
FIG 116 TYPICAL ALL-ELECTRIC CAM-SHAFT CONTROL EQUIPMENT (ENGLISH ELECTRIC)
(a) Line contactors (b) motor and gear box, (c) position regulator, (d), (e) contactors

if the armature is held up by the current in the operating coil (which is connected in the main circuit of one of the motors). When the cam-shaft advances (due to these contacts being closed), the pawl, together with the relay armature, is lifted by the cam (the contacts remaining closed), and, if the motor current is above the prescribed value for which the relay is set to operate, the armature is held while the pawl passes into the next notch in the cam. This operation opens the relay contacts, thereby interrupting the operating coil circuit of the cam-shaft motor relay, and stopping the motor. Observe that the cam-shaft motor is stopped by cutting off power from, and immediately short-circuiting, the armature, the field remaining excited. The cam-shaft motor will not again start until the circuit of its relay is re-established by the accelerating relay.

The "*by-pass*" or "*advance*" *feature* consists of a special lever (spring-biased to the "off" position) and contacts on the master controller, together with an auxiliary relay and a demagnetizing shunt coil on the accelerating relay. When the lever is placed in the "advance" position, the auxiliary relay is energized and the lifting of its armature connects the operating coil in series with the demagnetizing coil of the accelerating relay. The armature of the latter (if held up by a high motor current) is, therefore, released, and the cam-shaft is advanced one step.

CONTROL SYSTEMS FOR LOCOMOTIVE EQUIPMENTS (1,500 AND 3,000 VOLTS)

General. *A locomotive must be capable of operating under variable service conditions* which may include shunting service as well as the hauling of trains of various weights at slow and moderate speeds. Moreover, the conditions at starting are of an entirely different nature to those relating to motor-coach operation. In the latter case a high acceleration is essential, and the starting



Step		Contactors LB		1	2	3	4	5	6	7	8	9	10
S	1	●	●						●				
	2	●	●		●				●		●		
	3	●	●			●			●			●	
	4	●	●				●		●				●
	5	●	●	●			●						●
T		●	●	●									
		●	●	●				●		●			
P	6	●	●					●		●			
	7	●	●		●			●		●	●		
	8	●	●		●	●		●		●	●	●	
	9	●	●		●	●	●	●		●	●	●	●

FIG. 117. CONNEXIONS OF ENGLISH-ELECTRIC CAM-SHAFT CONTROL SYSTEM

period rarely exceeds 20 seconds. But with locomotive operation the acceleration must be limited to moderate values and the starting period may, under certain conditions, occupy several minutes. For example, in starting a long loose-coupled freight train, the initial tractive effort must be limited to a moderate value in order that the slack may gradually be pulled out of the couplings. This initial tractive effort must be increased by uniform increments until the normal accelerating tractive effort is reached. In passing from notch to notch the fluctuations in the tractive effort must be limited to relatively small values and must be uniform, both in order to avoid shock and to prevent slipping of the driving wheels.

Motor Combinations. With line voltages above 1,500 volts, two or more motors are connected permanently in series, and, for control purposes, are considered as equivalent to a single motor. Both series-parallel and double

series-parallel control are employed, the former being used *for locomotives* of moderate weight and the latter for heavy locomotives.

Transition is usually effected by the shunt method, as the total number of contactors required for control is then smaller than that required when bridge transition is adopted, although the number of contactors involved in the transition is greater with the former method. Considerable simplification of the control gear is possible by arranging the contactors effecting the combinations of the motors as a cam-operated group controlled electro-pneumatically, a typical example being shown in Fig. 118.

Shunt transition also enables all the field windings to be arranged on the earthed side of the system. This arrangement is advantageous for high-voltage

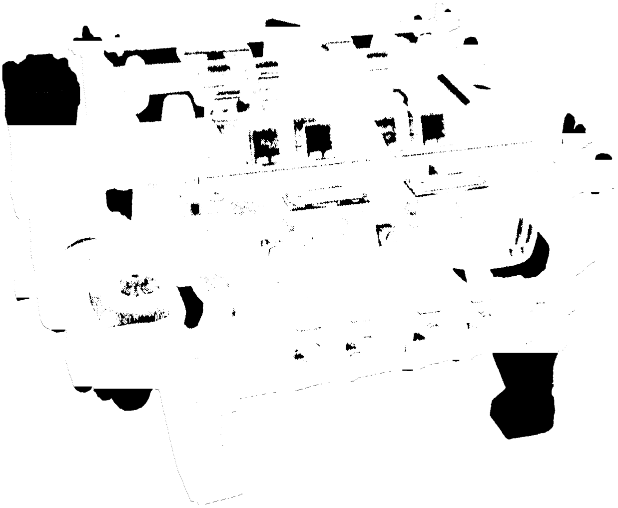


FIG. 118. CAM-OPERATED CONTACTOR GROUP FOR CHANGING COMBINATION OF MOTORS (METROPOLITAN-VICKERS)

circuits, as the reverser and field-tapping contactors may be of the low-voltage type. Moreover, as these switches and the field windings are subjected only to low voltages, the possibility of insulation breakdowns is eliminated.

Contactors. The special features of individual circuit-breaking contactors for high-voltage (3,000-V) circuits are (i) the insulation, (ii) the magnetic blow-out and arc chute.

A typical individual contactor is illustrated in Fig. 119. Both the fixed and moving contacts are insulated to enable two or more contactors to be connected in series when necessary. A strong blow-out field and a large narrow arc chute are provided for the suppression of the arc. The narrow arc chute is much more effective than a wide chute on account of the arc stream being brought more into contact with the sides of the chute and being thereby cooled more rapidly. In some cases a transverse barrier (called an "arc-splitter") is inserted in the arc chute for the purpose of increasing the cooling surface exposed to the arc stream.*

* Comprehensive data of magnetic blow-outs, together with photographs of arc streams, are given in a paper and discussion on "Air-break magnetic blow-outs," by J. F. Tritle, *Trans. A.I.E.E.*, XLI, 262–287.

(*a*) (*b*) (*c*)

FIG 119. ELECTRO-PNEUMATIC CONTACTOR FOR 3,000-VOLT CIRCUITS (METROPOLITAN-VICKERS)

(*a*) The complete contactor (*b*) Front and (*c*) side of contactor with arc chute removed

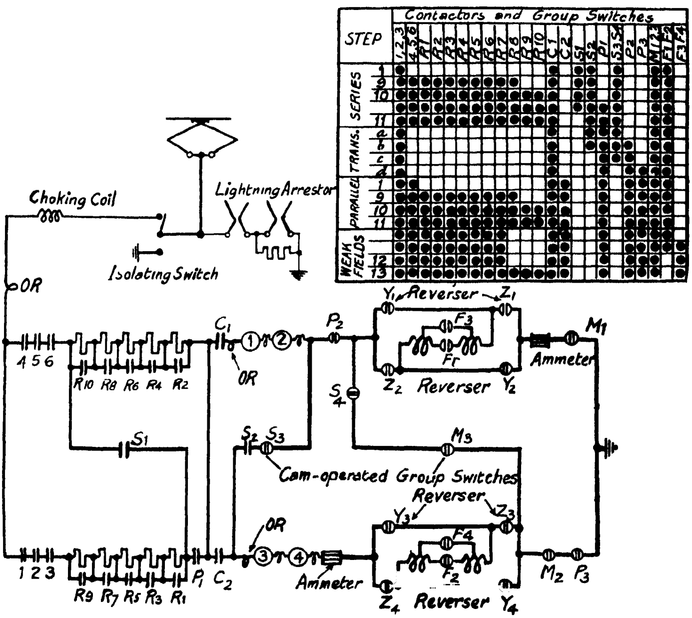


FIG. 120. MAIN-CIRCUIT CONNEXIONS FOR SERIES-PARALLEL CONTROL OF 3,000-VOLT LOCOMOTIVE EQUIPMENT (METROPOLITAN-VICKERS)

Switching operations on the rheostatic steps 2–8 inclusive involve closing, in successive order contactors R_1–R_7.

Example of Series-parallel Control for 3,000-volt Locomotive. This example refers to a 1,200-h.p. locomotive with Metropolitan-Vickers equipment. The electro-pneumatic unit-switch control system is employed and is arranged for regenerative braking, but the consideration of the regenerative-control features will be deferred for the present. Individual contactors, or unit switches, are used for the rheostatic steps and at other positions in the circuit where current has to be broken. But cam-operated contactor groups are employed for changing the motor combinations, field tapping, and reversing, as well as for changing over the main-circuit connexions from "motoring" to "generating."

A simplified diagram of the main-circuit connexions is given in Fig. 120.

The control is, of course, non-automatic, and, on account of the regenerative features, the master controller has three operating handles and contact drums (i.e. the combined reversing and motor-combination drum, the braking drum, and the rheostatic or accelerating drum) as shown in Fig. 121.

In operation the combined reversing and motor-combination handle is thrown to the "forward series" position and the accelerating handle is notched

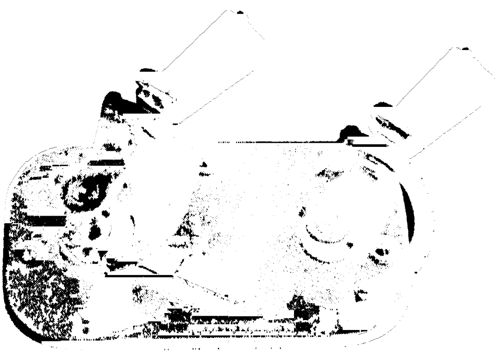


FIG. 121. CAP-PLATE AND OPERATING HANDLES OF MASTER CONTROLLER FOR LOCOMOTIVE (METROPOLITAN-VICKERS)

up to the full-series or weak-field position (tapped-field operation being possible with both combinations of the motors). Next the combined reversing and motor-combination handle is moved into the "forward parallel" position, and the accelerating handle is returned to the first notch, which causes transition to be effected automatically. The handle is then notched-up to the running position desired.

The manner in which transition is effected is as follows—

When the accelerating handle is returned to the first notch—the motor-combination handle being set for "forward parallel"—the opening of contactor R_1 energizes the "parallel" valve magnet of the pneumatic cylinders of the motor-combination cam-contactor group. As soon as contactor P_2 of this group closes, the circuits of the "parallel" valve magnet and the valve magnet of contactor S_2 are broken at the interlocking drum. When S_2 opens, the "parallel" valve magnet of the cam group is again energized—by means of interlocks on S_2—and the transition is completed by (i) closing contactor P_3, (ii)

opening contactors S_3, S_4, and (iii) energizing the valve magnets of contactors LS_4, LS_5, LS_6, C_2, the latter operation being effected by the interlocking drum of the cam group.

The valve-magnet coils of all the rheostatic contactors are connected to a special set of interlocking contacts under the control of an auxiliary relay, the magnet-coil of which is in circuit with the auxiliary contacts of the three overload relays. Hence the operation of any of the overload relays causes the whole of the rheostatic contactors to open, and in consequence the whole of the starting resistance is inserted before the line contactors open, the latter operation being effected by interlocks on contactor R_1. In this manner the arcing at the line contactors is much less severe than it would be if these contactors were opened directly by the overload relays.

Examples of Double Series-parallel Control for 1,500-volt 4-motor and 6-motor Locomotives. The motors of the four-motor locomotive are designed for the full line voltage, but those of the six-motor locomotive are designed for one half of the line voltage. With each equipment three speeds, with a given voltage and excitation, are obtained, these speeds being in the ratio of 1 : 2 : 4 with the four-motor equipment, and in the ratio of 1 : 2 : 3 with the six-motor equipment.

A schematic diagram of the main-circuit connexions of the four-motor equipment is shown in Fig. 122. There are five running steps for each combination of the motors, four being obtained by field weakening. The shunt method of transition is employed, and the exciting-field windings are all connected on the earthed side of the system.

A simplified diagram of the main-circuit connexions of the six-motor equipment is shown in Fig. 123, from which it will be observed that (i) two steps of shunted-field control—using inductive shunts*—are employed for each combination of the motors; (ii) the motor and control equipment is arranged in two halves.

APPENDIX TO CHAPTER IX

THE CALCULATION OF STARTING RHEOSTATS FOR DIRECT-CURRENT MOTORS

General Considerations. A reference to the connexion diagrams of series-parallel controllers will show that the starting rheostats must be arranged with a limited number of sections, and that some of the sections must be suitable for use with both the series and the parallel positions of the controller. The number of sections for use with a given motor equipment is governed by (i) the maximum tractive-effort which may be exerted during the starting period; (ii) the permissible variation of the tractive-effort. These considerations involve others, such as the adhesive weight of the train, locomotive, or car; the coefficient of adhesion; the mass to be accelerated; and the gradients (if any) which have to be negotiated.

With tramcars the adhesive weight is from 75 per cent to 100 per cent of the weight of the car, while with motor-coach trains the adhesive weight is usually of the order of 50 per cent of the weight of the train. Generally, in each of these cases, the mean accelerating tractive-effort during starting is much below that required to slip the driving wheels under normal conditions (i.e. with dry rails). Therefore, relatively large fluctuations in the tractive effort are permissible, so that only a few notches are required on the controller.

* An inductive shunt consists of a laminated-iron core with air gaps and a magnetizing winding, with a resistor connected in series.

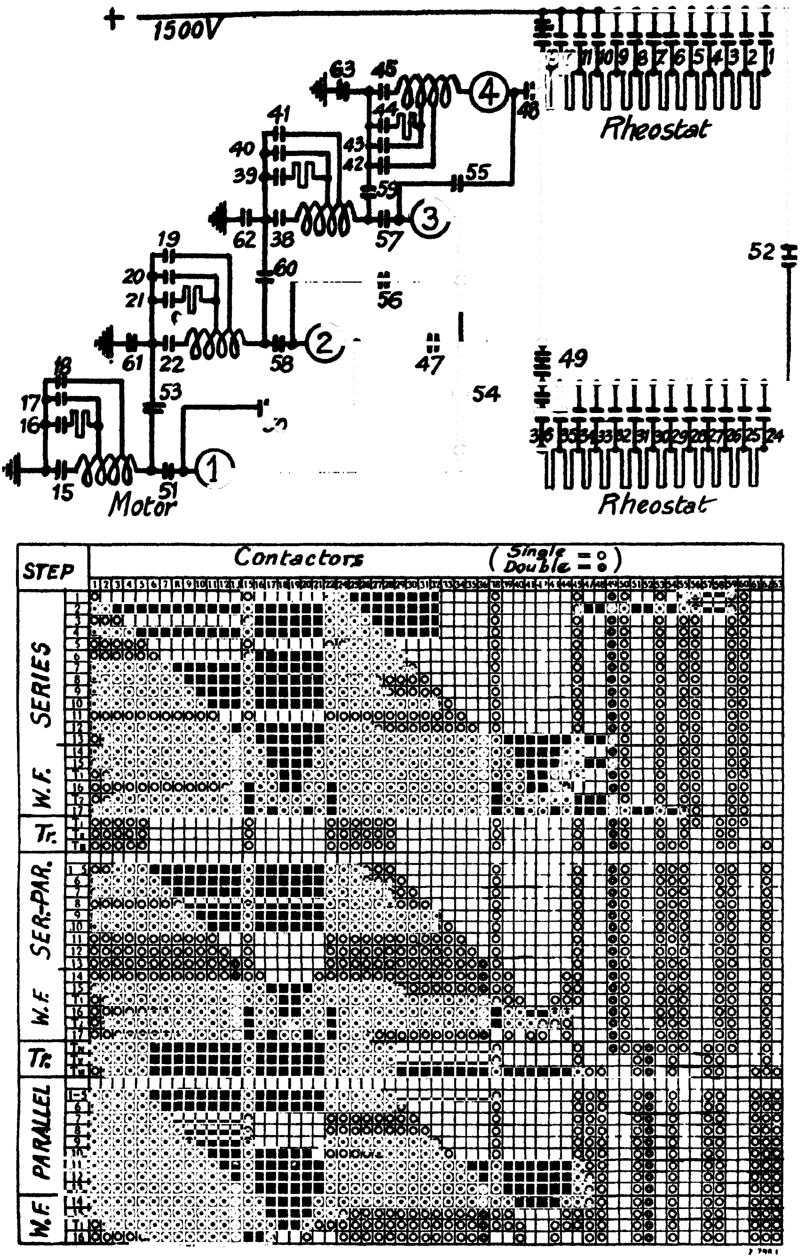


FIG. 122. MAIN-CIRCUIT CONNEXIONS FOR DOUBLE SERIES-PARALLEL CONTROL OF 4-MOTOR, 1,500-VOLT LOCOMOTIVE EQUIPMENT (BROWN-BOVERI)

On the other hand, with electric locomotives, the tractive-effort required for acceleration may approach the limiting value at which slipping of the driving wheels occurs; and in this case only a relatively small variation of the tractive-effort is permissible during starting, so that a large number of notches will be required on the controller.

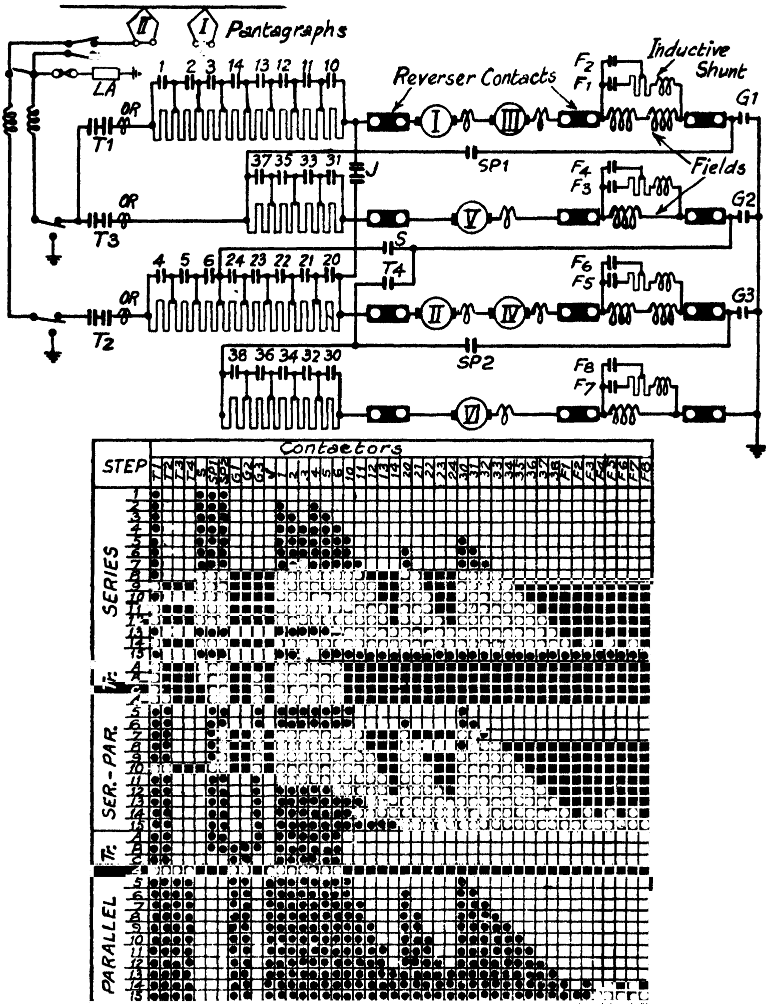


FIG. 123. MAIN-CIRCUIT CONNEXIONS FOR DOUBLE SERIES-PARALLEL CONTROL OF 6-MOTOR, 1,500-VOLT LOCOMOTIVE EQUIPMENT (GENERAL ELECTRIC CO.)

We shall show that the number of sections and also the resistances of the sections are both influenced by the slope of the speed curve between the limits of current during starting. In general, a motor possessing a steep speed curve (e.g. a single-phase motor, or a d.c. motor with either an unsaturated magnetic circuit or a high internal resistance) requires fewer rheostat sections (for a given percentage variation of tractive-effort) than a motor possessing a flat speed curve.

The data required for the calculation of the number of sections and the resistances of the sections are: the speed and tractive-effort curves of the motor (corresponding to the line voltage at which the start is to be made); the resistance of the motor; the limiting value of the tractive effort and the permissible variation of tractive effort during starting. The calculations involve the use of *two* coefficients—which are called "grading coefficients"—namely, the ratio of the lower and upper limits of the current during starting and the ratio of the tangents of the magnetization curve at these currents. They cannot, therefore, be effected without a knowledge of the magnetization curve (or, alternatively, the speed curve) of the motor. It is this fact that renders the calculations more complicated than those for the starting rheostat for a shunt motor (in which only one coefficient is involved).

The simplest solution is obtained by deriving a second relationship between the coefficients in *general terms* (i.e. independent specifically of any particular motor),

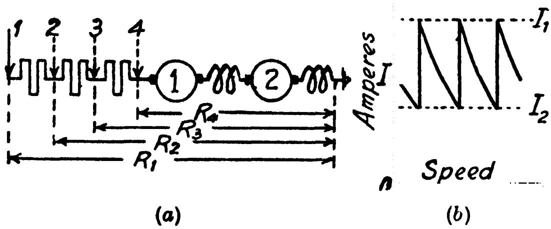


FIG. 124. CIRCUIT DIAGRAM AND STARTING CONDITIONS FOR CALCULATIONS OF STARTING RHEOSTATS

as with two relationships available, each of the two coefficients can be determined without difficulty.

The method is best explained by deducing from first principles, and in general terms, the equations relating to the starting period.

Calculation of Resistances of Sections for Series Steps. Consider a single starting rheostat to be connected in circuit with two motors as shown in the circuit diagram of Fig. 124 (*a*). Let n denote the number of sections in the rheostat; $(n + 1)$ the number of steps or notches in the series position of the controller; $R_1, R_2, \ldots R_n, R_{n+1}$, the resistances of the circuit on each of the several notches; R_m the resistance of each motor. If the starting conditions require an equal variation of tractive effort on each step, together with the same upper limit of tractive effort on all steps except the first (on which a lower tractive effort is required), the variation of current during the series portion of the starting period will be as shown in Fig. 124 (*b*). Denote the upper and lower limits of current by I_1 I_2, respectively, and the initial current (which is between I_1 and I_2) by I.

Then initially, $I = V/R_1$; where V denotes the line voltage which is assumed to be constant.

The resistance R_1 is kept in circuit until the current decreases to the lower limit, I_2, when the first section $(R_1 - R_2)$ is cut out so as to obtain the upper current limit I_1. The current change from I_2 to I_1 is assumed to take place instantaneously in all cases, i.e. the speed is unchanged.

Hence, since the counter-e.m.f. of any motor is proportional to the product of flux and speed, the counter-e.m.f's. of each motor immediately prior and subsequent to the change in the circuit conditions are

$$\tfrac{1}{2}(V - I_2R_1) = k\Phi_2, \text{ and } \tfrac{1}{2}(V - I_1R_2) = k\Phi_1, \text{ respectively.}$$

Whence
$$(V - I_1R_2)/(V - I_2R_1) = \Phi_1/\Phi_2.$$

Similarly, the resistance R_2 is kept in circuit until the current decreases again to its lower limit, I_2, when the second section $(R_2 - R_3)$ is cut out so as to obtain the upper limit, I_1. Hence, the counter-e.m.f's. immediately prior and subsequent to this change are

$$\tfrac{1}{2}(V - I_2R_2) = k\Phi_2, \text{ and } \tfrac{1}{2}(V - I_1R_3) = k\Phi_1, \text{ respectively.}$$

Whence $$(V - I_1R_3)/(V - I_2R_1) = \Phi_1/\Phi_2.$$

This process is continued until, finally, the last section $[(R_n - R_{n+1}) = (R_n - 2R_m)]$ is cut out. For this step we have

$$\tfrac{1}{2}(V - I_2R_n) = k\Phi_2, \text{ and } \tfrac{1}{2}(V - 2I_1R_m) = k\Phi_1.$$

Whence $$(V - 2I_1R_m)/(V - I_2R_n) = \Phi_1/\Phi_2.$$

Hence generally

$$\frac{\Phi_1}{\Phi_2} = \frac{V - I_1R_2}{V - I_2R_1} = \frac{V - I_1R_3}{V - I_2R_2} = \ldots\ldots = \frac{V - 2I_1R_m}{V - I_2R_n} \quad . \quad . \quad (31)$$

These, then, are the fundamental equations connecting the circuit resistances of the steps for the specified starting conditions. Observe that the ratio of the fluxes is involved as well as the limits of current.

When the rheostats consist of only two or three sections the resistances of these sections may be determined by solving the general equations (31) by trial and error. Thus the upper limit of current, I_1, being fixed, the flux Φ_1 is obtained from the magnetization curve (or, alternatively, from the speed curve if the magnetization curve is not available). The lower limit is then assumed, Φ_2 is obtained from the magnetization curve and R_2 is calculated, R_1 being known from the initial conditions. The value of R_2 is then substituted and a solution obtained for R_3, and so on, until finally the value obtained for R_m, when solving the final equation, should agree with the actual resistance of the motor, otherwise a fresh trial must be made with a different value for I_2.

Such a trial and error method, however, is not to be recommended on account of its tediousness and the amount of time consumed. The method is, of course, impracticable when the number of sections is large. Moreover, since the ultimate requirement is the resistances of the sections, rather than the circuit resistances of the successive steps, a method is desirable in which the former can be calculated directly. For these cases one of the simplest analytical solutions (which is universal in its application) is that developed by the author and given in a paper to the Institution of Electrical Engineers.*

Starting with the fundamental equations (31), the numerators are divided throughout by I_1 and the denominators by I_2, thus

$$\frac{\Phi_1/I_1}{\Phi_2/I_2} = \frac{V/I_1 - R_2}{V/I_2 - R_1} = \frac{V/I_1 - R_3}{V/I_2 - R_2} = \ldots = \frac{V/I_1 - 2R_m}{V/I_1 - R_n} \quad . \quad . \quad (i)$$

Let $$\lambda = (\Phi_1/I_1)/(\Phi_2/I_2); \ \gamma = I_2/I_1; \ \zeta = I_1/I.$$

Then $$V/I_1 = (V/I)(I/I_1) = V/I\zeta = R_1/\zeta.$$

$$V/I_2 = (V/I_1)(I_1/I_2) = V/I_1\gamma = R_1/\gamma\zeta.$$

Substituting these values in equations (i), we have

$$\lambda = \frac{R_1/\zeta - R_2}{R_1/\gamma\zeta - R_1} = \frac{R_1/\zeta - R_3}{R_1/\gamma\zeta - R_2} = \ldots = \frac{R_1/\zeta - 2R_m}{R_1/\gamma\zeta - R_n} \quad . \quad . \quad (ii)$$

* "A universal 'Chart' method of calculating starting rheostats for direct-current motors," *Journal I.E.E.*, Vol. 60, p. 867. For alternative methods see Vol. 58, p. 645, "Analytical determination of the steps in the starter of a series motor," by S. Parker Smith; Vol. 99, Pt. IV, p. 1, "A method of calculating the grading of resistors for electric traction equipments, with particular reference to multiple-unit trains using automatic acceleration," by A. F. Harvey.

Cross-multiplying, we have

$$R_1/\zeta - R_2 = \lambda(R_1/\gamma\zeta - R_1) = R_1\lambda(1/\gamma\zeta - 1) \quad . \quad . \quad \text{(iii}a\text{)}$$

$$R_1/\zeta - R_3 = \lambda(R_1/\gamma\zeta - R_2) \quad . \quad . \quad . \quad . \quad . \quad \text{(iii}b\text{)}$$

etc. etc.

Add $(R_1 - R_1/\zeta)$ to each side of the first equation, so as to eliminate R_1/ζ from the left-hand side, and simplify, thus

$$R_1 - R_2 = R_1\left\{\frac{1}{\zeta}\left(\frac{\lambda}{\gamma} - 1\right) + \left(1 - \lambda\right)\right\} = KR_1 \quad . \quad . \quad \text{(iv}a\text{)}$$

Reduce the *left-hand* sides of equations (iii$b$), (iii$c$), etc., to the form $(R_2 - R_3)$, $(R_3 - R_4)$, etc., by subtracting successively the upper from the lower equation of each pair (i.e. subtract equation (iii$a$) from (iii$b$), equation (iii$b$) from (iii$c$), and so on). Thus

$$R_2 - R_3 = \lambda(R_1 - R_2) = \lambda KR_1 \quad . \quad . \quad . \quad \text{(iv}b\text{)}$$

$$R_3 - R_4 = \lambda(R_2 - R_3) = \lambda^2KR_1 \quad . \quad . \quad . \quad \text{(iv}c\text{)}$$

$$R_n - 2R_m = \lambda(R_{n+1} - R_n) = \lambda^{n-1}KR_1 \quad . \quad . \quad \text{(iv}d\text{)}$$

Observe that these equations give directly the resistances of the sections of the rheostat, and that there is a common ratio (λ) between the resistances of successive sections. Observe, also, that the numerical value of λ depends upon the quotient of the ratio of fluxes and the ratio of currents corresponding to the current limits during starting (or alternatively upon the ratio of the tangents of the angles formed by lines drawn from the origin to the points on the magnetization curve corresponding to these currents). Thus the *value of λ depends upon the slope of that portion of the magnetization curve which lies between the current limits I_1, I_2.* Its maximum value is unity—which corresponds to a straight-line magnetization curve (i.e. a magnetic circuit worked at extremely low flux densities)—and its minimum value is equal to γ. In the latter case the flux is constant, and if the substitution $\lambda = \gamma$ be made in equations (iv) we obtain the resistances of the rheostat sections for a shunt motor.

Proceeding with the determination of a general relationship between λ and γ, we add together the whole of the left-hand sides of equations (iv$a$) to (iv$n$), inclusive, and perform a similar operation on the right-hand sides. The result is

$$\begin{aligned} R_1 - 2R_m &= KR_1(1 + \lambda + \lambda^2 + \ldots + \lambda^{n-1}) \\ &= KR_1(1 - \lambda^n)/(1 - \lambda) \\ &= R_1\left\{\frac{1}{\zeta}\left(\frac{\lambda}{\gamma} - 1\right) + \left(1 - \lambda\right)\right\}\frac{1 - \lambda^n}{1 - \lambda} \quad . \quad . \quad . \quad \text{(v)} \end{aligned}$$

Whence by rearrangement

$$\frac{2R_m}{R_1} = 1 - \left\{\frac{1}{\zeta}\left(\frac{\lambda}{\gamma} - 1\right) + \left(1 - \lambda\right)\right\}\frac{1 - \lambda^n}{1 - \lambda} \quad . \quad . \quad . \quad \text{(vi)}$$

But if e is the voltage drop in the *two* motors corresponding to the current I, then $e = 2R_mI$. Hence $2R_m/R_1 = (e/I)/(V/I) = e/V$.

The relationship between γ and λ can therefore be calculated for definite numbers of sections, and definite values of (e/V) and $\zeta(= I_1/I)$. Moreover, the results can be plotted in the form of universal curves for general use. Such calculations have been made by the author for a number of starting conditions, and the curves are published in the *Journal of the Institution of Electrical Engineers*, Vol. 60, pp. 870–880.

These curves give the *general* relationship between γ and λ/γ for values of n between 2 and 12, inclusive, for $\zeta = 1{\cdot}5$, and for appropriate values of (e/V). The relationship is given for γ and λ/γ, instead of for γ and λ, since $\lambda/\gamma = \Phi_1/\Phi_2$, and

therefore the co-ordinates represent the ratio of currents and the ratio of the corresponding fluxes, thereby enabling a second or *particular* relationship between these quantities to be plotted on the appropriate curve sheet (or preferably a sheet of superimposed tracing paper). The point of intersection of this (second) curve with the appropriate curve on the chart, therefore, gives both γ and λ.

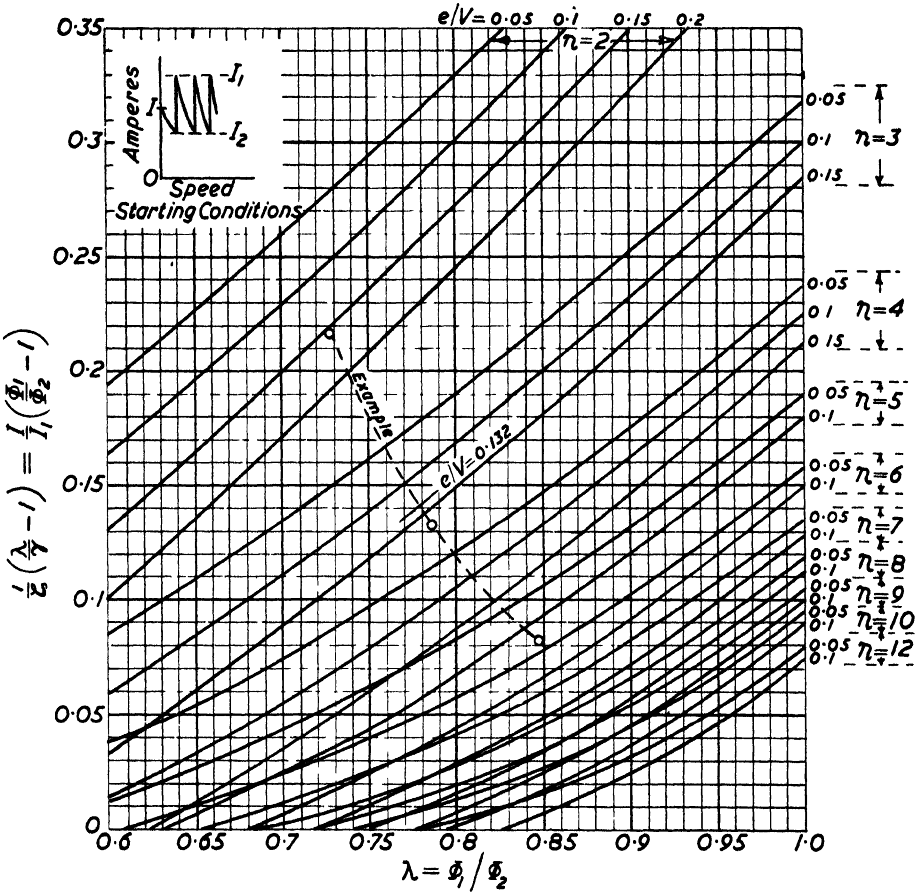


FIG. 125. CHART FOR CALCULATION OF SECTIONS OF STARTING RHEOSTATS (SERIES COMBINATION OF MOTORS)

Traction operating conditions, however, render it desirable for the rheostat designer to have freedom in the selection of the initial starting current, so that in this case the ratio I_1/I cannot be fixed for the purpose of calculating the general relationship between λ and γ. But the above principle can be adapted to include this case. Thus, by re-arranging equation (vi) after substituting e/V for $2R_m/R_1$, we obtain

$$\frac{1}{\zeta}\left(\frac{\lambda}{\gamma} - 1\right) = (1 - e/V)\frac{1-\lambda}{1-\lambda^n} - (1-\lambda) \qquad \text{(via)}$$

Therefore, the quantity $(\lambda/\gamma - 1)/\zeta$ $[= (\Phi_1/\Phi_2 - 1)/(I_1/I)]$ can be calculated in terms of λ for given values of n and e/V. The results are plotted in the curves of Fig. 125 and give the *general* relationship between λ and $(\lambda/\gamma - 1)/\zeta$. The *particular*

relationship between these quantities for the particular motor, starting conditions, and number of steps in the controller, is obtained as follows—

The upper limit of current I_1, is either given directly or is obtained from the current/tractive-effort curve of the motor for the specified upper limit of the tractive effort. Three values—say I_2', I_2'', I_2'''—are assumed for the lower limit of current over a range which should be a little greater than that actually expected during starting, but the range selected is immaterial, provided, of course, that all the assumed values are below the upper limit of current. The ratios of currents (i.e. I_2'/I_1, I_2''/I_1, I_2'''/I_1) are calculated.

The fluxes Φ_1, Φ_2', Φ_2'', Φ_2''', corresponding to these currents are obtained either directly from the magnetization curve or indirectly from the speed curve. Usually the latter procedure will have to be adopted. In this case, assuming that the given speed curve is that corresponding to the line voltage V, calculate the internal e.m.f's. corresponding to the currents I_1, I_2', I_2'', I_2''' and a terminal voltage (per motor) V. Obtain the speeds s_1, s_2', s_2'', s''', and calculate the flux ratios. Thus, since flux is proportional to (counter-e.m.f./speed), we have

$$\frac{\Phi_1}{\Phi_2'} = \frac{s_2'(V - I_1R_m)}{s_1(V - I_2'R_m)};\quad \frac{\Phi_1}{\Phi_2''} = \frac{s_2''(V - I_1R_m)}{s_1(V - I_2''R_m)};\quad \frac{\Phi_1}{\Phi_2'''} = \frac{s_2'''(V - I_1R_m)}{s_2(V - I_2'''R_m)}$$

where R_m is the resistance of the motor circuit appropriate to the speed curve in use.

Next calculate λ corresponding to the above ratios of fluxes and currents (e.g. $\lambda' = (\Phi_1/I_1)/(\Phi_2'/I_2')$, etc.), and from the given initial starting current calculate the quantities $(\lambda'/\gamma' - 1/\zeta)$, etc. These calculations are quickly carried through if arranged in tabular form as in the example which follows. Plot the results on a slip of tracing paper superimposed upon Fig. 125, and determine the point of intersection with the appropriate curve. λ and γ are then determined.

Determination of Number of Steps for Given Starting Conditions. When the initial starting current and the limits of currents during starting are fixed the number of steps in the starter for a given motor can be calculated by substituting appropriate values in equation (via) and solving for n. Thus

$$n = \log\left\{1 - \frac{(1-\lambda)(1-e/V)}{1-\lambda+(\lambda/\gamma-1)/\zeta}\right\}/\log\lambda \qquad . \qquad . \qquad \text{(vii)}$$

A simpler process, however, is to determine which of the appropriate curves of Fig. 125 has co-ordinates corresponding to the given values of λ and $(\lambda/\gamma - 1)/\zeta$. Thus values for the quantities λ and $(\lambda/\gamma - 1)/\zeta$ are calculated for the given starting conditions and the particular motor. Co-ordinates corresponding to these values are drawn in Fig. 125, and the appropriate e/V curve which passes through, or is nearest to, the point of intersection of the co-ordinates gives the value of n directly.

Example. Calculation of the sections of the starting rheostat for the series steps of a standard tramcar controller to be used with two 50-h.p., 525-volt motors. The resistance of each motor is 0·45 ohm. The initial tractive-effort per motor at starting is to be 1,100 lb and the maximum tractive-effort per motor is to be 1,750 lb. The relationship between current, speed and tractive-effort is—

Current (amp) . . .	60	70	80	105
Speed (m.p.h.) . . .	18·2	16·7	15·6	13·7
Tractive-effort (lb) . .	760	950	1,150	1,750

To comply with the starting requirements we select the initial starting current at 77A, and the upper limit of current at 105A. Hence $\zeta = I_1/I = 105/77 = 1{\cdot}36$. The voltage drop in the *two* motors at a current of 77A is $77 \times 2 \times 0{\cdot}45 = 69{\cdot}3$ V. Whence $e/V = 69{\cdot}3/525 = 0{\cdot}132$.

We next calculate the particular relationship between λ and λ/γ for a fixed

upper limit of 105A and lower limits of 80A, 70A, 60A. The calculations are arranged in tabular form

I_1	105						
I_2			80		70		60
$\gamma(=I_2/I_1)$		0·762		0·666		0·571	
Voltage drop per motor . .	47·2		36		31·5		27
Internal e.m.f. per motor ($=E$) .	477·8		489		493·5		498
Speed ($=s$)	13·7		15·6		16·7		18·2
$\lambda/\gamma(=E_1s_2/E_2s_1)$*		1·112		1·18		1·275	
$(\lambda/\gamma-1)/\zeta$		0·0822		0·132		0·202	
λ		0·848		0·787		0·728	

Plotting these results on Fig. 125 we obtain the point of intersection with the interpolated curve for $e/V = 0{\cdot}132$, $n = 3$, as $\lambda = 0{\cdot}78$, $(\lambda/\gamma - 1)/\zeta = 0{\cdot}142$. Whence $\lambda/\gamma = 1{\cdot}193$, $\gamma = 0{\cdot}654$, and $I_2 = 68{\cdot}6$A.

$$\text{Now } R_1 = 525/77 = 6{\cdot}82 \text{ ohms}$$

Hence substituting in equation (iv*a*) we have

$$R_1 - R_2 = R_1\{(\lambda/\gamma - 1)/\zeta + (1 - \lambda)\} = 6{\cdot}82 \times 0{\cdot}362$$
$$= 2{\cdot}47 \text{ ohms}$$

and from equations (iv*b*), etc.,

$$R_2 - R_3 = \lambda(R_1 - R_2) = 0{\cdot}78 \times 2{\cdot}47 = 1{\cdot}93 \text{ ohms}$$
$$R_3 - R_4 = \lambda(R_2 - R_3) = 0{\cdot}78 \times 1{\cdot}93 = 1{\cdot}5 \text{ ohms}$$

Calculation of Resistances of Sections for Parallel Steps. These calculations are effected by an extension of the above method, the same fundamental principles being involved. We shall consider the case in which rheostats are connected in the circuit of each motor, as is necessary when transition is to be made by the bridge method (Fig. 76). The equations deduced for this case can be applied to cases where a single rheostat is employed—as with tramcar controllers—but the values of the resistances obtained from the application of the equations must be halved. If transition is effected at the current I_2 (per motor), and I_{1p} is the upper limit of current (per motor) for the parallel steps, then, assuming the speed to be unchanged during the transition period, we have for the transition step

$$\Phi_{1p}/\Phi_2 = (V - I_{1p}R_{1p})/(\tfrac{1}{2}V - I_2R_m)$$

where R_{1p} is the resistance in circuit with each motor on the first parallel step. If the lower limit of current for the parallel steps is the same as that for the series steps,† and the resistances in circuit with each motor on the second and subsequent parallel steps are R_{2p}, R_{3p}, . . . $R_n{}'$, n' being the number of sections, the general equations for the parallel steps are—

$$\frac{\Phi_{p1}}{\Phi_2} = \frac{V - I_{1p}R_{2p}}{V - I_2R_{1p}} = \frac{V - I_{1p}R_{3p}}{V - I_2R_{2p}} = \ldots = \frac{V - I_{1p}R_m}{V - I_2R_n{}'} \quad . \quad . \quad (32)$$

Whence

$$\frac{\Phi_{1p}/I_{1p}}{\Phi_2/I_2} = \frac{V/I_{1p} - R_{2p}}{V/I_2 - R_{1p}} = \ldots = \frac{V/I_{1p} - R_m}{V - I_2R_n{}'} \quad . \quad . \quad . \text{(viii)}$$

Similarly for the transition step

$$\frac{\Phi_{1p}/I_{1p}}{\Phi_2/I_2} = \frac{V/I_{1p} - R_{1p}}{\tfrac{1}{2}V/I_2 - R_m} \quad . \quad . \quad . \quad . \quad . \quad \text{(ix)}$$

* E_1, s_1 denote the internal e.m.f. and speed, respectively, at the current I_1; E_2, s_2 denote these quantities for the assumed lower current limits, I_2.

† This condition corresponds to the majority of cases of automatic control equipments for motor-coach trains. The method can be readily adapted to cover the cases in which the lower limits of current differ.

Denoting $(\Phi_{1p}/I_{1p})/(\Phi_2/I_2)$ by λ_p; I_2/I_{1p} by γ_p; V/I_{1p} by R_1' $(= \gamma_p R_o)$; V/I_2 by $R_o (= R_1/\gamma\zeta)$; substituting in equations (viii) and (ix) and cross-multiplying, we have

$$R_1' - R_{1p} = \lambda_p(\tfrac{1}{2}R_o - R_m) \quad . \quad . \quad . \quad . \quad (\text{x}a)$$

$$R_1' - R_{2p} = \lambda_p(R_o - R_{1p}) \quad . \quad . \quad . \quad . \quad (\text{x}b)$$

$$R_1' - R_{3p} = \lambda_p(R_o - R_{2p}) \quad . \quad . \quad . \quad . \quad (\text{x}c)$$

etc. etc.

From equation (xa) we obtain

$$R_{1p} = R_1' - \lambda_p(\tfrac{1}{2}R_o - R_m) = R_o\gamma_p - \lambda_p(\tfrac{1}{2}R_o - R_m) \quad . \quad . \quad (\text{xi}a)$$

and by successive subtraction of equations (xa), (xb), etc., we have

$$\begin{aligned} R_{1p} - R_{2p} &= \lambda_p\{(R_o - R_{1p}) - (\tfrac{1}{2}R_o - R_m)\} \\ &= \lambda_p(\tfrac{1}{2}R_o + R_m - R_{1p}) \quad . \quad . \quad . \quad . \quad . \quad (\text{xi}b) \\ R_{2p} - R_{3p} &= \lambda_p(R_{1p} - R_{2p}) \\ &= \lambda_p^2(\tfrac{1}{2}R_o + R_m - R_{1p}) \quad . \quad . \quad . \quad . \quad . \quad (\text{xi}c) \end{aligned}$$

etc. etc.

Whence

$$\begin{aligned} R_{1p} - R_m &= (\tfrac{1}{2}R_o + R_m - R_{1p})(\lambda_p + \lambda_p^2 + \ldots \lambda_p^{n'}) \\ &= (\tfrac{1}{2}R_o + R_m - R_{1p})\,\lambda_p\,(1 - \lambda_p^{n'})/(1 - \lambda_p) \quad . \quad (\text{xii}) \end{aligned}$$

Substituting for R_{1p} from equation (xi$a$), and denoting $\lambda_p(1 - \lambda_p^{n'})/(1 - \lambda_p)$ by α, we have, after re-arranging and simplifying equation xii,

$$\frac{R_m}{R_o} = \frac{(1 + \alpha)(\gamma_p - \tfrac{1}{2}\lambda_p) - \tfrac{1}{2}\alpha}{(1 + \alpha)(1 - \lambda_p)} \quad . \quad . \quad . \quad (\text{xiii})$$

Now $R_m/R_o = (e/I)/(V/I_2) = (e/V)(I_2/I)$, where e is the voltage drop per motor corresponding to the current I.

Hence

$$\frac{e}{V} \cdot \frac{I_2}{I} = \frac{(1 + \alpha)(\gamma_p - \tfrac{1}{2}\lambda_p) - \tfrac{1}{2}\alpha}{(1 + \alpha)(1 - \lambda_p)} \quad . \quad . \quad . \quad (\text{xiv})$$

Hence the general relationship between λ_p and γ_p can be calculated for various numbers of sections and for appropriate values of the product (e/V) (I_2/I). A set of curves can then be plotted to give the general relationship between λ_p/γ_p and γ_p. Such a set of curves is given in Fig. 126*.

The curve giving the particular relationship between λ_p/γ_p and γ_p is obtained in the same manner as for the series steps, but in the present case the lower limit of current I_2 is fixed, and therefore values are assumed for the upper limit.

Starting Rheostats for Locomotives. The problems connected with the design of rheostats for locomotive service involve the consideration of a number of special features. Thus a locomotive may be required to haul trains of various weights, and also to run light.

Again, when starting a heavy train up a gradient, the tractive-effort required may approach the limiting value at which slipping of the driving wheels occurs. Under these conditions it is imperative that the limiting tractive-effort be not exceeded on any notch, and the controller must be held on a notch until the current has decreased to the minimum value. The rheostats should therefore be proportioned to allow of considerable time being spent on each notch. Thus with control equipments for heavy freight locomotives it is standard practice to install rheostats having a 5-minute rating, while under exceptionally severe operating conditions continuously-rated rheostats are installed. These equipments form a striking contrast to those for motor-coach trains, in which a 20-second rating for the rheostats is general practice.

* Tabulated values of α are given in the Author's paper (*Journ. I.E.E.*, Vol. 60, p. 871).

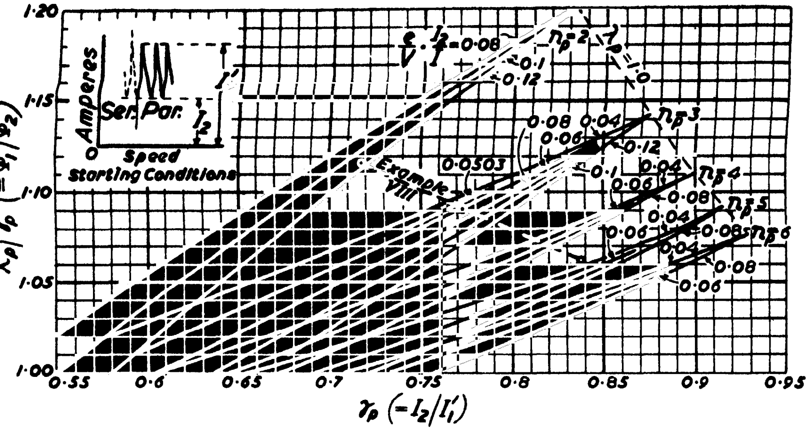


Fig. 126. Chart for Calculation of Sections of Starting Rheostat (Parallel Combination of Motors)

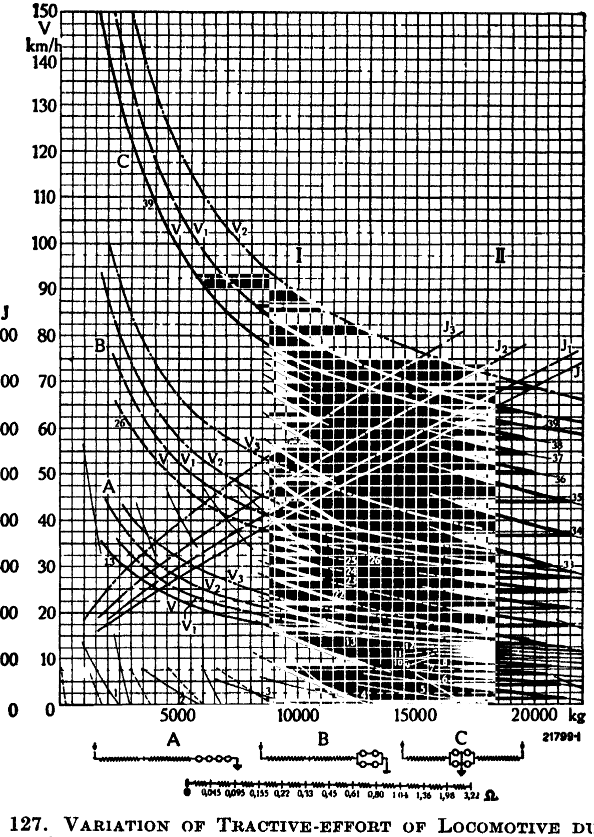


Fig. 127. Variation of Tractive-effort of Locomotive during Starting Heavy and Moderately-heavy Loads

The locomotive is equipped with four motors and is controlled according to the scheme shown in Fig. 122. The speed curves V, V_1, V_2, V_3 correspond, in order, to 100, 78, 61, 43 per cent field excitation. The numbers 1–39, against the tractive-effort/speed curves, indicate the positions of the master controller.

Moreover, in starting a long loose-coupled freight train the initial tractive effort must be limited to a low value, and must be increased in small increments to pull the slack gradually out of the couplings. In this case it is very important that the variation of tractive effort during starting shall not exceed a definite value.

The starting and accelerating operations are, therefore, effected with the aid of an ammeter connected in the circuit of one motor, as the driver then has an indication of the extent of the variation of the tractive effort on the several steps.

On account of the variable conditions of operation, transition cannot always be effected at the same current, and, in consequence, full advantage cannot be taken of the bridge method. Hence, for series-parallel control, shunt transition is usually employed, and, to avoid a sudden restoration of normal tractive-effort when the motors are first connected in parallel, one or two extra resistance notches must be provided in the parallel-transition positions of the controller.

The calculation of the resistances of the rheostat sections is, therefore, not so simple as in the cases previously considered. The problem is difficult and complicated; it is probably best tackled by arranging the grading of the rheostats to give uniform variations of tractive-effort, together with the same upper limit of tractive-effort on each step, when starting the normal or average load. Additional steps are then arranged to meet the special starting conditions.

In these circumstances the current-speed curve will not follow the regular "saw tooth" shape (Fig. 124) during the early portions of the starting period even when the normal load is being started, and irregularities will occur with other loads. But if both the variation and the maximum value of the tractive-effort are within the prescribed limits the starting performance may be considered as satisfactory. The calculations for the resistances of the additional sections are best effected by a graphical trial and error method.

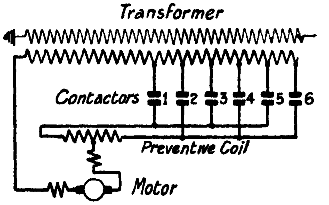
In Fig. 127 are given the speed/tractive-effort and current/tractive-effort curves for a 4,000-h.p., 1,500-volt, locomotive equipped with four motors of the type illustrated in Fig. 14 and controlled on the double series-parallel system according to the scheme shown in Fig. 122. The heavy "saw-tooth" lines show the starting performances when the locomotive is starting heavy and moderately-heavy loads. In the former case all the combinations of the motors are employed. The initial tractive-effort is 14,000 kg (13·77 tons) the mean tractive-effort is approximately 18,500 kg, and the variation of tractive-effort during the initial steps is 4,000 kg. With the moderately-heavy load only the series-parallel and parallel combinations are employed. The initial tractive-effort is 9,000 kg, the mean tractive-effort is approximately 10,000 kg, and the variation of tractive-effort during the initial steps is about 4,000 kg. In both cases the tapped-field steps are used only in the parallel combination of the motors.

CHAPTER X

THE CONTROL OF SINGLE-PHASE RAILWAY MOTORS

THE control of the speed and torque of single-phase series railway motors is effected by variation of the applied voltage. Since the operating voltage of the motors is much lower than the line voltage, a transformer forms an essential part of the equipment, and therefore the various voltages required for starting and speed control may be obtained from tappings on this transformer. Hence not only are rheostats dispensed with, but *each control point becomes a running point*, so that a number of economical speeds are available.

The *regulation of the voltage* is effected by either (i) a group of contactors or (ii) a tap-changing switch. In each case precautions must be taken, when



Sequence of Contactors

Notch	Contactors 1	2	3	4	5	6
1	●	●				
2		●	●			
3			●	●		
4				●	●	
5					●	●

FIG. 128. CONNEXIONS FOR CONTACTOR METHOD OF TAP-CHANGING

changing the voltage, to avoid the successive short-circuiting of the sections of the transformer-winding between the tappings.

Contactor Method of Tap Changing. The connexions are shown diagrammatically in Fig. 128. The contactors (1, 2, . . . 6) are arranged in two groups (i.e. 1, 3, 5; 2, 4, 6), with the common terminals of each group connected to a preventive coil, the centre-point of which is connected to the motor. Two contactors, normally connected to adjacent tappings, are closed on each notch. Each contactor, therefore, carries approximately one half of the motor current. Transition from one voltage to the next is effected by opening one contactor and closing another belonging to the same group.

Division of Current in Preventive Coil. It will be of interest to investigate the currents in the two portions of the preventive coil and the motor voltage during transition.

Under normal conditions the currents in the coil and the sections of the transformer are distributed in the manner shown in Fig. 129 (*a*), the vector difference between the currents in the two portions of the coil being equal to twice the normal magnetizing current. Fig. 129 (*b*) shows the vector diagram for these conditions. The vectors OE_1, OE_2 represent the voltages of the tappings to which the coil is connected: the vectors OI, OI_0 represent the motor current and the normal magnetizing current of the coil respectively. The currents in the two portions of the coil are therefore represented by OI_1 and OI_2.

The motor voltage is obtained by subtracting vectorially the voltage drop in the right-hand portion of the coil from OE_1. Oa represents the induced voltage in this portion of the coil, Ob the internal voltage drop due to resistance, and Oc—the resultant of Oa and Ob—the total internal voltage drop. The motor voltage is given by OV, the resultant of OE_1 and Oc.

When one-half of the coil carries the motor current the whole of the ampere-turns are expended in the magnetic circuit, and a considerable choking effect will be produced unless the magnetic circuit is designed with a high reluctance.

The vector diagram for this case is shown in Fig. 130 (*b*), in which *OI* represents the motor current, *Oa* the induced voltage, *Ob* the internal voltage drop due to

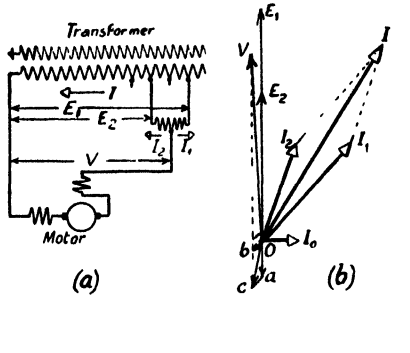


Fig. 129.

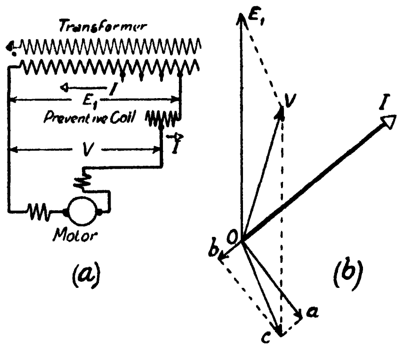
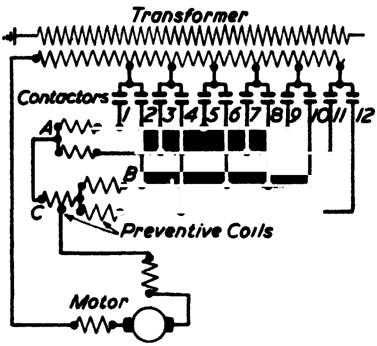


Fig. 130

Vector Diagrams showing the Effect of Preventive Coil on Motor Voltage

resistance, and *Oc* the total internal voltage in the half-coil which is carrying the current. Hence the motor voltage is given by *OV*.

Contactor Methods of Tap Changing for Large Currents. Owing to certain difficulties in the design of a.c. contactors for large currents (above 1,000 A) it is now the practice, when contactors are to be employed for tap changing, to divide the current between a number of contactors.



Step	Contactors											
	1	2	3	4	5	6	7	8	9	10	11	12
1	●	●	●	●								
2		●	●	●	●							
3			●	●	●	●						
4				●	●	●	●					
5					●	●	●	●				
6						●	●	●	●			
7							●	●	●	●		
8								●	●	●	●	
9									●	●	●	●

Fig. 131. Contactor Method of Tap Changing for Large Currents

Fig. 131 shows a method in which the motor current is divided between four contactors. Three preventive-coils (also called "bridging coils" and "centre-tapped reactors") are necessary to ensure the correct division of current between the contactors. Although nine steps are obtained with six tappings and four contactors closed on each step, on locomotives three additional steps may be obtained initially—at low tractive-effort—by closing contactors 1, 2, 3 successively.

Tap Changing by Double-contact Sliding Switch. This method can be applied to either the low-voltage or high-voltage side of the transformer. It requires one tapping for each operating position or motor voltage. Tap-changing on the high-voltage side is necessary when a large number of operating

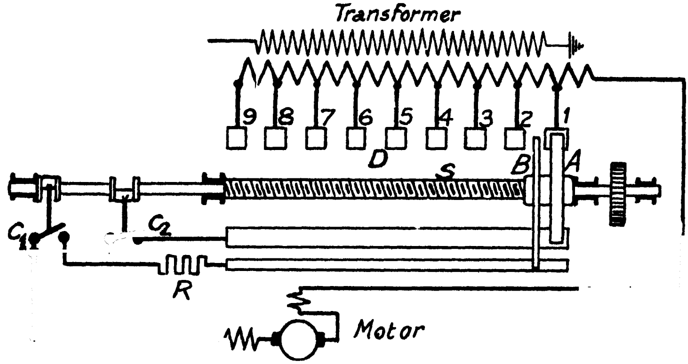


FIG. 132. TAPPING-SWITCH METHOD OF TAP-CHANGING (BROWN BOVERI)

voltages is required and/or the secondary current is large (e.g. above 5,000 A).

Fig. 132 shows the connexions for the simplest case. The sliding contacts A, B are insulated from each other and are fitted to a carriage which is operated by the screw S, the pitch being equal to that of the fixed contact blocks, D, which are connected to the tappings of the transformer. The contactors C_1,

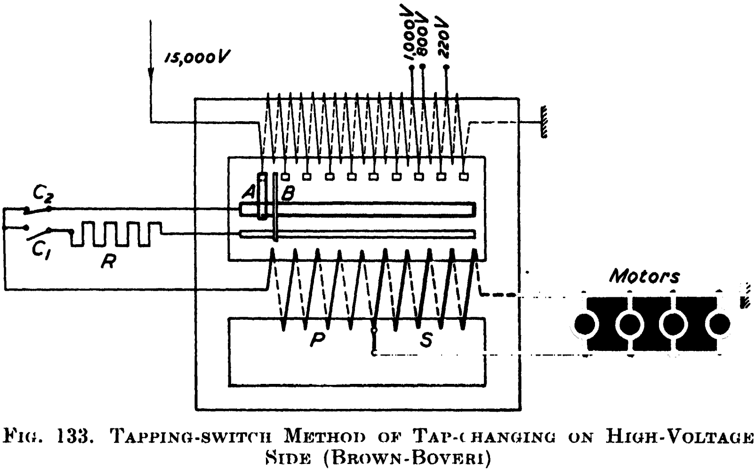


FIG. 133. TAPPING-SWITCH METHOD OF TAP-CHANGING ON HIGH-VOLTAGE SIDE (BROWN-BOVERI)

C_2 are operated by cranks fitted to an extension of the screw S. One contactor, C_1, is connected to a preventive resistor, R, and the other contactor, C_2, short circuits this resistor together with the contacts of contactor C_1. When either of the sliding contacts A, B is fully on one of the fixed contacts, contactor C_2 is closed and C_1 is open. When both of the sliding contacts are on adjacent fixed contacts (i.e. during transition), both contactors are closed, and the preventive resistance is connected between the sliding contacts.

The cycle of operations in changing from one tapping to the next is as follows—

Assuming the switch to be in the position shown in Fig. 132 and contactor C_2 to be closed, the forward rotation of the screw causes B to make contact with block 2. Contactor C_1 is then closed and contactor C_2 is opened before A leaves contact block 1. Contactor C_1 remains closed while switch B is passing

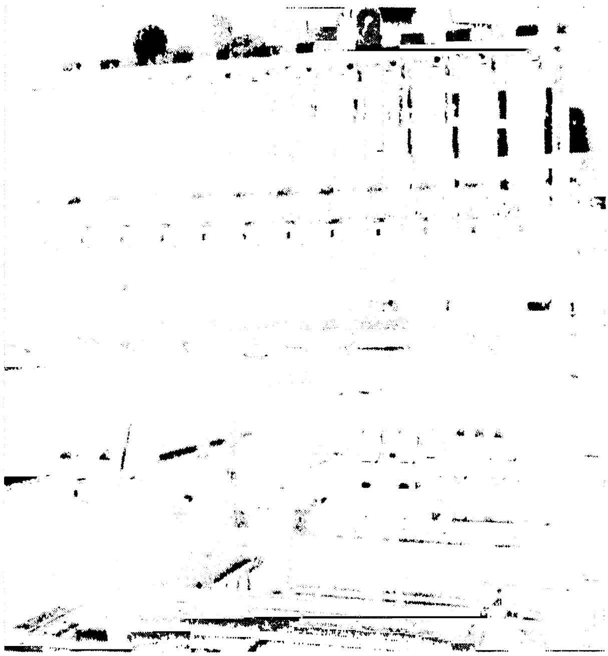


FIG. 134. TYPICAL CONTACTORS AND AUXILIARY CONTROL EQUIPMENT (SÉCHERON)

across block 2. Contactor C_2 closes immediately after A makes contact with block 2, and C_1 opens just before B leaves this block.

A similar type of switch is employed for tap-changing on the high-voltage side, but in this case the tappings are arranged on an auxiliary auto-transformer from which the primary of the main transformer is supplied. The scheme is shown in Fig. 133, the cycle of operations in tap changing being similar to that described above. The sliding contact switch is oil immersed and fitted to the side of the transformer tank, but the contactors C_1, C_2 are of the air-break type.

CONTROL APPARATUS

Contactors. The electro-pneumatic type is employed in modern equipments both in Europe and America, but the electromagnetic type was formerly employed in Germany and Scandinavia.

The *electro-pneumatic contactor* for a.c. circuits closely resembles, and in some cases is almost identical with, its prototype for d.c. circuits. It possesses a number of advantages over the electromagnetic type. Thus (i) the higher contact pressure enables a smaller width of contact to be employed for a given

current, thereby resulting in a lighter moving contact and a lower eddy-current loss in the contacts; (ii) the operating mechanism is simpler and lighter than that of an electromagnetic contactor; (iii) the power required for operation and control is very small.

The electromagnetic valves are usually identical with those for d.c. equipment and are supplied with direct current at a low voltage. Fig. 134 shows a group of typical contactors, together with auxiliary control equipment, installed in a locomotive of the Swiss Federal Railways.

The *electromagnetic contactor* for a.c. circuits differs in a number of features from a similar contactor for d.c. circuits. In general, the former is much

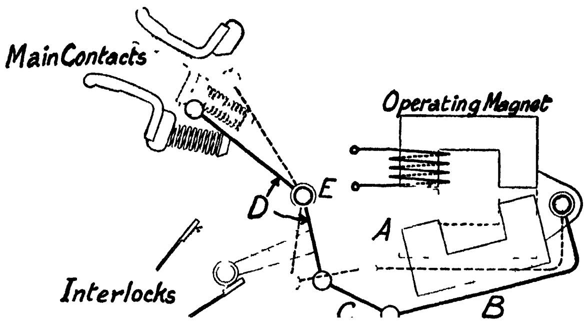


FIG. 135. MAGNET AND LEVER SYSTEM OF SIEMENS-SCHUCKERT ELECTROMAGNETIC CONTACTOR

A—armature. *C*—link.
B—lever fixed to armature *D*—bell-crank lever pivoted at *E*
Full lines show contactor in open position; dotted lines show contactor closed.

heavier than the latter and requires more power for its operation. Moreover, a number of difficulties are encountered in its design which are non-existent in the d.c. contactor. For example, (i) to obtain rapid action the moving contact must be operated through a system of levers from a relatively light moving armature; (ii) the magnetic circuit must be laminated; (iii) chattering or vibration of the moving armature due to the alternating flux must be prevented; (iv) the main contacts must be laminated or divided to reduce eddy currents (which would be generated in solid contacts by the alternations of the blow-out field); (v) the current in the operating coil varies with the position of the moving armature and the ratio of maximum to minimum current—corresponding to the open and closed positions, respectively—may be of the order of 5 to 1.

A typical lever system is sketched in Fig. 135. The design is such that the load on the armature of the magnet (due to the weight of the moving parts) increases as the air gap decreases, but becomes very small when the air gap approaches zero. Chattering is prevented by short-circuited coils (called "shielding coils") of copper in each of the pole faces, each coil embracing approximately one-half of a pole face. The circulating currents due to the alternating flux cause a phase difference between the fluxes in the two portions of the pole faces, with the result that the variation of the resultant flux is relatively small and a practically steady pull is obtained.

The contact pressure is obtained by a compound compression spring, which is designed to give a high pressure both initially and finally. Rapid opening is thereby ensured.

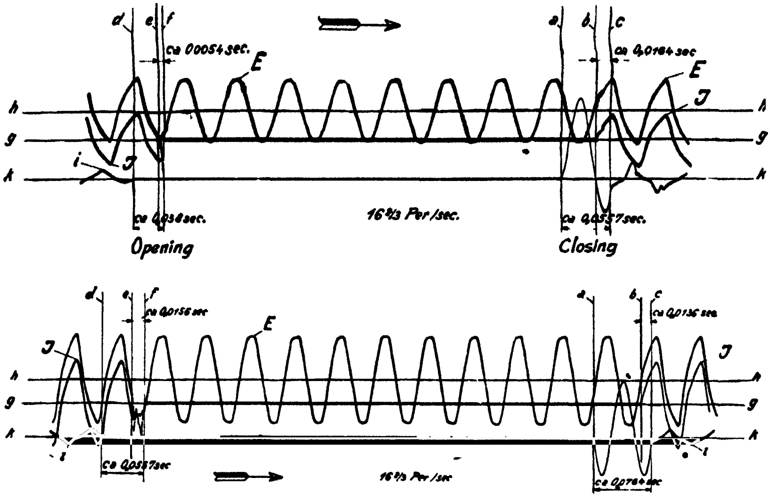


Fig. 136. Oscillograms showing Operation of Siemens-Schuckert Electromagnetic Contactor on 16⅔ c/s System

Upper oscillogram—Contactor breaking 1,294 amp (r.m.s.), J, at 76 3 volts, E. *Lower oscillogram*—Contactor breaking 2,560 amp (r.m.s), J, at 125 volts, E: a, operating coil energized at 170 volts, 16⅔ c/s; b, main contacts touch, c, armature (of magnet) closes, d, voltage removed from operating coil; e, main contacts open; f, arc ruptured; g, zero line, main current; h, zero line, voltage, i, current in operating coil, k, zero line, operating-coil current.

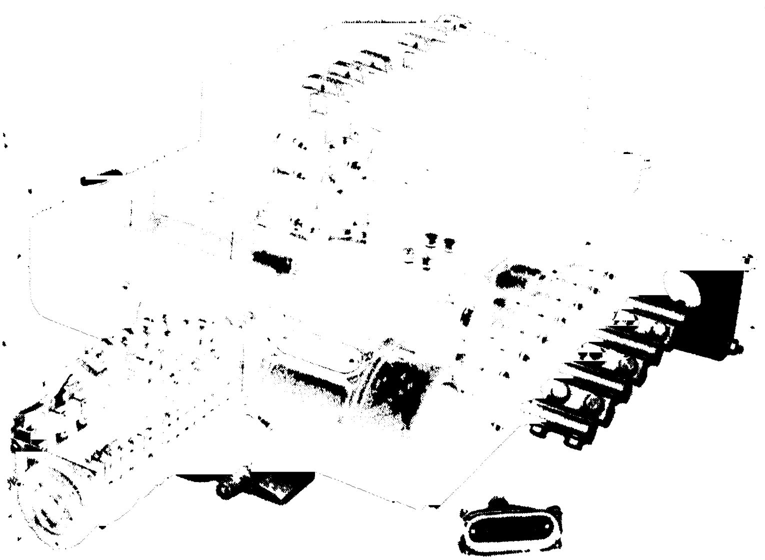


Fig. 137. Cam-shaft Contactor Group for Motor-coach (Brown-Boveri)

The position regulator is shown in the foreground. The shields between the first two pairs of contactors have been removed so as to expose the contacts.

The strong blow-out field is confined, by pole-pieces of special design, to the space occupied by the contact tips and arcing horns, and a sharp rupture of the arc is obtained. This feature is shown in the reproduced oscillograms shown in Fig. 136. These oscillograms show also the variation of current in the operating coil during the closing of the contactor.

Cam-shaft Contactors. The cam-shaft principle of contactor operation has recently been applied to a tap-changer for light motor-coach equipments. An example is shown in Fig. 137.

The contactors are arranged in two rows on either side of a central cam-shaft. The fixed contacts are provided with terminals for connecting to the tappings on the transformer, and the moving contacts are connected, by flexible

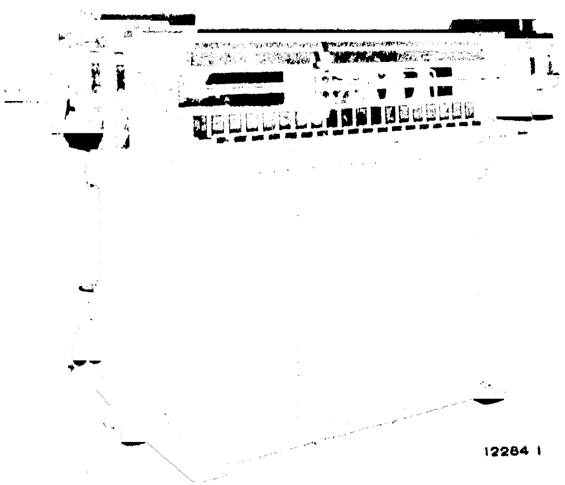


FIG. 138. BROWN-BOVERI TAPPING SWITCH MOUNTED ON TRANSFORMER

leads, to two bus-bars, to which the preventive coil is connected. The arcing horns are of special shape and material to enable arcs to be ruptured effectively without a magnetic blow-out.

The cam-shaft is driven, through gearing, by a small motor in conjunction with a "position regulator" for determining the notches.

Tap-changers of the type shown in Fig. 137 are built, with a maximum of 20 notches, in two sizes with ratings of 750 A and 1,000 A per contactor.

Tapping Switches. A typical switch, mounted in position on the transformer, is illustrated in Fig. 138.

The screw shaft operating the switch is chain driven by a small d.c. motor, and the arcing switches, or contactors, are actuated by cranks fitted to this shaft. A slip-coupling is interposed between the motor and the driving chain-wheel.

The sliding contacts are usually of the laminated-brush type, but in some cases solid contacts are employed. With the laminated contact the laminations are

set obliquely to the direction of motion so as to prevent the formation of grooves in the fixed contact bars and blocks. The sliding surfaces are lubricated automatically by felt pads.

Switches of the type illustrated in Fig. 138 are made with a maximum number of 18 switch positions, as although the switch itself could be arranged

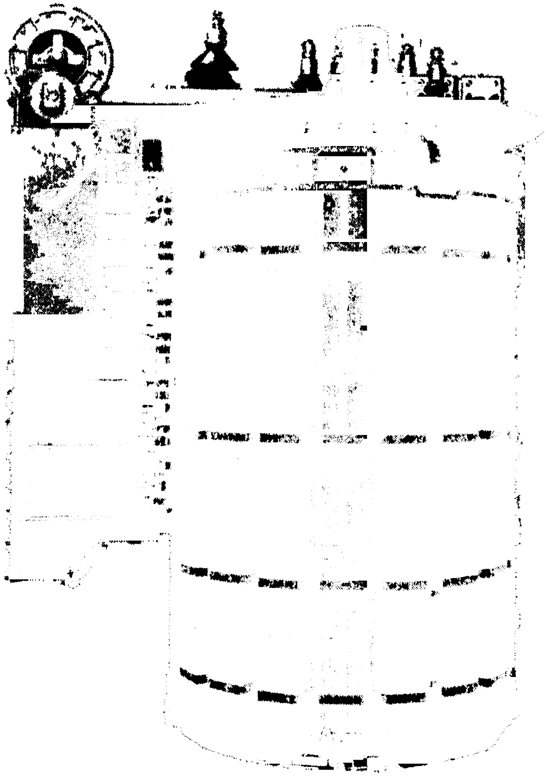


FIG. 139. BROWN-BOVERI TRANSFORMER (RADIALLY-LAMINATED-CORE TYPE, REMOVED FROM TANK) AND HIGH-VOLTAGE TAPPING SWITCH

The tapped winding (*see* Fig. 133) is located between the horizontal yokes *A* and *B*, and the main primary and secondary windings (*P* and *S*, Fig. 133) are between the yokes *B* and *C*.

The high-voltage (15-kV) terminal is shown to the left of the cover-plate, and the copper-strip terminals for the motor connexions are at the right; the other group of five terminals (connected to tappings on the lower winding) are for train heating and auxiliary supplies.

with a larger number of positions, difficulties are encountered in providing a larger number of tappings on the transformer.

These difficulties and those due to heavy currents are overcome by tap-changing on the high-voltage side. In this case the tapping switch is arranged in a vertical position on the side of the transformer as shown in Fig. 139, the contacts being oil immersed and enclosed in a sealed tank which is bolted to the side of the tank enclosing the transformer (also oil immersed). The contactors (C_1, C_2, Fig. 133) are of the air-break type, and the moving contacts

are connected to the operating mechanism by insulated rods as shown in Fig. 140. The operating screw-shaft of the tapping switch is gear driven by a small d.c. motor as in the previous case.

Reversers. These switches have to perform the same functions as the reversing switches in d.c. equipments. Usually a throw-over drum-type switch is employed and is operated either electrically (by electromagnets) or electro-pneumatically.

Master Controllers. In equipments with individual contactors the control circuits are supplied at low-voltage and as non-automatic control is employed

FIG. 140. OPERATING MECHANISM AND AIR-BREAK CONTACTORS (LEFT) FOR BROWN-BOVERI HIGH-VOLTAGE TAP-CHANGER

no special features are necessary in the master controllers. As the motors are permanently connected in parallel (or in series-parallel in some cases) only two operating handles are required, i.e. a reversing lever and, usually, a handwheel.

With tapping switches of the sliding-contact type provision has to be made for stopping automatically the operating motor when the required tapping, corresponding to the position of the master controller, is reached. This was originally effected by a follow-up mechanism in the master controller, but in later equipments with high-voltage tap-changing the automatic control and selector gear is housed in a compartment adjacent to the tap-changing switch.

Transformers. The single-winding ("auto") type is usually employed and tappings are provided not only for the motor circuit but also for the auxiliary and train heating circuits. Oil cooling is usual with European equipments and air cooling, by means of a blower, with American equipments. In many

cases with oil cooling an external oil cooler and a circulating pump are necessary as sufficient cooling surface cannot be provided on the tank for the cooling of the oil by natural circulation. With motor-coach equipments an air blast is necessary for the oil cooler (Fig. 201), but on large locomotives the cooling tubes may be mounted along one side of the body so as to be exposed to air currents.

Typical oil-immersed transformers for locomotive equipments are shown in Figs. 141A, 141B, 142.

The transformer shown in Fig. 141A is for a locomotive equipped with 50-c/s commutator motors, and because of the heavy currents the tap-changing

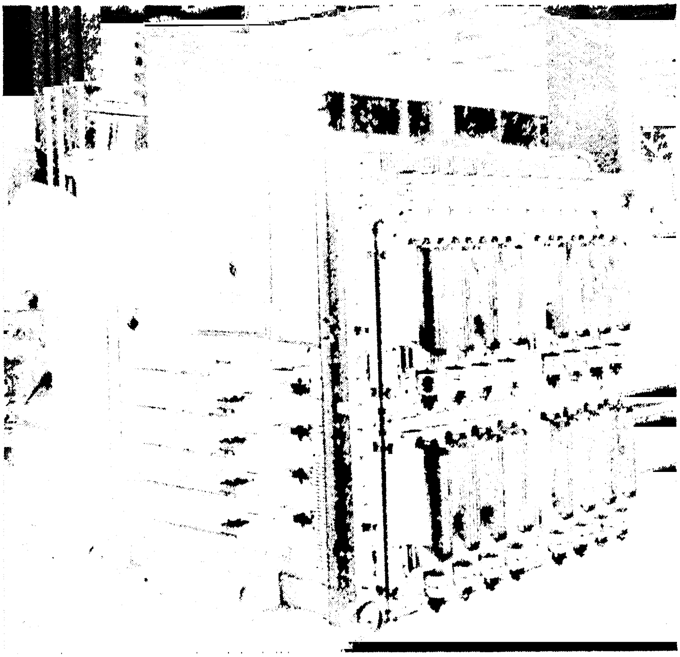


FIG 141A. 1,400 KVA LOCOMOTIVE TRANSFORMER FOR 22-KV, 50-C/S SUPPLY (A C.E.C , CHARLEROI)

With electro-pneumatic tap-changing contactors and air-blast oil cooler (at rear) mounted on tank

contactors are mounted on the tank. The oil pump and the air-blast oil cooler are also fixed to the tank so that the transformer and cooling system form a self-contained unit.

The cores of the transformers shown in Figs. 141A and 141B are of the conventional "shell" type with the central limb and yokes of rectangular cross-section, but the core of Fig. 142 is of the "cylindrical shell" type with a central portion of cylindrical cross-section (the laminations being arranged radially) and a number of distributed yokes consisting of packets of laminations arranged radially.

This form of core—originally due to A. F. Berry—is particularly advantageous for single-phase transformers and enables a considerable saving in

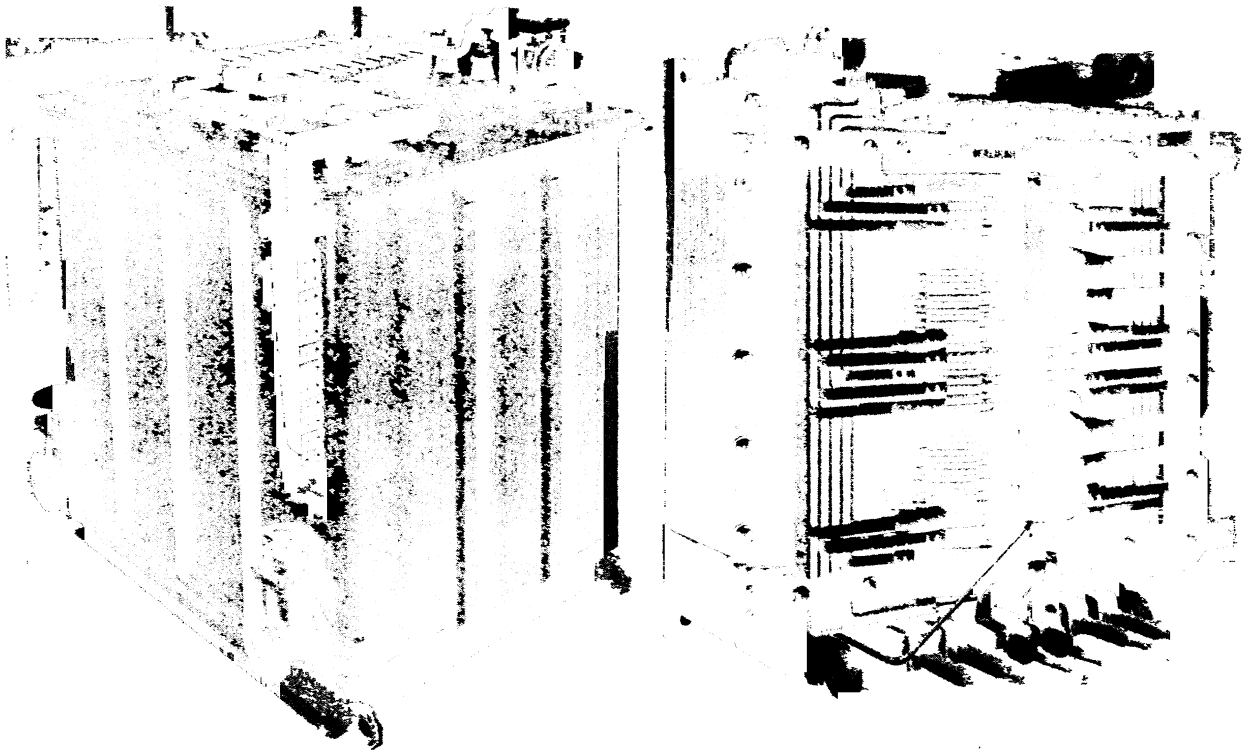


FIG 141B OERLIKON 2,500 KVA, $16\frac{2}{3}$ C S OIL IMMERSED TRANSFORMER FOR LOCOMOTIVE SERVICE

The normal position of the core is horizontal and not vertical as shown

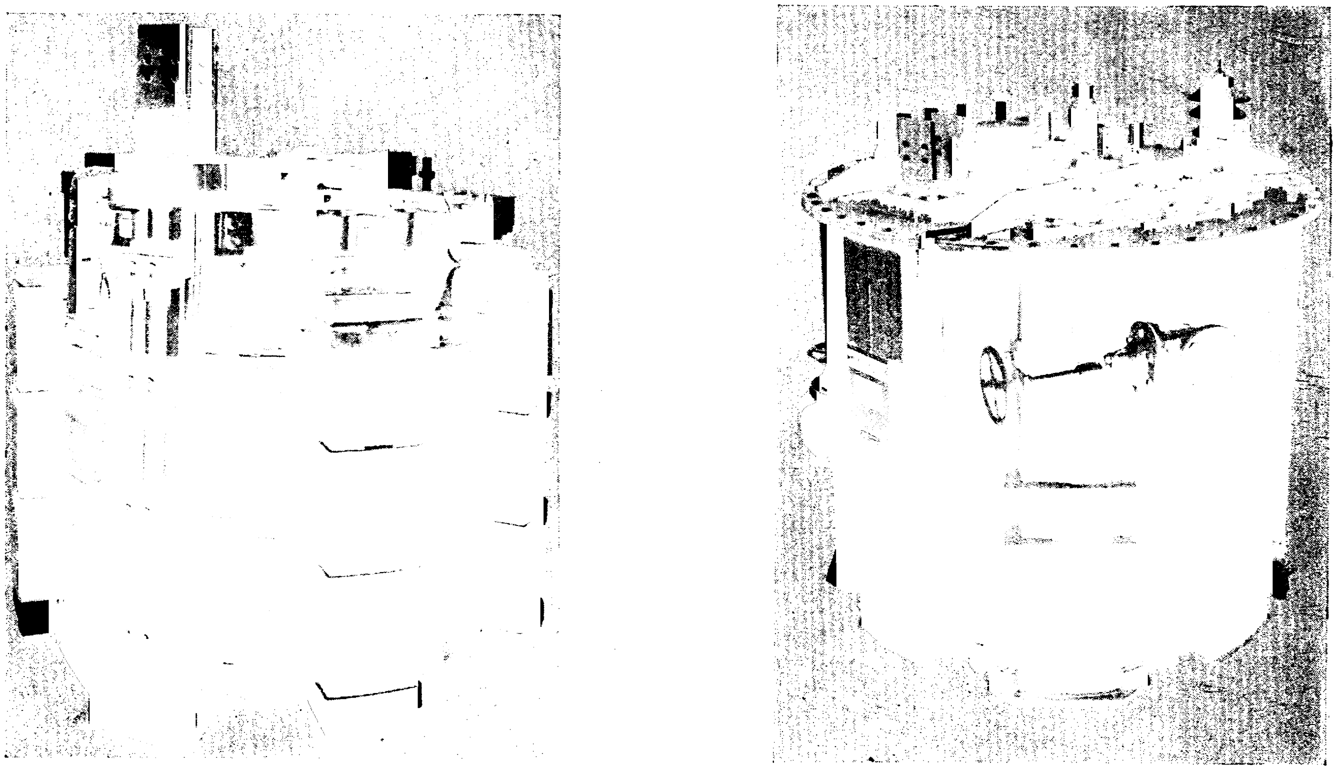


FIG. 142. BROWN-BOVERI 1,600-KVA, $16\frac{2}{3}$-C/S, OIL-COOLED TRANSFORMER WITH RADIALLY-LAMINATED CORE AND TAPPINGS ON LOW-VOLTAGE SIDE

space and weight to be effected in comparison with the conventional form. Modern spot welding technique has enabled a robust construction to be obtained for the radially-laminated central core and the radial yokes.

The coils are of cylindrical form, concentrically arranged, and the connexions between the tappings and the terminals are accommodated in the recesses between the radial yokes so that only a small clearance is necessary between the core and tank.

This construction is also advantageous for high-voltage tap changing as the main and regulating (tapped) windings may be arranged axially around the central core; the main windings (concentric coils) being uppermost and separated from the regulating winding by a packet of annular laminations, located at B in Fig. 139, which carries the differential flux. Similar packets of laminations, located at A and C, connect the radial yokes with the lower and upper extremities of the central core.

Calculation of Voltage Steps for Starting. If the magnetic reactions due to circulating currents are ignored and the reactance voltage of the motor at a given current is assumed to be independent of the terminal voltage, the simple vector diagram of Fig. 35 and the e.m.f. equation (15) may be used as the basis of the calculations. In this case if the variation of current during starting is between the fixed upper and lower limits I_1 and I_2 respectively, and the initial current is I, as represented in Fig. 124 (b), the fundamental equations are

$$V_1 = I\sqrt{(R^2 + X^2)} = \sqrt{(R^2I^2 + V_1^2 \sin^2\phi_1)} = \sqrt{(R^2I^2 + V_n^2 \sin^2\phi_n)} \quad . \quad \text{(i)}$$

$$\frac{\Phi_{m1}}{\Phi_{m2}} = \frac{\lambda}{\gamma} = \frac{V_2\cos\phi_2' - I_1R}{V_1\cos\phi_1'' - I_2R} = \ldots \frac{V_n\cos\phi_n' - I_1R}{V_{n-1}\cos\phi_{n-1}'' - I_1R} \quad . \quad . \quad . \quad \text{(ii)}$$

where n denotes the number of steps in the controller; $V_1, V_2, \ldots V_n$, the voltages applied to the motor on the several steps; $\cos\phi_1'$, $\cos\phi_2'$, ... $\cos\phi_n'$, the power factors corresponding to the current I_1, at the several voltages $V_1 \ldots V_n$; $\cos\phi_1''$, $\cos\phi_2''$, ... $\cos\phi_n''$, the several power factors corresponding to the current I_2; $\cos\phi_1$, $\cos\phi_n$, the power factors for the current I at the voltages V_1 and V_n respectively; R the effective resistance of the motor (mean value over the range of currents I_1 to I_2) and X the reactance of the motor at the current I.

In practice the general equations (ii) cannot be reduced to a simple equation involving λ, γ, n, V_1, V_n, etc.,* as in the direct-current case (p. 167). But if n and V_1 (or I) are not confined to definite values, and V_n, together with the current limits are given, a simple solution may be obtained for the voltage steps. Thus since $\Phi_{m1}/\Phi_{m2} = \lambda/\gamma$ we have

$$V_1\cos\phi_1'' = (\gamma/\lambda)V_2\cos\phi_2' - RI_2(1/\lambda - 1)$$
$$V_2\cos\phi_2'' = (\gamma/\lambda)V_3\cos\phi_3' - RI_2(1/\lambda - 1)$$
$$V_{n-1}\cos\phi_{n-1}'' = (\gamma/\lambda)V_n\cos\phi_n' - RI_2(1/\lambda - 1)$$

Now

$$V_1\cos\phi_1'' = \sqrt{[V_1^2 - (V_1\sin\phi_1'')^2]} = \sqrt{(V_1^2 - V_n^2\sin^2\phi_n'')};$$
$$V_2\cos\phi_2'' = \sqrt{(V_2^2 - V_n^2\sin^2\phi_n'')}; \text{ etc.}$$

and

$$V_2\cos\phi_2' = \sqrt{(V_2^2 - V_n^2\sin^2\phi_n')}; \text{ etc.}$$

* Such an equation, however, can be obtained in the hypothetical case of a motor with an unsaturated magnetic circuit (i.e. $\lambda/\gamma = 1$).

Whence

$$V_{n-1} = \sqrt{\{[\gamma/\lambda\sqrt{(V_n^2 - V_n^2 \sin^2 \phi_n')} - RI_2(1/\lambda - 1)]^2 + V_n^2 \sin^2 \phi_n''\}} \quad . \text{ (iii)}$$

$$V_1 = \sqrt{\{[\gamma/\lambda\sqrt{(V_2^2 - V_n^2 \sin^2 \phi_n')} - RI_2(1/\lambda - 1)]^2 + V_n^2 \sin^2 \phi_n''\}} \quad . \text{ (iv)}$$

The voltage steps are therefore calculated successively, starting from the final step corresponding to normal voltage, V_n. In practice difficulties may arise in obtaining the exact voltages for the steps owing to the impracticability of providing suitable tappings on the transformer.

Example. Calculation of the voltage steps for a railway motor, the normal voltage being 335 V and the current limits 650 A and 450 A. The power factors corresponding to these currents at 335 V are 0·97 and 0·98 respectively, and the speeds are 1,530 r.p.m. and 1,900 r.p.m. respectively. The effective resistance of the motor is 0·05 ohm and the reactance is 0·135 ohm.

Solution. Evaluating the constant terms in equation (iii) we have $\gamma/\lambda = \Phi_{m2}/\Phi_{m1}$ = 1,530(335 − 0·05 × 450)/1,900(335 − 0·05 × 650) = 0·836; γ = 450/650 = 0·693; λ = 0·693/0·836 = 0·829; $(1/\lambda - 1)$ = 0·206; $RI_2(1/\lambda - 1)$ = 0·05 × 450 × 0·206 = 4·96; $\sin \phi_n' = \sqrt{(1 - 0{\cdot}97^2)}$ = 0·243; $\sin \phi_n'' = \sqrt{(1 - 0{\cdot}98^2)}$ = 0·2; $V_n^2 \sin^2 \phi_n'$ = 6,620; $V_n^2 \sin^2 \phi_n''$ = 4,490.

Hence substituting in equation (iii) we obtain V_{n-1} = 275 V; and for the following steps we have V_{n-2} = 225 V; V_{n-3} = 183 V; V_{n-4} = 148 V; V_{n-5} = 119 V; V_{n-6} = 95·3 V; V_{n-7} = 76·3 V.

Now $Z = \sqrt{(0{\cdot}05^2 + 0{\cdot}135^2)}$ = 0·144, and therefore the voltages at standstill to give currents of 450 A and 650 A are 64·8 V and 93·6 V respectively. Hence the voltage of 76·3 V, calculated for V_{n-7}, is suitable for the first step, and the initial starting current is (76·3/0·144 =) 530 A.

CHAPTER XI

THE CONTROL OF THREE-PHASE RAILWAY MOTORS

General. The methods of obtaining a range of speeds from polyphase railway motors are: (i) rheostatic control, (ii) control by changing the number of poles, (iii) cascade control, (iv) combined cascade and pole-changing control.

The *multi-speed methods of control* (i.e. the pole-changing and cascade combinations) can be considered as the adaptation (and extension) of series-parallel control to polyphase motors. Thus the simple rheostatic and the two-speed changeable-pole, or cascade, control of two three-phase motors correspond, respectively, to the rheostatic and series-parallel control of two d.c. motors. The diagrams given in Fig. 72 (p. 111), showing the losses in the starting rheostats for rheostatic and series-parallel control, also represent approximately the relative losses in the rheostats for the a.c. cases.

Rheostatic Control. This is the simplest but least efficient of the methods of regulating the speed of polyphase motors. Only one economical running speed is obtained, and approximately one-half of the energy supplied to the motors during the accelerating period is wasted in the rheostats. Owing to these features, the applications of simple rheostatic control are limited to those light locomotives and motor-coaches for which a single economical speed is sufficient and energy consumption is not of importance.

The general relationship between the torque and slip of a polyphase motor (operating at constant voltage and frequency) is given by the equation

$$\mathfrak{T} = \frac{KsR}{R^2 + s^2X^2} = \frac{K}{X}\left(\frac{R/sX}{1 + (R/sX)^2}\right) \quad . \quad . \qquad (20a)$$

where $\mathfrak{T}$ is the torque, K a constant, s the slip, R the total resistance per phase of the rotor circuit (i.e. the normal resistance, per phase, of the rotor winding *plus* the external resistance), X the total reactance per phase of the rotor circuit at standstill ($s = 1$)—i.e. X is equal to the normal reactance per phase of the rotor winding at standstill together with any additional reactance that may be introduced by the external resistance and the connecting cables.

The general interpretation of this equation is shown graphically in Fig. 143, in which are given slip-torque curves for various values of the ratio R/X. Observe that for each value below unity of this ratio there is a particular slip at which the torque is a maximum (this slip being given simply by the ratio R/X*), and that the maximum torque has the same value throughout.

* Thus if equation (20a) is differentiated with respect to s, and the first differential coefficient is equated to zero, we have

$$\frac{d\mathfrak{T}}{ds} = \frac{d}{ds}\left(\frac{KsR}{R^2 + s^2X^2}\right) = \frac{KR(R^2 + s^2X^2) - KsR(2sX^2)}{(R^2 + s^2X^2)^2} = 0$$

whence $\quad R^2 + s^2X^2 = 2s^2X^2$

i.e. $\quad R = sX$

Thus the torque is a maximum at the slip for which the reactance (sX) is equal to the resistance.

The starting torque is given by

$$\mathfrak{T}_s = \frac{K}{X}\left(\frac{R/X}{1 + (R/X)^2}\right)$$

and is, therefore, a maximum when $R = X$. Observe that the starting torque corresponding to a given value of R/X is inversely proportional to X—the reactance of the rotor circuit.

Therefore, when external resistance has to be inserted in the rotor circuit for the purpose of regulating the speed, it is important that no additional reactance be introduced into the circuit. This matter is of especial importance when the cascade connexion is employed, as reactance in the secondary motor circuit adversely affects the power factor of the primary motor.

Of the two types of rheostat available—i.e. metallic (or grid) and liquid—the liquid type, on account of its non-inductiveness, is preferable to the

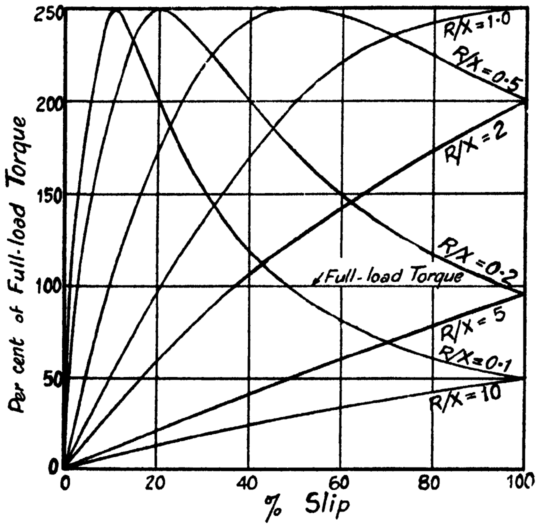


FIG. 143. TORQUE-SLIP CURVES FOR POLYPHASE INDUCTION MOTOR

metallic type for the control of polyphase railway motors, especially when cascade working is to be adopted. Moreover, with a liquid rheostat the resistance can be cut out in such a manner that a uniform torque is obtained throughout the whole period of rheostatic acceleration. Against these advantages there are the following disadvantages: for a given equipment a liquid rheostat is heavier than a metallic rheostat; provision has to be made for cooling and circulating the liquid, or electrolyte, and for replacing loss due to evaporation; the electrodes require renewal periodically.

Control by Changing the Number of Poles. This method of control is the simplest of the multi-speed methods. With two-speed machines having slip-ring rotors the regulation of the torque and speed during starting and acceleration is effected by rheostatic control, liquid rheostats being employed.

Two-speed Cascade Control. The motors are operated in cascade for the lower speed and in parallel for the higher speed. For parallel operation each

motor must have the same ratio of transformation—since the rotors are connected *in parallel* as well as the stators—but for the cascade operation the ratio of transformation of the secondary motor must be unity if the same starting rheostat is to be used for both parallel and cascade operation.

With high-voltage motors the ratio of transformation is greater than unity. Hence if both motors are designed for the same ratio of transformation, cascade operation will necessitate re-grouping the stator winding of the secondary motor for a 1 : 1 ratio of transformation.

The switch for re-grouping the stator winding is usually of the drum type and is mounted on the frame of the motor. Fig. 144 gives the connexions and development of a suitable switch for re-grouping the windings of a 12-pole

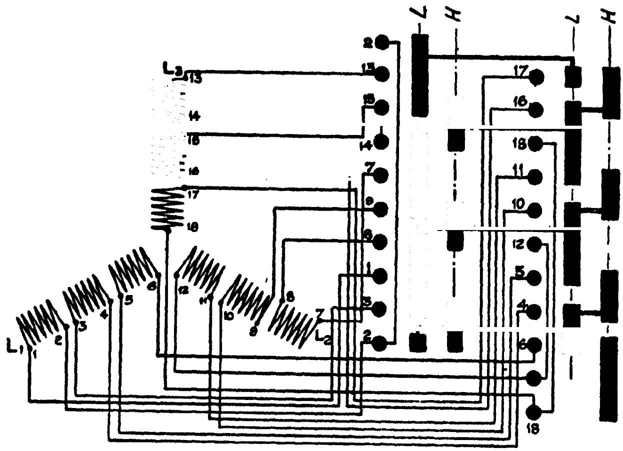


FIG. 144. CONNEXIONS AND DEVELOPMENT OF RE-GROUPING SWITCH FOR 12-POLE MOTOR

L, H—switch positions for low and high voltages respectively.

motor. Each phase of the stator winding consists of six groups of coils—one group per pair of poles. For cascade working these coils are connected in three parallel sets (each set comprising two groups of coils in series) and the phases connected in delta; while for parallel operation the coils are connected in series and the phases connected in star.

The change of connexions from cascade to parallel, and vice versa, is effected by a change-over switch, which is usually of the drum type. This switch and the re-grouping switch are operated pneumatically.

Combined Cascade and Pole-changing Control. In this method the primary and secondary motors are each wound for the same number of poles, but the windings are so arranged that the number of poles can be changed.

In Chapter VI we described two pole-changing windings to give 8 and 6 poles. The simpler winding (Fig. 59) requires a three-phase supply when connected to give 8 poles and a two-phase supply when connected to give 6 poles. The other winding (Fig. 58) operates throughout with three-phase current.

Considering first the winding which operates throughout with three-phase

current, cascade operation will involve either the re-grouping of the stator winding of the secondary motor for a ratio of transformation of unity, or the use of the inverted cascade connexion and the re-grouping of the sections of the (metallic) rheostat. With high-voltage motors the inverted cascade connexion necessitates the insulating of the rheostats to withstand the line voltage.

With the alternative pole-changing winding—operating with three-phase current for 8 poles and two-phase current for 6 poles—cascade operation will involve a suitable three-phase/two- (or four-) phase rotor winding and a suitable rheostat. A single rotor winding to meet the requirements is described in Chapter VI, and a liquid rheostat solves satisfactorily the rheostat problem.

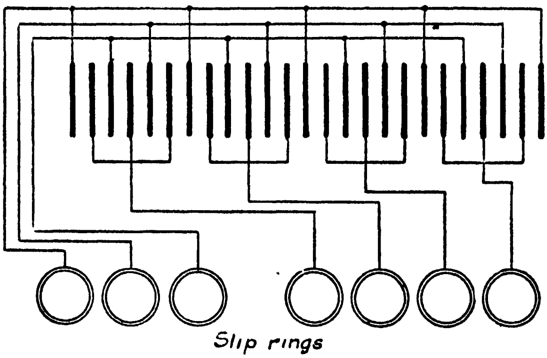


FIG. 145. CONNEXIONS OF RHEOSTAT ELECTRODES TO SLIP-RINGS FOR THREE-PHASE AND TWO-PHASE OPERATION WITH ROTOR WINDING OF FIG. 60

The seven slip rings (Fig. 61) are connected to the electrodes of the liquid rheostat in the manner shown in Fig. 145.

The rheostat must be suitable for either the three-phase or the two-phase rotor circuits. A reference to Fig. 61 will show that when the motor is operating with three-phase current (i.e. 8 poles), there will be no potential difference between the two- (or four-) phase slip-rings—since these are connected to the four neutral points of the winding—while, with two-phase operation (i.e. (6 poles), all the three-phase slip-rings will be at the same potential.

The two-phase current for supplying the stators of the motors may be obtained from the three-phase supply by means of two auto-transformers, connected according to the Scott (or "T") system.

CONTROL APPARATUS FOR MULTI-SPEED MOTORS

Control apparatus for multi-speed three-phase equipments has been developed almost exclusively for use on electric locomotives. Notwithstanding the apparent complexity of the combinations to be effected by this apparatus in some of the multi-speed methods of control (e.g. the two-speed and four-speed cascade-parallel methods), the control equipment is characterized by its general simplicity and the small number of parts.

This simplification is obtained by the adoption of high-voltage motors and automatic liquid rheostats, together with the use of compressed air for performing the principal operations of the control apparatus.

Therefore no switches are required to break heavy currents, and by confining the interruption of the main primary circuits to the reverser, the pole-changing, change-over, and re-grouping switches have to be designed with reference to current-carrying capacity and insulation only.

Automatic Liquid Rheostat. The most interesting portion of the control equipment is the automatic liquid rheostat. The liquid rheostat was first applied to railway purposes (in about 1901) by Ganz & Co. in connexion with the equipment of the Valtellina line of the Italian State Railways. The experience obtained on this line proved that a liquid rheostat possesses several advantages for the control of three-phase motors on the cascade system.

The *general features of liquid rheostats* for three-phase railway motors are—

A number of electrodes, of iron plate, are suspended in the upper part of a tank into which a solution of carbonate of soda can be pumped from a storage tank at a lower level, the flow of liquid being controlled by a valve operated by the driver, either directly or by a servo mechanism.

Pole-changing Switches. These are usually of the drum type and are operated pneumatically. The switches are not intended for breaking the circuit, which operation is performed either by a double-pole oil switch combined with the pole-changing switch, or by the reverser (which, in this case, is designed for circuit breaking).

Re-grouping and Change-over Switches for Cascade Control. For two-speed cascade-parallel equipments, the change-over switch may be combined with the regrouping switch when the latter is of the drum type. Hence, with the usual method of pneumatic operation, only one double-acting pneumatic cylinder is required for effecting the cascade and parallel combinations of the motors and rheostat.

Reversers. To reverse the direction of rotation of a three-phase motor, two of the line wires must be interchanged with respect to two terminals or phases of the stator winding. Since in all three-phase traction systems one phase of the supply and one terminal of each motor is permanently earthed, the reversal of direction of rotation involves simply the inter-change of the connexions between the trolley wires and the corresponding terminals of the motor.

With cascade and combined pole-changing-cascade equipments, the reverser usually fulfills the combined functions of a reversing and circuit-breaking switch. A capstan-type, air-break switch is used on the 3,300-volt locomotives of the Italian State Railways.

Master Controller. In multi-speed control equipments with a combined circuit breaker and reverser the master controller has only to control the valve magnets of the pole-changing switches and the regulator valve of the liquid rheostat. The controller is therefore of simple form.

Phase Converter for Split-phase System

Chief Constructional Features. The synchronous type of machine is now employed: it consists of a stator, with primary and secondary windings, a rotating field magnet, an exciter and a starting motor.

The single-phase primary winding is supplied from the traction system and may

operate at the distribution voltage (15 kV) so as to dispense with a transformer. The secondary winding is a polyphase winding to suit the pole-changing requirements of the motors. The rotating field magnet is of the cylindrical type (i.e. similar to that of a turbo-alternator).

In order to reduce size and weight, liquid cooling of both stator and rotor has been adopted by Ganz and Co. The stator is cooled by oil circulated through the air gap and special slots in the stator core. The rotor is cooled by water circulated through pipes located in channels in the teeth. Both oil and water pass through tubular coolers with air cooling.

Principle. When the converter is running at synchronous speed, the single-phase currents in the primary winding produce an alternating field. This may be split up into two component rotating fields which are equal in magnitude but rotate at synchronous speed in opposite directions.

The backwardly rotating component field is neutralized by a low-resistance squirrel-cage winding on the rotor to prevent e.m.f's. of double supply frequency being induced in the secondary winding and rotor core.

The forwardly rotating component field induces e.m.f's. of supply frequency in the secondary winding and its reaction with the rotor field produces the torque necessary to maintain rotation.

The excitation of the rotor controls the power factor by neutralizing wholly or partially the wattless component of the input current. For example, at no load the magnetizing ampere-turns for passing the flux through the magnetic circuit could be supplied either wholly by the rotor—in which case the power factor would be unity—or jointly by the rotor and the primary winding—in which case the power factor would be below unity due to the wattless (magnetizing) component in the supply current.

When the secondary winding is loaded the magneto-motive force produced has a component directly opposing that of the rotor. Hence, if the power factor is to be maintained at its former value, the direct component of the secondary ampere-turns must be neutralized by an increase in the excitation.

CHAPTER XII

REGENERATIVE BRAKING

General Considerations

Mechanical Regenerative Braking. The energy output from the motors of an electric train, operating on a level track, is expended in (i) accelerating the train, and (ii) supplying the losses due to the resistances to motion. When the train is running at constant speed, the kinetic energy which it possesses is equal to the energy expended in acceleration. During coasting a portion of this energy is utilized for propulsion. Hence, coasting may be considered as a form of *mechanical regenerative braking* or recuperation of energy. Generally, the greater the ratio of the coasting period to the total running period the lower will be the energy consumption. But prolonged coasting will result in a low schedule speed, and, if the original schedule speed is to be maintained, an increase in the acceleration will be necessary, which will usually involve the use of larger motors, so that the saving in energy consumption may be neutralized by the increased train weight and the additional cost of the equipments. With modern

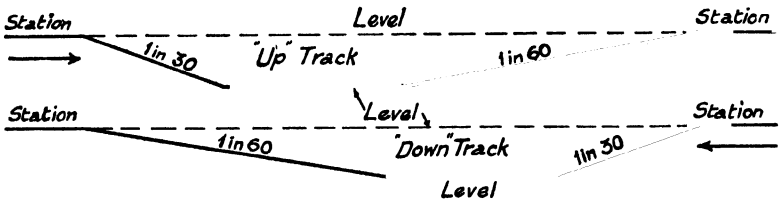


Fig. 146. Grading of Track on London Underground Railway (Central Line)

urban and suburban traffic conditions, the coasting period is from 20 per cent to 50 per cent of the total running period, and consequently a large percentage of the acceleration energy has to be wasted in the brakes.

By suitably *grading the track* the kinetic energy of the train may also be utilized in doing work against gravity. When the train is brought to rest it will, therefore, possess a certain amount of potential energy which can be utilized during the descent of the train to the level. This form of mechanical regenerative braking is adopted on some sections of the London tube railways, the tracks being graded in the manner shown in Fig. 146, which refers to the original Central London Railway. The energy consumption of the original 120-ton trains was 43 watt-hours per ton-mile for a schedule speed of 14 m.p.h., power being cut-off when the train was on the level track and the brakes applied on entering the station. This consumption is approximately 75 per cent of that estimated for the same train and service on level track.

The total economy, however, is not simply represented by the decreased energy consumption of the train, because the service can be run with smaller equipments than a similar service on a level track.

But these economies will be sacrificed if the service is run in any other than the predetermined manner for which the graded sections of the track is designed.

Hence if an increase in schedule speed becomes necessary, owing to traffic requirements, a higher acceleration, together with a higher maximum speed, must be employed, which, in general, will entail alterations in the train equipments.

Electric Regenerative Braking Possibilities on Level Track. Although graded track construction is quite practicable for tube railways, it is scarcely practicable for surface railways. Hence, if reductions are to be effected in the energy consumption of level-track surface lines by means of regenerative braking, the electrical equipment must be utilized for this purpose, the motors being operated as generators during the period of retardation, and the power generated being returned to the supply system.

The *energy which it is possible to recuperate* under these conditions will depend upon (i) the train speeds at which regenerative braking commences and ceases, (ii) the efficiency of the electrical equipment and line conductors to the point of utilization of the recuperated energy, (iii) the train resistance. The train speed at which regenerative braking ceases will depend upon a number of factors connected with the motor equipment and the system of control. With the booster and metadyne systems (p. 207) regenerative braking is possible, theoretically, until zero speed is reached, but in practice the final braking to standstill would be effected mechanically, and regeneration would cease at a low speed (about 5 m.p.h.).

If the retardation, β_r m.p.h.p.s., and the specific train resistance, r, during regeneration are assumed to be constant, a simple expression may be derived for the relationship between the recuperated energy (in watt-hours per ton) and the speeds at which regenerative braking commences and ceases, V and V_1 respectively. Thus, applying the principles of Chapter III—

The kinetic, or stored, energy at a train speed of V m.p.h. is equal to $0{\cdot}0283V^2 \times 1{\cdot}1$, or $0{\cdot}031V^2$ watt-hours per ton of train weight.

The energy expended against train resistance is equal to $2rD'$ watt-hours per ton of train weight, where D' is the distance (in miles) run during recuperation.

[NOTE. $D' = \frac{1}{2}(V^2 - V_1^2)/3{,}600\beta_r$.]

Hence the energy available for recuperation is equal to

$$0{\cdot}031(V^2 - V_1^2) - r(V^2 - V_1^2)/3{,}600\beta_r$$

watt-hours per ton of train weight, and the energy returned to the supply system is equal to

$$\eta(V^2 - V_1^2)(0{\cdot}031 - r/3{,}600\beta_r)$$

watt-hours per ton of train weight.

For example, if regenerative braking commenced at a speed of 30 m.p.h. and ceased at 5 m.p.h., then, assuming an average value of 75 per cent for the equipment efficiency, a constant retardation of 2 m.p.h.p.s. and a constant train resistance of 14 lb per ton, the recuperated energy would be

$$0{\cdot}75(30^2 - 5^2)\{0{\cdot}031 - 14/(3{,}600 \times 2)\} = 21{\cdot}8$$

watt-hours per ton of train weight. Hence with a service having 0·75 stops per mile (for which the specific energy consumption without regenerative braking may be of the order of 70 watt-hours per ton mile) the approximate energy consumption may be $70 - 21{\cdot}8 \times 0{\cdot}75 = 53{\cdot}6$ watt-hours per ton mile.

Disadvantages and Advantages of Regenerative Braking on Level Routes. In practice a number of difficulties and disadvantages are involved in the

application of regenerative braking to level routes. The disadvantages, in so far as d.c. equipments are concerned, are briefly—

The motors are larger, heavier, and more costly than those for ordinary equipments, thereby resulting in more costly mechanical parts (e.g. trucks), an increase in the weight of the train, and possibly an increase in the number of motors. Additional equipment is necessary for the purpose of controlling and safeguarding the regenerative action of the motors and to obtain suitable regenerative characteristics. These features result in increased first cost of the trains, increased maintenance charges on the electrical equipment, and increased complication in the control and method of operation. Moreover, difficulties in the operation of the sub-stations may occur should the recuperated energy exceed the energy output from the sub-station.

To offset the disadvantages there are the following advantages—

Reduced energy consumption; reduced wear of brake shoes and wheel tyres; lower maintenance costs for these items; relatively small amount of brake dust produced when the mechanical brakes are applied.

In the majority of cases, however, and especially with motor-coach trains, the increased cost of the train equipments and the additional features necessary to obtain regenerative braking, combined with the increase in the maintenance costs of the electrical equipment, may entirely off-set the economies in the energy consumption and the other items.

Practical Results. A number of trains with metadyne equipments are in service on the Circle Line in London. Tests on a six-coach train (comprising three 2-coach units, each unit having four motors and a metadyne converter set) in normal service showed that during regenerative braking the recuperated energy was about 25 to 30 per cent of the energy input to the motors.*

A similar result was obtained on the Paris Metropolitan Railway some years ago on a motor-coach train with booster equipments. The energy consumption of this train in normal service was 48 watt-hours per ton mile, while that of a train with standard series-parallel non-regenerative equipment was 60 watt-hours per ton mile.

In both cases the regenerative equipment is heavier than a standard series-parallel equipment, the increase being about 3½ tons for the metadyne equipment (four 160 h.p. motors) and about 1½ tons for the booster equipment (two 175 h.p. motors).

Experience with *regenerative equipments on tramways and trolleybus routes* has shown that on level routes the energy consumption is about 10 per cent lower than that of a standard (series motor) equipment, the operating conditions being similar in each case. With undulating routes the saving may be of the order of 20 per cent.

Electric Regenerative Possibilities on Main-line and Mountain Railways. The operating conditions on main line railways having long gradients and on

* Details of the equipments are given in the following papers—"The Metadyne and its Application to Electric Traction" by G. H. Fletcher and A. Tustin, *Journ. I.E.E.*, Vol. 85, p. 370; "Railway Traction Control Equipment on Suburban London Transport," by E. Webster, *Proc. I.E.E.*, Vol. 96, Pt. II, p. 537.

mountain railways are very favourable to electric regenerative braking* owing to (i) the relatively large amount of energy available during the descent of the gradients, (ii) the exclusive use of electric locomotives, (iii) the operating conditions permitting the use (when desirable) of motors having constant-speed characteristics. In these cases, even when d.c. series motors are employed, the additional equipment necessary for regenerative braking adds but a small percentage to the cost of the locomotive.

The *advantages* due to regenerative braking on these railways are greater than those obtained on level routes. Thus, in addition to the saving of energy, there are large savings in the maintenance of the mechanical brakes and wheel tyres. Moreover, owing to the mechanical brakes being used only to a small extent—and, in some cases, not all all—during the descent of gradients, the danger of overheating of the brake shoes and wheel tyres (which may be a serious menace with mechanical brakes) is eliminated, thereby conducing to greater safety in operation and more uniform braking. Further, higher operating speeds on the gradients become possible and heavier trains can be taken down the gradients.

In these circumstances regenerative braking results in a considerable reduction in the operating costs compared with mechanical braking.

Practical Results. The Giovi-Genoa lines of the Italian State Railways† form a striking example of the advantages of electric regenerative braking on a railway with heavy gradients. With electric traction the capacity of the lines has been trebled, due to the heavier trains which can be run on the down gradients and the higher speeds permissible with electric braking. The running costs have been found to be only about 75 per cent of those when the lines were operated with steam locomotives, although the plant of the generating station is not fully utilized. These low costs are the result of electric recuperation of energy on the down gradients, the recuperated energy being of the order of from 60 per cent to 80 per cent of the energy consumption for the up journey with the same train.‡ Considerable saving is also effected in the brake shoes, wheel tyres, and rails, as the mechanical brakes are only used for "slow-downs" and stops.

General Conditions Relating to Electric Regenerative Braking on Main-line and Mountain Railways. In the application of any system of electric regenerative braking to these railways three important conditions have to be fulfilled—

(i) The speed-torque characteristics of the regenerative equipment must be such that mechanical stability is obtained over the whole range of operating speeds, i.e. an increase in speed must be accompanied by an increase in the braking torque.

(ii) The electrical or volt-ampere characteristics of the equipment must ensure electrical stability over the whole range of operating speeds, i.e. sudden fluctuations in the line voltage must not cause flash-overs or sudden fluctuations in the braking torque.

(iii) If and when the recuperated energy exceeds the energy demand of

* Generally, regenerative braking is desirable, and necessary, in any electrification scheme for lines having long gradients exceeding 0·6 per cent (1 in 166).

† For a description of this electrification see *Tramway and Railway World*, Vol. 27, p. 345; Vol. 35, p. 184.

‡ In all cases of regenerative control the efficiency of the equipment has an important bearing on the economical results. It is only with the use of large gearless three-phase motors that results of the above order can be obtained.

other trains operating on a given section of the distributing system, the substation converting plant must be capable of returning the excess energy to the primary supply system, and the latter must be capable of absorbing this energy. Hence, in the case of railways which are supplied from a separate generating station which has no other load, provision must be made for dissipating any excess energy in loading rheostats (which are usually of the water type); otherwise dangerous operating conditions, both at the generating station and at the trains descending gradients, would occur.

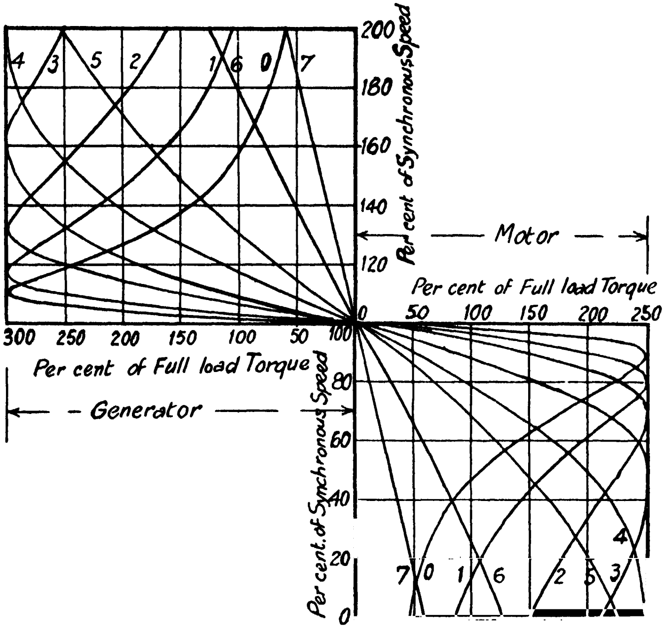


FIG 147. SPEED-TORQUE CURVES FOR MOTOR AND GENERATOR OPERATION OF POLYPHASE INDUCTION MACHINE

ELECTRIC REGENERATIVE BRAKING SYSTEMS

I. Regenerative Braking with Three-phase Motors. This is the simplest system of regenerative braking, especially when automatic liquid rheostats are employed. No additional apparatus or equipment is necessary, and no extra notches or special operating positions are required on the master controller.

The essential feature in the control of the three-phase induction motor for regenerative braking depends upon the property of this machine to operate as a non-synchronous (induction) generator when driven at speeds above synchronism. The machine, however, is not self-exciting as a generator and must be connected to a system supplied by synchronous generators, this system supplying the excitation and determining the frequency at which the induction generator operates.

The relationship between torque and slip for generator operation is similar to that for motor operation (except for a slight difference in the magnitudes of the maximum torque in the two cases), i.e. the slip (which is negative*)

* Thus if n = speed of rotor in r.p.m., n_s = synchronous speed in r.p.m., the slip $= (n_s - n)/n_s = -(n/n_s - 1)$.

with constant resistance in the rotor circuit is approximately proportional to the torque, while for constant torque the slip is approximately proportional to the resistance of the rotor circuit.

Fig. 147 shows typical torque-speed curves for both motor and generator operation. Observe that with no external resistance in the rotor circuit the speed varies only very slightly over the whole range of the torque, and that the effect of adding external resistance to the rotor circuit is to increase the speed for a given braking torque. Hence, when braking with zero external resistance in the rotor circuit the speed will be practically unaffected by the steepness of the gradient, and will also be practically independent of the load, i.e. weight of the train. But if increased speeds are necessary with light loads they may be obtained by inserting resistance in the rotor circuit. Alternatively, multi-speed motors or cascade control may be employed.

The curves of Fig. 147 show also that the maximum braking torque available from the machine is greater than the maximum motoring torque. The former is usually of the order of three times the continuous-load torque, and is always considerably above the torque necessary to slip the driving wheels.

Regenerative braking on level track can be obtained only with multi-speed equipments. Thus with the two-speed cascade system, regenerative braking is obtained at speeds above the cascade-synchronous speed by cascading the motors, the control of the braking retardation being effected by regulating the resistance in the rotor circuit of the secondary motor. Hence, with this (two-speed) equipment, the economy obtained from regenerative braking on level track will be small. Even a four-speed equipment will not show marked economy on account of the unavoidable losses in the rheostats.

Regenerative braking on gradients, however, will show considerable economy, as the machines can be operated without external resistance in the rotor circuit, i.e. under conditions of high efficiency. The energy returned to the distribution system is equal to the work done by gravity on the descending train, less the energy expended against train resistance and the losses in the motors. Thus the output from the machines (which is practically proportional to the braking torque) is determined by the weight of the train and the gradient. The speed—with zero external resistance in the rotor circuit—is determined by the speed-combination of the machines, and will be slightly higher than that corresponding to the motor speed for this combination. In general, therefore, the speed-combination which is used when ascending a given gradient will also be used when descending this gradient. But in the latter the train weight which can be handled by a given locomotive is greater than that in the former case.

When two or more locomotives have to be coupled together for the purpose of handling a heavy train down a gradient, it is desirable that the locomotives should share the load equally in spite of any differences in the diameters of the driving wheels. This condition is easily obtained with equipments provided with automatic liquid rheostats. The rheostats on the locomotive with the largest driving wheels are short-circuited, while the rheostats on the other locomotives are adjusted (from the driving master controller) to maintain a load on these locomotives equal to that on the former locomotive.

II. **Regenerative Braking with d.c. Motors.** In this case the voltage generated in the armatures when braking must exceed the supply voltage by an amount which is equal to the voltage drop (due to resistance) in the machines and

connexions. Moreover, the generated voltage must be maintained at this value independent of the speed and braking torque. These conditions necessitate either shunt- or separately-excited field windings.

The speed and braking torque are controlled by regulating the excitation. They could also be controlled by a variable resistor, or, alternatively, a booster, connected in the armature (main) circuit.

The range of speed and braking torque obtainable from a given equipment with this method of control (i.e. by regulating the excitation of the traction motors) is governed by considerations of heating and commutation. For example, the torque corresponding to a given combination of the motors is limited, at the lower speeds, by the heating of the armature and field windings. At the higher speeds (i.e. when the machines are operating with weakened fields) the torque is limited by commutation. Thus, to obtain satisfactory commutation the ratio of field ampere-turns to armature ampere-turns must always exceed a minimum value (which is of the order of 0·4).*

Regenerative braking with shunt machines cannot be effected with the simplicity which this method of excitation would suggest, as difficulties are encountered in the series-parallel operation of the machines, both as generators and motors. Moreover, shunt machines are inherently sensitive to voltage fluctuation and would be extremely unstable on a traction circuit. But satisfactory operation, so far as the motors are concerned, can be obtained with compound excitation by a suitable choice of shunt and series ampere-turns, and by arranging that the series and shunt ampere-turns oppose each other during regenerative braking.

Such motors are employed on trolleybuses when regenerative equipments are required, and motors with series and separate (adjustable) excitation are employed on many 3,000-V d.c. main-line locomotives in service in Italy. In the latter case the separately excited winding (which is supplied from a low-voltage exciter) is regulated to give speed control during motor operation and a limited amount of regenerative braking on "down" gradients, large scale regenerative braking being unnecessary.

Modern developments in large-scale regenerative braking on d.c. railways have been entirely with standard series traction motors. Suitable characteristics for regenerative braking are obtained by separately exciting the field windings,† and electrical stability is obtained either by differentially compounding the exciter or by connecting the exciting circuit in parallel with a resistance inserted in the main current. The braking torque is controlled by regulating the excitation of the exciter.

The exciter may be driven either by a separate motor (which is usually series wound and is connected to a permanent mechanical load, such as a fan or blower) or from one of the axles of the locomotive. In the latter case the exciter is usually an axle-mounted generator and is driven from one of the trailing axles. In some cases—e.g. with locomotive equipments having a large number (six or more) of motors—one of the driving motors is used as an exciter.

Elementary diagrams of connexions for these cases are given in Fig. 148. In diagram (*a*) the exciter is differentially compounded, and is driven by a

* Other factors are discussed in a paper by O. H. Hahn, "Theory and Practice of Regenerative Braking of d.c. Locomotives with particular reference to Multiple-unit Operation." *Proc. I.E.E.*, Vol. 95, Pt. II, p. 85.

† The term "field windings" here refers to the exciting windings and does not include the commutating-pole windings.

series motor, to which is also connected a permanent mechanical load. The shunt winding of the exciter is separately excited from an auxiliary supply: the differential series winding is connected in the main circuit and carries the recuperated current. The exciter has to carry the sum of the armature and field currents of the traction motors. Control of the braking torque is effected by a rheostat connected in the shunt-field circuit of the exciter.

Diagram (*b*) also refers to a motor-driven exciter, but in this case the exciting circuit (i.e. the exciter armature together with the field winding of the traction motor) is connected in parallel with a "stabilizing" resistance in the main circuit. The exciter armature carries only the exciting current of the traction motor, but its voltage must be equal to the sum of the voltage drops in the stabilizing

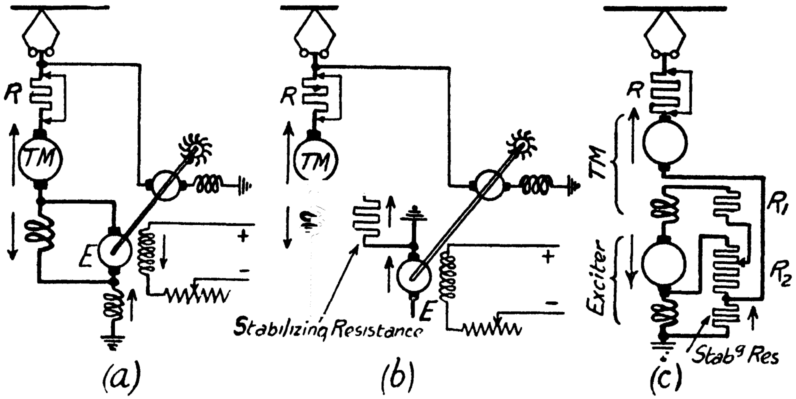


FIG. 148. ELEMENTARY DIAGRAMS OF METHODS OF OBTAINING REGENERATIVE BRAKING WITH D.C. MOTORS

resistance and the traction motor field winding. The same scheme of connexions may be employed with an axle-driven exciter.

Diagram (*c*) shows a method of utilizing one of the traction motors as an exciter. In this case the "exciter" operates as a self-excited series generator, and supplies not only its own excitation but also that of the other machines which are generating. Control of the excitation and braking torque is obtained by both shunting the field winding of the "exciter" and varying the resistance in the exciter-armature circuit.

The speed-torque characteristics obtained from each of these methods have the same general form, but, for given conditions, the variation of speed with torque is generally smaller with a stabilizing resistance than with a differentially compounded exciter. Typical characteristics are shown in Fig. 149. Such characteristics can be calculated when the requisite data are available, but the calculations are more tedious than those for the motor speed-torque characteristics, as the shape of the magnetization curve of the exciter and armature reaction have to be taken into account.

It is important to observe that for each value of excitation, the torque increases as the speed increases until a definite torque is reached, after which an increase of speed results in a decrease of torque (i.e. the braking system becomes unstable mechanically). The value of the maximum torque depends upon the excitation, the largest value being obtained with the largest excitation.

Comparison of Above Methods. With the differentially-compounded exciter method (Fig. 148 (*a*)) the traction motors must first be connected to the line as motors when regenerative braking is required, the change from motoring to braking being effected by increasing the excitation. Stability against surges of current due to sudden fluctuations of line voltage is obtained by the differential compounding of the exciter in combination with armature reaction and voltage drop in the exciter armature. For example, should a sudden decrease in the line voltage occur, the tendency for the recuperated current

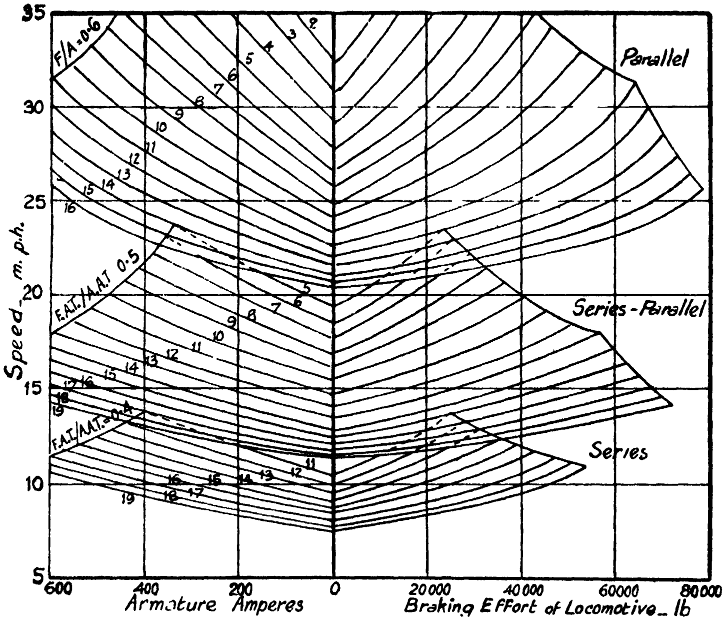


FIG. 149. SPEED/CURRENT AND SPEED/TRACTIVE-EFFORT CURVES FOR ELECTRIC LOCOMOTIVE WHEN BRAKING REGENERATIVELY

Motor-driven exciter and 0 04-ohm stabilizing resistance.

to increase is checked by a reduction of the exciter voltage, which decreases the excitation of the traction motors.

With the motor-driven exciter method employing a stabilizing resistance the traction motors may be connected to the line as generators, and, on account of the inherent stability of this method, the voltage of the machines at the time of their connexion to the line does not require exact adjustment. In the event of a sudden decrease in the line voltage the tendency for the recuperated current to increase is checked by the decreased excitation due to the increased voltage drop across the stabilizing resistance.

This method possesses the advantages of greater simplicity and a smaller exciter for a given motor equipment than the preceding method, and the losses in the stabilizing resistance are to some extent balanced by the decreased commutator losses of the exciter and the maintenance of this machine.

With the traction-motor exciter method shown in Fig. 148 (*c*), the "exciter" carries only the field current of the traction motors. The stabilizing resistance

carries the sum of the armature and field currents. Electrical stability is therefore obtained by a method similar to that employed in Fig. 148 (*b*). The control of the excitation of the exciter and the traction motors is effected by utilizing a portion of the main starting rheostats and contactors. Hence, regenerative braking entails very little additional equipment. On the other hand, the range of speeds over which braking is possible is less than that obtainable with the preceding methods (with which two or more combinations of the motors are possible).

Example of Control Equipment for Regenerative Braking. This example refers to a 3,000-volt, 1,200-h.p. locomotive with Metropolitan-Vickers equipment. The control equipment for motor operation is arranged on the series-parallel system and is described in Chapter IX (p. 161). For regenerative braking a motor-driven exciter with stabilizing resistance is employed, and both combinations of the motors can be used if desired. Transition is effected by opening the main circuit.

Simplified diagrams of the motor and control circuits—in so far as regenerative operation is concerned—are given in Fig. 150. The exciter is driven by a double-armature, 3,000-volt, motor which is mechanically connected to a blower. The motor has a series winding (which normally supplies the majority of the excitation) and a light shunt winding which is separately excited from an auxiliary source (100 V). The exciter has a main shunt winding, which is separately excited, and a light series (cumulative) winding which is connected in series with the motor. The object of this (series) winding is to compensate for gradual or semi-permanent fluctuations in the line voltage. Thus if the line voltage increases, the speed of, and the current input to, the motor both increase. Hence the voltage generated in the exciter increases, thereby increasing the excitation of the traction motors. A decrease in the line voltage produces the opposite effect.

The change of connexions from motoring to braking (which involves the cutting-in of the exciter and stabilizing resistances) is effected by a cam-operated group of contactors.

The control operations are—

The combined reversing and motor-combination handle of the master controller (Fig. 121) is placed in either the series or parallel position, the braking handle is moved to the first notch, and the accelerating handle is moved to the first notch.

The braking handle is then adjusted until the ammeter shows that recuperation has commenced. The current is maintained approximately constant at a small value, by manipulating the braking handle, while the starting rheostats are cut out, after which operation the braking current is adjusted to the desired value by manipulation of the braking handle (which controls the amount of resistance in the shunt field of the exciter).

III. **Regenerative Braking with Single-phase Motors.** The problem of obtaining regenerative braking with single-phase series motors presents difficulties which are considerably greater than those discussed above in connexion with d.c. equipments.

These difficulties are concerned chiefly with the prevention of self-excitation (i.e. the building-up of the machine as a self-excited generator at a frequency different from that of the supply system) and the attainment of a high power

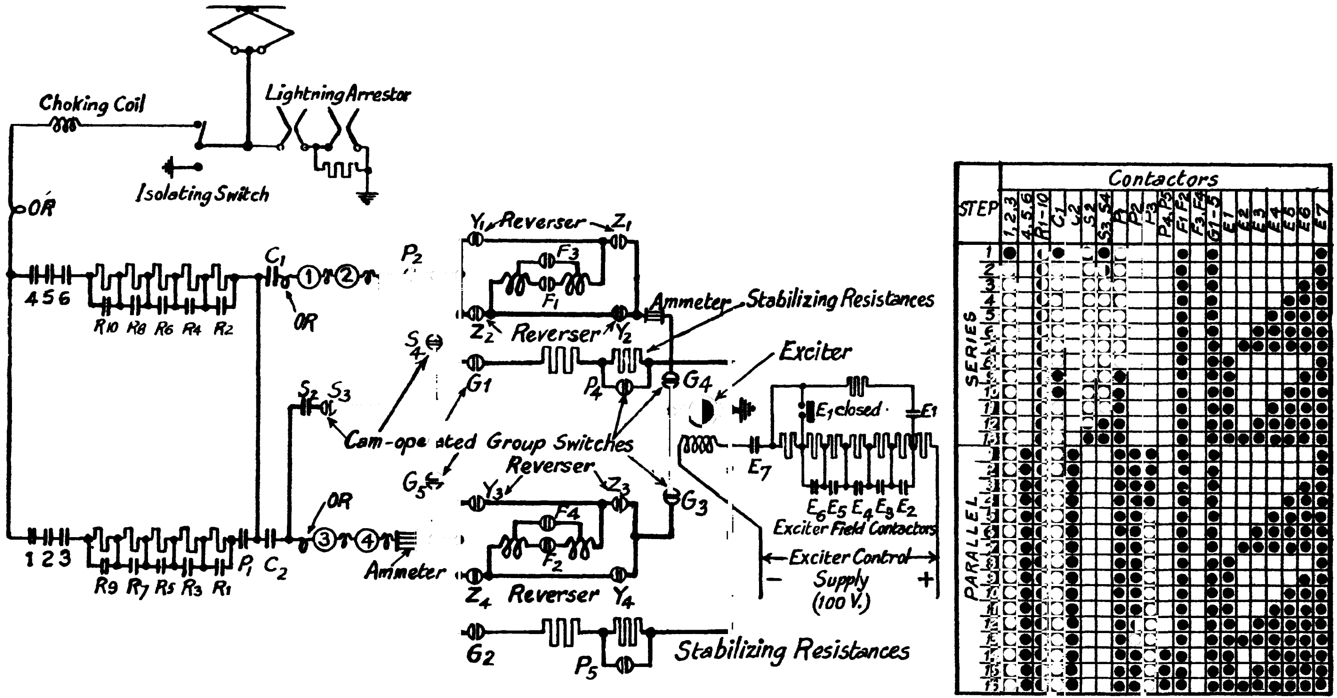


Fig. 150. Main-circuit Connexions for Regenerative Braking with Series-parallel Locomotive Equipment (Metropolitan-Vickers)

factor. Circuits for recuperation at high power factor, however, usually suffer from self excitation, and those which are inherently stable and free from self excitation usually operate at a low power factor. Much development work to overcome these difficulties has been carried out during recent years by the Oerlikon Co., and a number of schemes have been evolved and tried out in service.*

Shunt Connexion with Series Reactor. The simplest scheme (due to Behn-Eschenburg and applied to about 100 locomotives and motor-coaches of the Swiss railways) is shown diagrammatically at (*a*), Fig. 151, and a simplified vector diagram is shown at (*b*). The exciting winding, *E*, of the traction motor

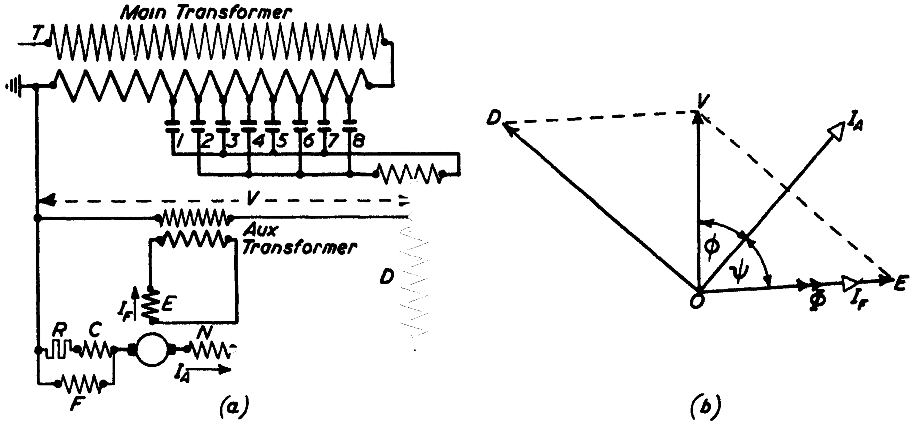


FIG. 151. CIRCUIT AND VECTOR DIAGRAMS FOR BEHN-ESCHENBURG METHOD OF REGENERATIVE BRAKING (OERLIKON)

is excited from the secondary winding of an auxiliary transformer connected to the tap-changer of the main transformer, to which the armature is connected with a choking coil or iron-cored reactor, *D*, in series. (With multi-motor equipments the exciting windings of all the motors are connected in series and supplied from the auxiliary transformer; the several armatures are connected in the same combination as when operating as motors, a reactor being connected in each circuit.) To obtain the correct (leading) phase of the commutating flux a resistor, *R*, is connected in series with the commutating-pole winding, *C*, and the combination is shunted with an iron-cored reactor, *F*.

The current, I_F, in the exciting winding has a phase-difference of approximately 90° (lagging) from the voltage, *V*, at the tap-changer; and the e.m.f. *E*, generated in the armature is in phase with this current. The vectors representing these qualities are OI_F, OV and OE. The voltage across the reactor is, therefore, represented by OD (which is the vector difference between OV and OE) and the armature current, I_A, is represented by OI_A, lagging approximately 90° from OD, its magnitude being proportional to OD if the reactor has a straight-line characteristic.

For a given voltage at the tap-changer the braking torque is proportional

* Some of these schemes are discussed at length in an article by P. Leyfraz on "Recent developments of regenerative braking for single-phase railways," *Bulletin Oerlikon*, Nos. 283, 284.

to $I_A \cos \psi$ and the power returned to the supply system is, ignoring losses, proportional to $I_A \cos \phi$.

From the vector diagram we deduce that, for constant excitation, (i) the braking torque is approximately constant at all speeds, (ii) the volt-amperes absorbed by the reactor D exceed those generated in the armature, (iii) the power returned to the supply system has a low-power factor (averaging about 0·7 in practice) and is only a fraction of the full-load rating of the motor.

In practice the last item is not a serious disadvantage with passenger trains

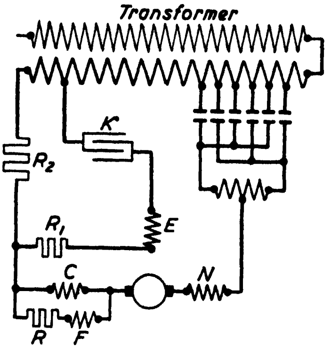


FIG. 152. CAPACITOR CONNEXION FOR REGENERATIVE BRAKING (BROWN-BOVERI)

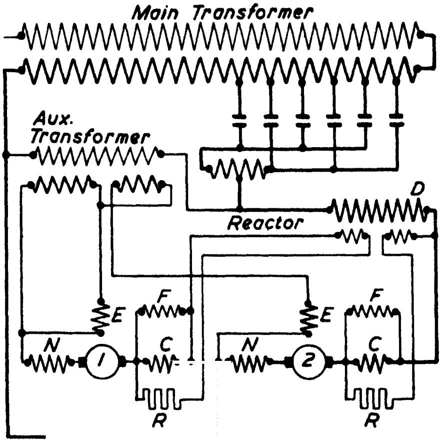


FIG. 153. EXCITER-MOTOR CONNEXION FOR REGENERATIVE BRAKING (OERLIKON)

as the full braking power of the locomotive is not usually required, 15 to 40 per cent being sufficient on many Swiss main lines.

The disadvantage of a low power factor is compensated by the simplicity of operation, stability, reliability and freedom from self excitation.

The weight of the main and auxiliary reactors and the resistors for the commutating poles is about 8 lb per kW of rated braking power at the treads of the wheels.

Capacitor Connexion. A high power factor may be obtained by connecting a capacitor (condenser) in series with the exciting winding of the motor of such capacitance that a resonant circuit is obtained. The power factor of this circuit is then unity and the field current is in phase with the voltage at the terminals of the transformer. Hence current fed into the transformer from the armature will have a power factor of practically unity. Such a scheme has been developed by Brown Boveri & Co. and has been applied to a number of motor-coach equipments.

A schematic diagram of connexions for the simplest case is shown in Fig. 152. The exciting winding of the traction motor is excited from a low-voltage tapping of the transformer with a variable capacitor, K, and a limiting resistor, R_1, in series, the object of the latter being to obtain a resonance curve with a rather broad peak. A stabilizing or decompounding resistor, R_2, is common to the field and armature circuits. Correct excitation for the commutating poles is obtained by means of an inductive shunt, R, F.

During braking the machine operates as a separately excited generator with light differential compounding, and the braking characteristics (speed/torque) resemble those shown in Fig. 149, i.e. the speed is only slightly affected by variations in the braking torque.

In practice relays, and modifications, to the simple circuit of Fig. 152 are necessary to maintain high power factor and efficiency over the working range of speeds and to suppress self excitation.*

Exciter-motor Connexion. This scheme, which also operates at a high power factor, is suitable for multi-motor equipments (e.g. four or more motors) and has been applied by the Oerlikon Co. to a number of motor-coaches and

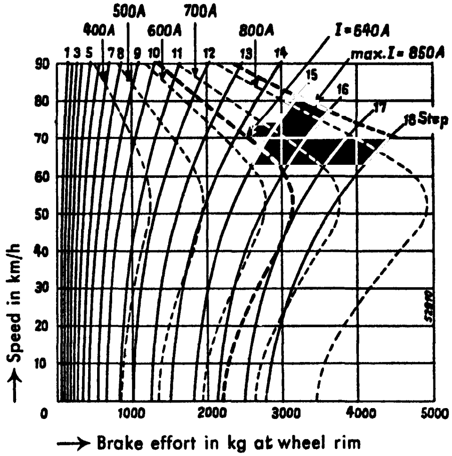


FIG. 154. TYPICAL BRAKING CHARACTERISTICS FOR EXCITER-MOTOR CONNEXION (OERLIKON)

The full-line curves show the speed/braking-effort characteristics for each of the 18 steps of the controller, and the dotted curves show the characteristics for constant currents.

locomotives. The speed/torque characteristics are similar to those of the Behn-Eschenburg connexion. Typical characteristics are shown in Fig. 154.

A schematic diagram of connexions for two motors is shown in Fig. 153, in which machine No. 1 acts as an exciter and No. 2 as the regenerative generator. The reactor, D, in the armature circuit of No. 2 equalizes the phase difference between the armature e.m.f. and the voltage at the tap-changer. Stability and suppression of self excitation are obtained by means of differential compounding (using a tertiary winding on the auxiliary transformer) and suitable excitation of the commutating poles, two shunts—one inductive, F, and one non-inductive, R,—being employed. The non-inductive shunts are supplied by auxiliary windings on the reactor D.†

APPENDIX TO CHAPTER XII

THE METADYNE SYSTEM OF OPERATION FOR D.C. MOTORS

Metadyne Converter. The metadyne converter is a single-armature d.c. machine

* See *Brown Boveri Review*, Vol. 31, p. 125; *Bulletin Oerlikon*, p. 2038.

† Further details, including theory and performance are given in *Bulletin Oerlikon*, Nos. 275, 284.

which converts input power at constant voltage into output power at variable voltage, or vice versa. The variable output voltage is fed to a group of traction motors. Regenerative braking to standstill is obtained by feeding power from the motors (now acting as generators) to the output terminals, the power being returned to the supply system through the input terminals.

The metadyne belongs to the "cross-field" class of machines in which armature reaction is utilized for excitation and self regulation. The number of groups of brushes is double that in an ordinary machine, and to obtain commutating zones for these brushes each main pole is divided into two equal parts.

Principle of Simple Metadyne. Consider the armature of a bipolar d.c. machine to be mounted in a four-pole magnet frame without windings but with brushgear (four groups) as shown in Fig. 155. The brushes are assumed to make contact with the active conductors so that the magnetic axis due to a current in the winding coincides with the axis of the brushes supplying the current.

If, with the armature stationary, a low-voltage supply is connected to the brushes A_1A_2 and the brushes B_1B_2 are open-circuited, the flux distribution will be as shown

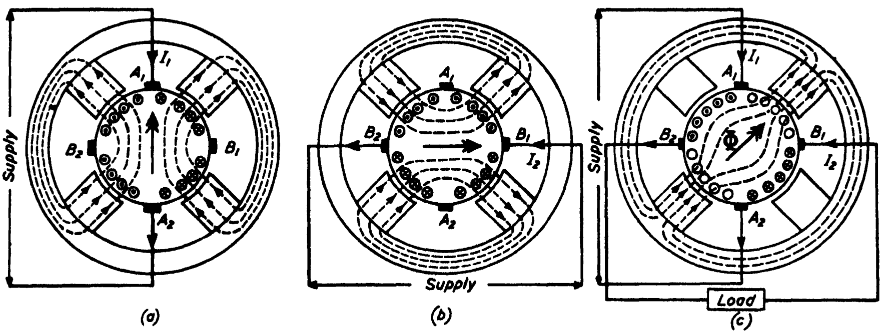


FIG. 155. DIAGRAMS TO ILLUSTRATE PRINCIPLE OF SIMPLE METADYNE

at (*a*), Fig. 155. If the supply is transferred to the brushes B_1B_2, and brushes A_1A_2 are open circuited, the flux distribution will be as shown at (*b*), Fig. 155.

If the armature is driven at constant speed and a current, I_1, is supplied to the brushes A_1A_2, an e.m.f., E_2, will be induced in the armature winding between the brushes B_1B_2, the magnitude of this e.m.f. being proportional to the flux due to I_1. No e.m.f. will be induced in the A_1A_2 axis, and therefore the voltage at the brushes A_1A_2 will be equal to the voltage drop due to the current I_1.

If a load is connected to the brushes B_1B_2 the resultant flux distribution, due to the load current I_2 and the supply current I_1 (assumed unchanged), will be as shown at (*c*), Fig. 155. The resultant flux, Φ, may be considered to have two components —one component, Φ_1, along the A_1A_2 axis and the other component, Φ_2, along the B_1B_2 axis. The rotation of the armature in the component Φ_2 induces an e.m.f., E_1, in the A_1A_2 axis which opposes the supply voltage. Hence if the current is to be maintained at its original value, I_1, the supply voltage must be increased by an amount to balance E_1. The torque due to I_1 and Φ_2 will balance the torque due to I_2 and Φ_1, as the rotational losses are supplied by the driving motor.

Under steady-state conditions with unsaturated magnetic circuits we have

$$E_1 = K\Phi_2 = K_1I_2 \text{ and } E_2 = K\Phi_1 = K_1I_1$$

Hence $E_1/I_2 = E_2/I_1$, or $E_1I_1 = E_2I_2$, i.e. the machine operates as a d.c. transformer.

If the load is entirely resistive and the internal resistance of the machine is zero, the e.m.f. E_2 will build up to a value such that $E_2 = I_2R$, where R denotes the resistance of the load. But if the load consists of a series, or separately-excited,

motor, the steady-state condition occurs when the output of the motor balances the mechanical power required by the load, the current input to the motor remaining constant.

Control of Output Characteristic. The simple metadyne gives a fixed and inflexible characteristic, e.g. with constant supply voltage the output current is constant.

Control of the output characteristic may be effected by means of separately excited windings on the poles, the excitation being controlled in such a manner as to give the characteristics desired. One winding—called the secondary *variator*—

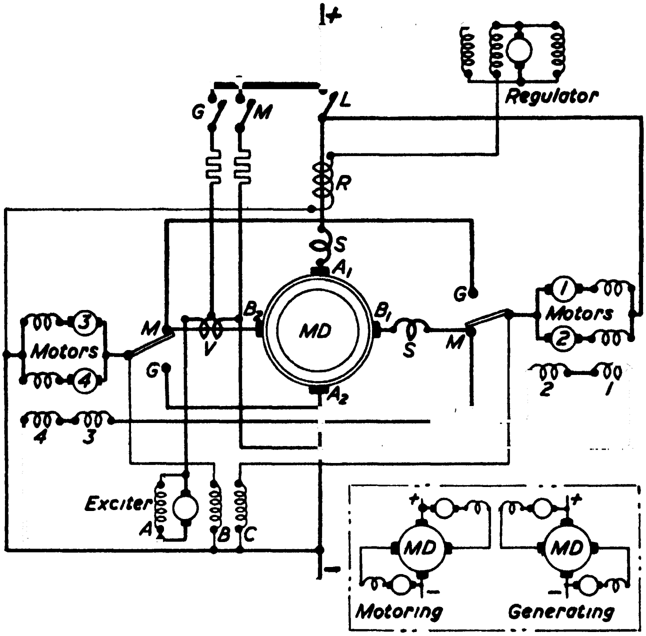


FIG. 156. SIMPLIFIED MAIN-CIRCUIT CONNEXIONS OF METADYNE FOR 4-MOTOR MOTOR-COACH EQUIPMENT (METROPOLITAN-VICKERS)

The metadyne armature, *MD*, is direct-coupled to the armatures of the regulator and the exciter. The field windings of the metadyne are represented by—*S*, light series stability (primary and secondary), *R*, regulator; *V*, variator. *M* and *G* refer to the positions of the change-over switch (and the appropriate contactors) for motor and generator operation respectively.

produces a m.m.f. along the B_1B_2 axis, and the other winding—called the *regulator* (or primary variator)—produces a m.m.f. along the A_1A_2 axis. Hence since the resultant m.m.f. along the B_1B_2 axis must be constant—to maintain Φ_2 constant—the m.m.f. produced by the variator winding causes a change in the load current I_2. The regulator winding is necessary to maintain the balance of the torques, i.e. $\Phi_2 I_1 = \Phi_1 I_2$. In the metadynes in service on the London Transport lines the regulator winding is connected in series with a shunt machine—called the regulator machine—which is coupled to the metadyne and connected to the supply.

Scheme of Operation.* In practice to reduce the size of the metadyne the booster connexion is employed, the armatures of the traction motors—arranged in

* Only a brief outline can be given here. Fuller details are given in the following: *Metropolitan-Vickers Gazette*, Vol. 17, p. 400; *Journ. I.E.E.*, Vol. 85, p. 370; Vol. 96, p. 537.

two groups—being connected to the brushes A_1B_1 and A_2B_2 as shown in Fig. 156. The voltage across each group of armatures is therefore equal to—supply voltage $\pm$ output voltage of metadyne.

The main-field windings of the traction motors are connected in series with the variator winding and supplied by an exciter which has three field windings, i.e. a self-excited shunt winding and two separately-excited windings (acting on separate magnetic circuits) one of which is supplied from the brushes A_2B_1 and the other from the brushes A_2B_2.

The metadyne has four field windings, (i) regulator, (ii) variator in two sections, (iii) primary stability, (iv) secondary stability. The two latter are light series windings to give stability with fluctuations of supply voltage.

Connexion of Metadyne to Supply. The metadyne is started by operating the regulator machine as a motor, a series starting winding being provided for this purpose. When normal speed is reached the variator winding is separately excited

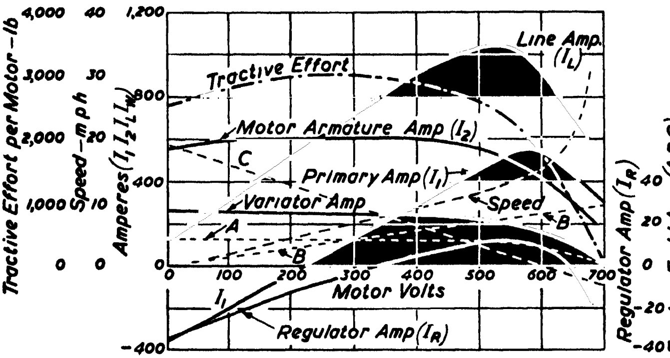


FIG. 157. PERFORMANCE CURVES OF METADYNE AND FOUR TRACTION MOTORS (METROPOLITAN-VICKERS)

Line voltage 600 V, wheel dia 36 in gear ratio 65/16, 4 motors each of 152 h p

from the supply so that a back-e.m.f. equal to the supply voltage is built up between the A_1A_2 brushes, which may then be connected to the supply.

Control of Traction Motors. The motors are started by supplying current, from the exciter, to the main-field windings, which operation also transfers the variator winding to this circuit, connects the regulator winding in circuit with the regulator machine and cuts out the series starting winding. Control of the motor current during acceleration is effected automatically by the joint action of the three field windings of the exciter. Definite balancing speeds on level track are obtained by means of rheostats in the separately-excited field windings of the exciter.

Typical characteristics are shown in Fig. 157. The negative values of current in the regulator winding indicate that the regulator machine is acting as a motor, while positive values indicate action as a generator. Negative values of primary current, I_1, indicate that the voltage across the B_1B_2 brushes opposes the supply voltage.

Regeneration. For regenerative braking the armatures of the traction motors must be transferred to the quadrants A_1B_2 and A_2B_1, this operation being effected by a change-over switch when the metadyne is disconnected from the supply. The metadyne is re-connected to the supply and the braking torque is controlled automatically by the exciter.

CHAPTER XIII

AUXILIARY ELECTRICAL EQUIPMENT FOR TRAMCARS AND TROLLEYBUSES

In addition to the motors and controller a tramcar or trolleybus requires—(i) a current collector; (ii) a starting rheostat; (iii) apparatus to protect the equipment against excessive currents and over-voltages due to surges or lightning; (iv) a lighting, and in some cases a heating, installation; (v) an air compressor, when air brakes are employed. Trolleybuses operating in Great Britain also require—a leakage testing socket; apparatus to suppress radio interference; a battery and generator for low-voltage lighting (which is compulsory with an all-metal body).

Current Collectors. Tramcars and trolleybuses derive their power from overhead conductors suspended about 22 feet above the road surface. The current collector must therefore be mounted on the roof of the vehicle and must adapt itself to variations in the level of the wire.

The *trolley collector* is universally employed on trolleybuses. With this collector contact with the wire is made by either a grooved wheel, or a grooved slider with carbon insert, Fig. 158, carried at the end of a light pole which is hinged to a swivelling base fixed to the roof of the vehicle. As two trolley wires are necessary for a trolleybus a separate trolley collector is provided for each wire, the bases being mounted side by side as shown in Fig. 159. Each collector can therefore adapt itself to variations in level of the wire concerned.

Trolley collectors must operate in a trailing position, and if the direction of motion of the vehicle is reversed the collector must be rotated through 180 degrees.

The head—whether fitted with a wheel or slider—is always of the swivelling type. The wheel is of gunmetal, and the diameter at the bottom of the groove is about 3 inches: it runs on a lubricated spindle fitted in a guard, which is mounted on a vertical spindle carried in a bearing in the body of the head, the thrust being taken either by a ball or a flat surface in conjunction with a friction washer.

In the slider type of collector a renewable carbon block, *a*, Fig. 158, is fixed to a holder, *b*, which is mounted on a lubricated spindle, *c*, fitted to a swivelling guard, *d*, carried in a light frame of welded steel clamped to the trolley-pole. The swivelling bearing is fitted with a renewable bush, *e*, and a friction washer, *f*. This head gives quieter operation and less sparking than the wheel type.

The pressure between the head and trolley wire is approximately 20 lb for a wheel collector and 35 lb for a carbon-inset slider. The lever and spring system at the base is designed to maintain this pressure approximately constant over the operating range of the head.

Bow and pantograph collectors—in addition to the trolley collector—are employed on tramcars. The pantograph collector has large applications in Europe and has superseded the bow collector which was formerly employed.

The pantograph collector is in general use on locomotives and motor-coaches operating from overhead contact wires, and is described in Chapter XV. For tramcars a lighter construction can be employed, and considerable simplification can be made to the mounting (due to the low voltage).

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The advantages of the pantograph over the trolley collector are—(i) its suitability for operation in either direction of motion, (ii) no risk of the collector leaving the wire at junctions, (iii) considerable simplification of overhead construction at junctions due to the absence of points and grooved crossings.

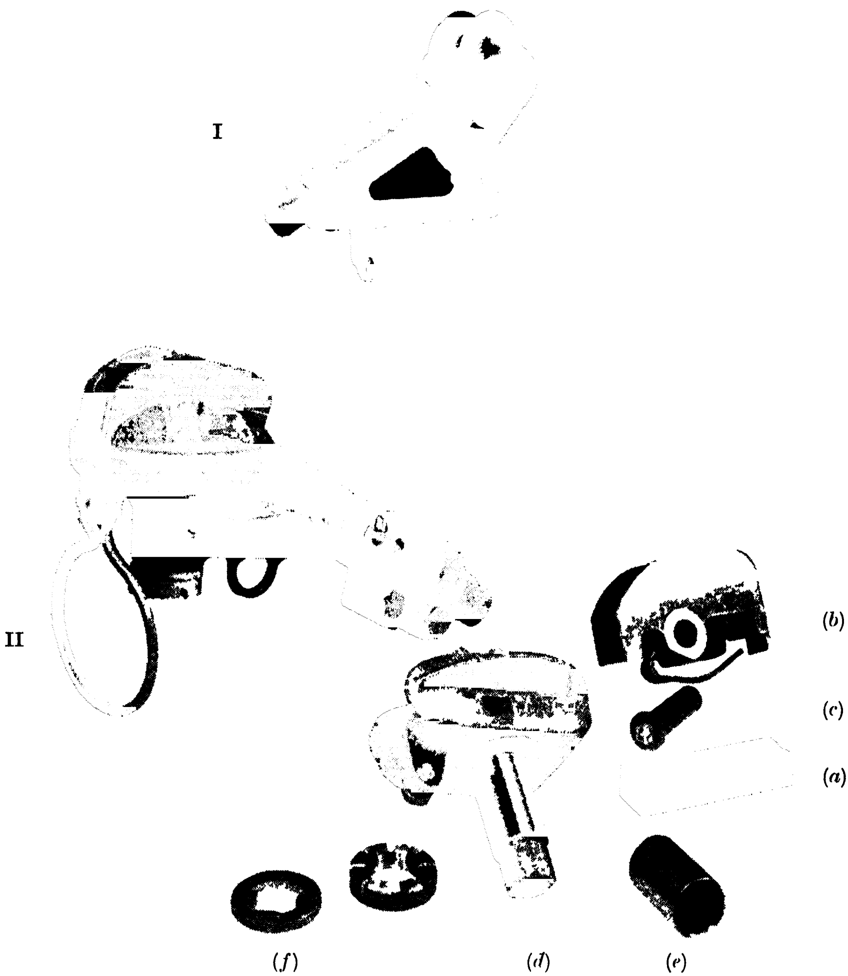


FIG. 158. I. WHEEL COLLECTOR (BRECKNELL-WILLIS); II. LIGHT-WEIGHT WELDED-STEEL TROLLEY-HEAD WITH SLIPPER COLLECTOR (BRITISH INSULATED CALLENDER'S CONSTRUCTION CO.)

Rheostats. Robustness and light weight are essential in rheostats for trolleybuses, light weight being particularly important for buses operating in Great Britain, where the weight of the fully-laden vehicle is limited by statutory regulations.

A typical rheostat fulfilling these requirements is shown in Fig. 160. The

resistor elements consist of coils of high-resistivity stainless steel strip wound on edge: the individual turns are supported by ceramic insulators mounted on sheet steel supports, the ends of which are carried on mica-insulated bars

FIG. 159. COLLECTORS FOR TROLLEYBUS MOUNTED ON ROOF OF VEHICLE (METROPOLITAN-CAMMELL-WEYMAN)

fitted between end frames. Lugs for connexions are welded to the ends of each coil. The resistor material and the ceramic supports can withstand temperatures of 450°C without deterioration, while the edgewise winding provides good cooling with a relatively small overall dimension.

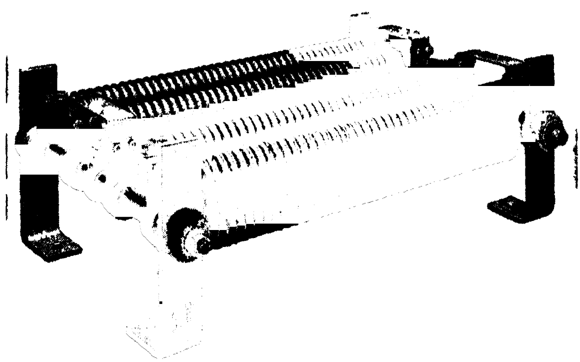


FIG. 160. RHEOSTAT WITH EDGEWISE-WOUND STRIP ELEMENTS (B.T.-H.)

Fig. 161 shows an alternative form in which the strip is formed into loops, and rigidity is obtained by spot welding transverse strips, *A*, to the loops at intervals and supporting these on mica-insulated rods with ceramic insulators as spacers.

Protective Devices. 1. *Circuit Breaker.* The equipment is protected against excessive overloads by an automatic circuit breaker, which is connected

between the current collector and the main wiring. Two circuit breakers are employed on a trolleybus, one in each trolley circuit.

Typical circuit breakers are shown in Fig. 162. The main contacts are located in an arc chute and are provided with a magnetic blow-out, the blow-out coil actuating also the tripping mechanism. The handle is arranged to trip the circuit breaker when it is moved to the "off" position. Hence the circuit breaker can be operated by hand in the same manner as a switch.

2. *Radio Interference Suppressor.* On trolleybuses, due to the insulation of the vehicle from earth, arcing or sparking at the collector, controller, motors, etc., may cause e.m.f's. of radio frequencies to be induced in the wiring and

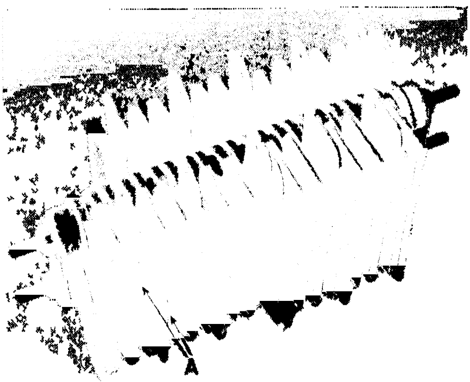


FIG 161. RHEOSTAT ELEMENT REMOVED FROM FRAME (RHEOSTATIC CO)

radiated from the trolley wires. The most serious interference occurs at the contacts of the master controller in systems with electromagnetic contactors operated at line voltage. Suppressors* take the form of (i) two radio-frequency chokes, one connected in each supply lead to the master controller, (ii) a capacitor filter, comprising three star-connected capacitors each about $0{\cdot}1\mu F$ capacitance, connected between the trolley cables and chassis as shown in Fig. 96.

3. *Lightning Arrester or Surge Diverter.* This apparatus is usually only fitted to vehicles which operate in districts liable to lightning storms. One type consists of a small cylindrical block of carborundum held between two electrodes one of which, with a small air gap in series, is connected to the positive trolley, and the other to earth or the negative trolley. A discharge which breaks down the air gap is dissipated in the numerous air spaces throughout the block, the high resistance of which prevents the current being maintained by the line voltage.

Car Wiring. *Power Circuits.* With modern tramcar equipments the wiring for the power circuits consists of two multi-core cables (one along each side of the car), with distinctive colours to the separate cables. These cables interconnect the controllers, and tappings are brought out at suitable points for connexion to the motors, rheostats, and brakes.

* Details are given in British Standard No. 827.

With trolleybus equipments the motors, rheostats, controller, and circuit breakers are all arranged at the forward end of the vehicle. The connexions are made by flexible cables (e.g. 248/·018, 140/·01) of the tough-rubber-sheathed type.

Lighting Circuits. These circuits have to provide for the interior lighting of the car as well as for the lights required by traffic regulations. When lighting at line voltage is permissible (e.g. on tramcars and also trolleybuses with wooden and composite bodies) the circuits consist of a number of low-voltage lamps

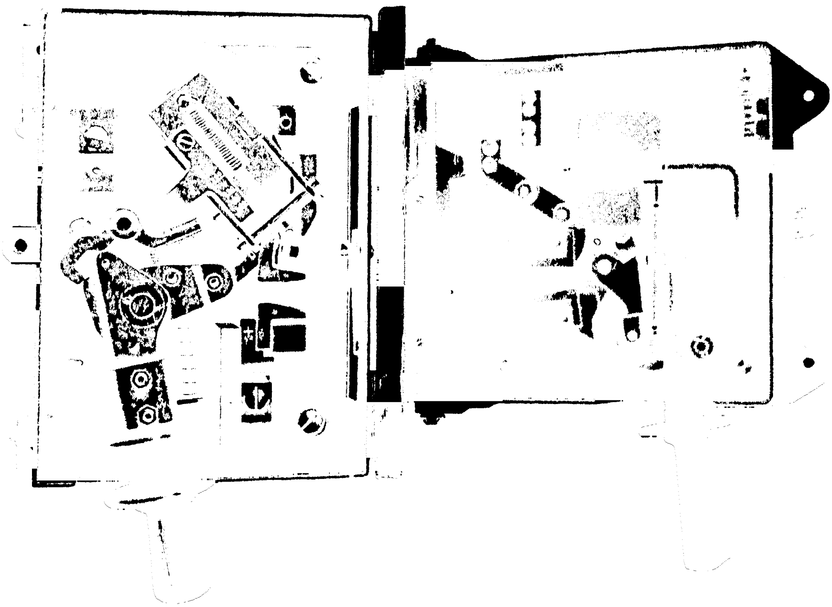


FIG. 162. TRAMCAR AND TROLLEYBUS CIRCUIT-BREAKERS
Left B.T.-H. *Right* Metropolitan-Vickers

connected in series. Formerly 105-V filament lamps (five in series) were employed, but the "long series" system—with circuits of either fifteen 40-V lamps, each 40 or 60 W, or ten to twelve 50-V lamps, each 40 or 60 W—is preferable, each lamp having a short-circuiting device which comes into action on the failure of a lamp and therefore enables the faulty lamp readily to be located. Each lampholder, however, must also be fitted with a short-circuiting device which comes into action on the removal of a lamp from its holder.

Fluorescent lighting has been adopted on some tramcars and trolleybuses. The lamps (2-ft, 20-W, tubular, normal circuit voltage 110 V) may be supplied either with d.c. at line voltage (in which case a number are connected in series with a ballast resistor) or with a.c. (at a frequency of about 800 c/s) from a small motor-alternator set. Details are given in Chapter XIV. In each case a small 12-V battery must be provided for supplying the traffic or "legal" lights.

Low-voltage Lighting. The lighting circuits of trolleybuses with all-metal bodies must be supplied at low voltage (24 or 30 V).

The supply is obtained from either a generator coupled to the traction

motor—in conjunction with a battery, automatic switch and voltage regulator —or a motor-generator with a floating battery. In the former case the generator is rated at 1,600 watts and is designed for a "cut-in" speed of about 4 m.p.h.

The motor-generator is of the two-bearing type, and consists of a series motor and a shunt generator rated at 1,200 watts. The speed at light load is limited by the ventilating fan, and the voltage of the generator is controlled by an automatic regulator. As statutory regulations require double insulation between the high-voltage and low-voltage parts, the motor armature is built

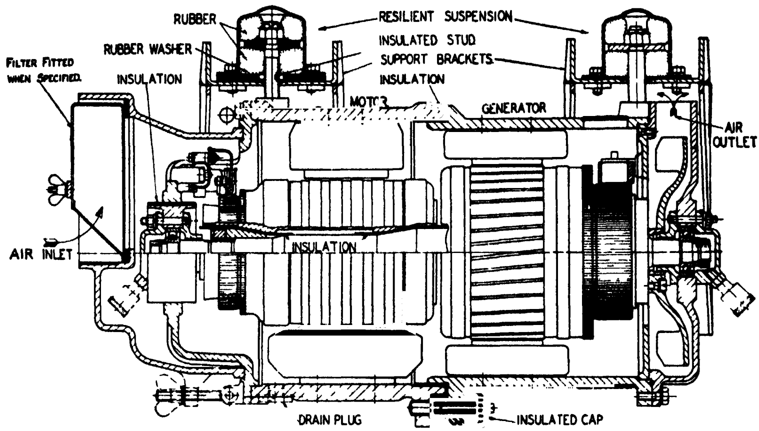


FIG. 163. LONGITUDINAL SECTION OF MOTOR-GENERATOR FOR TROLLEYBUS (METROPOLITAN-VICKERS)

on a sleeve which is insulated from the shaft, and the two magnet frames are insulated from each other. Fig. 163 shows a typical arrangement.

Air Compressor. For trolleybuses the air compressor is of the two-cylinder type with a capacity of 5 cubic feet per minute; it is direct coupled to a $\frac{3}{4}$-h.p. series motor operating from the traction circuit and controlled by a pressure-operated contactor.

Leakage Testing Socket. To facilitate the carrying out of the daily leakage test on a trolleybus with an all-metal body a multi-point socket is provided, the contacts of which are connected to the chassis and to each of the frames of all the electrical equipment which is insulated from the chassis. The test is made by energizing the whole equipment at line voltage and measuring the leakage current from (i) the chassis to earth, (ii) the frames of the equipment to earth, a special plug and milli-ammeter enabling these measurements to be made in quick succession. The first reading gives the total leakage over both primary and secondary insulation, and the second reading gives the leakage over the primary insulation.

Dewirement Indicator. This is necessary on trolleybuses with low-voltage lighting and is located in the driving cab in view of the driver: it gives both visual and aural signals of loss of voltage at the trolleys. The essential features are (i) a relay connected to the traction circuit; (ii) an indicating lamp, and

(iii) a buzzer. The lamp and buzzer are supplied from the battery through the contacts of the relay which are closed when the relay is de-energized (i.e. when the line voltage fails).

A simpler type, giving only a visual signal, consists of a neon lamp with protective fuses.

CHAPTER XIV

AUXILIARY ELECTRICAL EQUIPMENT FOR ELECTRIC LOCOMOTIVES AND MOTOR-COACHES

THE auxiliary electrical equipment required by locomotives and motor-coaches includes: (i) current collectors; (ii) motors and control gear for driving (*a*) the compressors, or exhausters, for the power brakes, (*b*) the blowers for ventilating the main motors (if these are of the forced-ventilated type); (iii) the motor-generator for supplying the lighting, control, and auxiliary circuits of high-voltage d.c. equipments at suitable voltage.

CURRENT COLLECTORS

The type of current collector depends upon the position of the conductor from which the locomotive or motor-coach obtains its supply. With conductor-rail distribution systems the current collector consists of a cast-iron, or cast-steel, collector shoe, which is maintained in contact with the conductor rail either by its own weight or by means of a spring. With overhead distribution systems the current collector consists of a bow- or pan-shaped sliding contact carried on a light framework and maintained in contact with the trolley wire by means of springs.

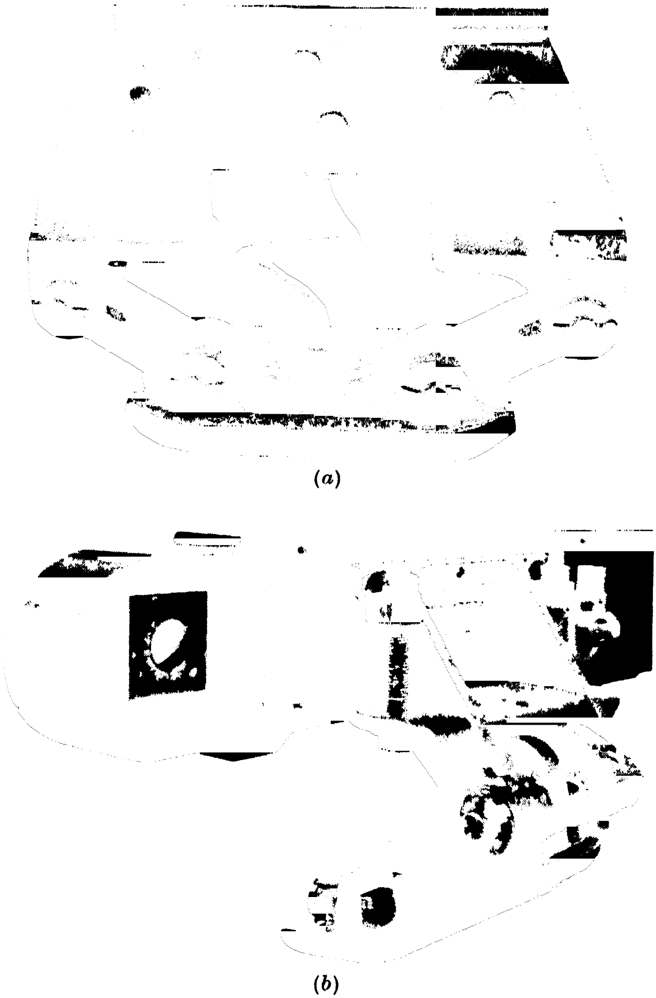
Collector Shoes. In this country the top-contact conductor rail is generally used, and typical collector shoes are shown in Fig. 164.

When the conductor rails are located outside the track rails, the collector shoes are attached to an oak beam, which is fixed to the axle-boxes, or to a part of the truck frame directly connected to the axle-boxes (e.g. the equalizer bars in an equalized bogie). In the other position of the conductor rails (i.e. between the track rails) the collector shoe is either attached to an oak block (which is bolted to a bracket fixed to the motor frame) or to an oak beam at the end of the truck, this beam being connected to longitudinal channels carried from the axle-boxes. It should be noted that, when passing round curves, the transverse movements of the shoes will be greater the greater the distance the shoes are from the pivotal centre of the truck.

Bow and Pantograph Collectors. An essential condition in the collection of current from an overhead conductor is that the collector shall maintain contact with the trolley wire at all speeds. Hence the collector must follow instantaneously any irregularities in the level of the trolley wire, and therefore, with an ordinary suspension, a collector of very small inertia (e.g. a trolley wheel or a light bow) must be employed. But, although the trolley-wheel collector* is used (on account of its cheapness) to some extent on inter-urban railways in America, its use on a large railway system could not be tolerated, both on account of the complication involved in the overhead conductors and the liability of the trolley wheel to leave the trolley wire. In practice, therefore, a bow or pantograph collector, together with a level trolley wire, must be

* The current-collecting capacity of the larger wheels may be as high as 800 amperes at low speeds. With a level trolley wire, currents of 200 amperes have been collected at speeds of from 50 to 60 m.p.h.

employed. The bow collector has the smaller inertia, but is not so readily adaptable to the collection of large currents as the pantograph collector. Moreover, the bow collector must always be run trailing. Hence for reversible operation, either duplicate bows or a reversing bow must be employed. On the



(a)

(b)

FIG. 164. COLLECTOR SHOES FOR TOP-CONTACT CONDUCTOR RAILS
(a) B.T.-H. (b) London Transport Executive (for positive rail).

other hand, a pantograph collector is reversible, but, on account of its greater inertia, requires a greater pressure to maintain contact with the trolley wire than a bow collector.

The collector, whether bow or pantograph, is usually maintained in contact with the trolley wire by means of springs, while the raising and lowering operations are performed by air cylinders.

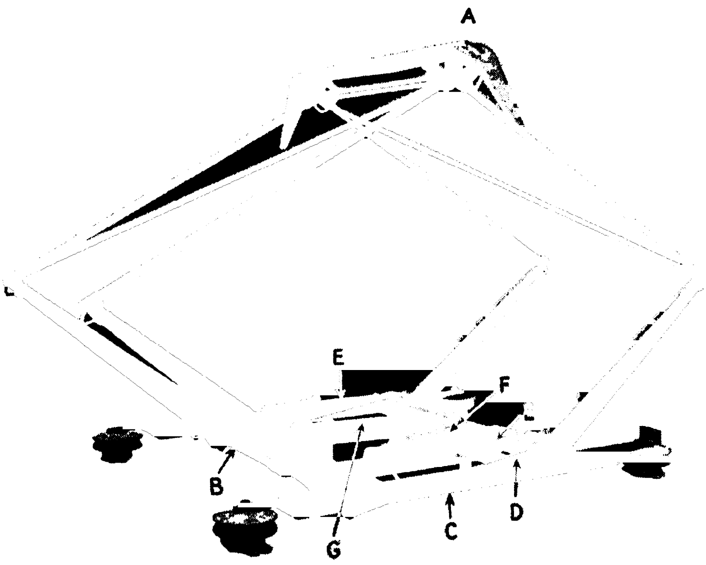


FIG 165 METROPOLITAN VICKERS PANTOGRAPH COLLECTOR IN NORMAL POSITION

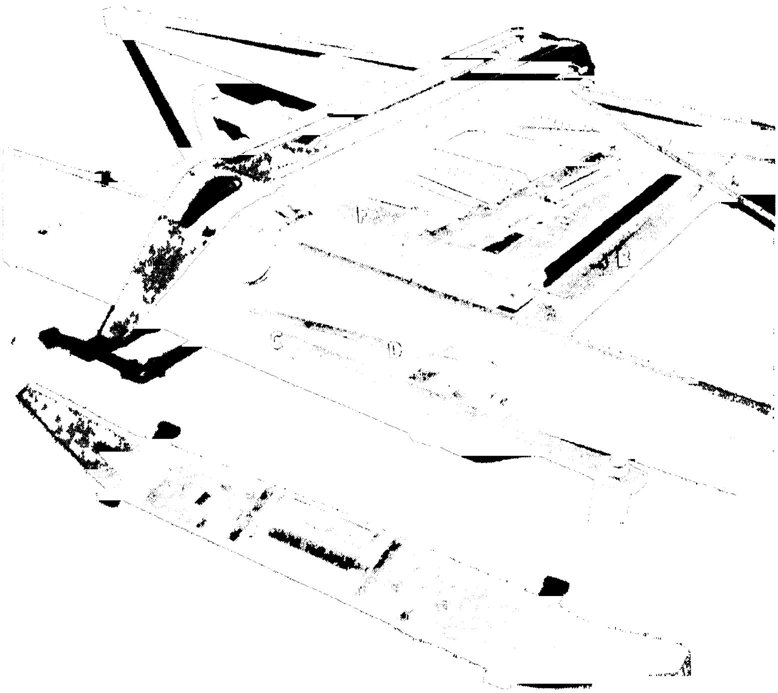


FIG. 166. METROPOLITAN VICKERS PANTOGRAPH COLLECTOR IN LOWERED POSIT...
Detail of the underside of the head is shown below.

A *pantograph collector for* 1,500- *or* 3,000-V *locomotives* or motor-coaches is shown in Figs. 165, 166. The contact portion consists of a pressed steel pan, A, of channel section, fitted with renewable copper wearing strips, which is supported by springs at the apex of a hinged pentagonal framework, these springs enabling the pan to follow small irregularities in the level of the trolley wire. The lower sections of the framework are fitted to horizontal shafts, B, which are carried in ball bearings fitted to the main frame, C. These shafts

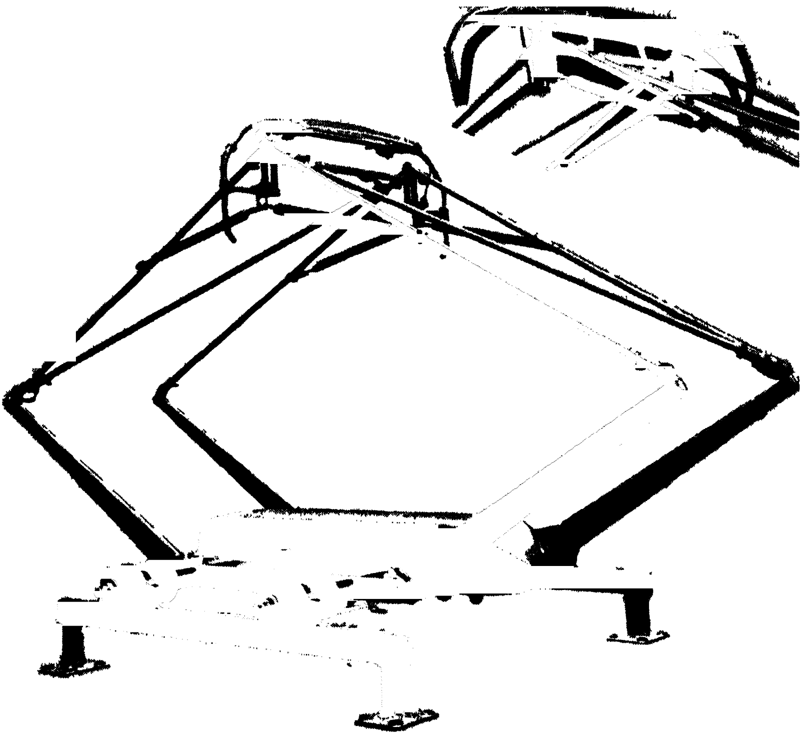


FIG. 167. BROWN-BOVERI PANTOGRAPH COLLECTOR FOR SINGLE-PHASE LOCOMOTIVE

Inset shows type of head for 3,000-volt, d c locomotive

are interconnected by rods D and springs E, the latter being adjusted to balance most of the dead weight of the movable framework.

The pantograph is maintained in the normal (raised) position by compressed air in a cylinder, F, the piston rod of which is connected by a system of levers to a central crank on one of the shafts. This action stresses the central spring, G. Hence when air is exhausted from the cylinder the pantograph is lowered by the action of this spring and is secured in this position by a latch.

The pantograph shown in Fig. 165 has a single head and that shown in Fig. 166 has a duplex head suitable for heavier currents, a detailed view of the underside of the head being also shown.

The cranks and springs are so designed that the variation of pressure on

the trolley wire does not exceed 5 lb over the whole working range of the collector. The average working pressure is 16 lb, but other values may be obtained by varying the stroke of the pistons by means of adjustable collars inside the cylinders.

The extremities of the pans are of the inverted-horn shape to prevent the pans fouling the trolley wires at crossings, and also to prevent a sagging trolley-wire becoming entangled with the pans.

A *pantograph for single-phase* 15,000-V *locomotives* is shown in Fig. 167. The contact strip is of aluminium and is lubricated.* The inset view shows the head employed for 3,000-V d.c. locomotives and currents of 800 A.

For d.c. railways, operating at 1,500 and 3,000 volts, the pan type of pantograph collector is usually employed on account of its greater current-collecting capacity. Pans of the form illustrated in Fig. 166 can readily be designed for collecting the currents required by the largest d.c. locomotives. For example, currents of 2,000 amperes have been collected at speeds of 40 m.p.h. by a collector having two pans (with inserted copper bars), each pan being 5 in. wide by 42 in. long.

Power Supply for Brakes and Ventilating Apparatus

Power brakes† require either a supply of compressed air or means for creating and maintaining a vacuum. In the former case an electrically-driven compressor and air reservoirs are required, together with means for controlling the motor so as to maintain the air pressure within prescribed limits. In the latter case a vacuum pump, or exhauster, is required, together with means for controlling the speed of the motor to give the degree of vacuum desired.

Compressors. The compressor is usually of the single-stage two-cylinder, single-acting type, and is directly connected to the motor. With compressors for motor-coaches, a moderate speed motor is adopted in order to reduce the weight, and the compressor is driven through double-helical spur gearing.

The single-stage compressors for use on motor-coaches and locomotives are built in capacities up to 100 cu. ft (piston displacement) per minute at normal air pressure (80 to 90 lb per sq. in.). Larger compressors are of the two-stage type, with three or four cylinders and an inter-cooler between the low- and high-pressure cylinders.

Automatic starting and stopping of the motor to maintain the air pressure within prescribed limits is effected by a pressure-operated switch, the moving contact of which is actuated, through a system of levers and a toggle mechanism, by the air pressure on a spring-loaded diaphragm.

Exhauster. The exhauster or vacuum pump (which is used with vacuum brakes) is usually driven by a slow-speed motor through a spring coupling. In some cases the motor is run continuously at slow speed (for the purpose of maintaining the vacuum), and is automatically switched over to full speed when the brake valve is moved to the "off" or "release" position. In other cases the motor is controlled by an automatic governor, which is arranged to start the motor when the vacuum falls to 15 in.

* When sparking at the contact strip would cause radio interference the aluminium strip is replaced by carbon.

† A description of the mechanical equipment for compressed air and vacuum brakes is given at the end of Chapter XVI.

Ventilating Plant. Blowers are required on locomotives when forced-ventilated motors are used. In some cases a single blower is employed which delivers air at a small pressure (6 to 8 in. of water) into a central duct—built into the underframe of the locomotive body—from which it is distributed to the motors. In other cases, with large frame-mounted motors, a separate blower is provided for each motor. The blowers are then mounted on the motor frames, and two or more blowers are coupled together and driven by a single motor.

Power Supply for Auxiliary Circuits

The auxiliary circuits requiring a supply of current are: (i) control; (ii) motors for driving the compressors, exhausters, and blowers; (iii) lighting; (iv) heating.

With *single-phase equipments* these circuits are supplied from tappings on the main transformer, a voltage of about 220 volts being employed for the auxiliary motors, and also for the control and lighting circuits, except in cases where a low-voltage, d.c., train-lighting system is already in existence. The train-heating circuits are usually supplied at a voltage between 600 and 1,000 volts, in order to limit to moderate values the current to be carried by the heating bus-line cable and couplers, as in some cases from 300 to 400 kW may be required for heating purposes. Usually three tappings are provided in order that the degree of heat may be varied when desired, the control switch being located in the driver's cab.

With equipments operating at 25 c/s and 50 c/s a three-phase supply is preferable for the auxiliary motors as these may then be of the three-phase induction type with squirrel-cage rotors. This supply is obtained either from an alternator driven by a single-phase induction motor or a phase converter operating on the Arno principle.*

With *high-voltage, d.c. equipments* the compressor, exhauster, and blower motors are usually supplied directly from the traction circuit when the voltage does not exceed 1,500 volts, but for higher voltages these machines are supplied at low voltage (100 to 125 V) from a motor-generator set, which also supplies the control and lighting circuits.

Alternatively, on 3,000-V circuits, when a relatively large amount (50 to 60 kW) of power is required for air conditioning, ventilation, etc., a three-phase supply for the auxiliary services (other than the control circuits) is preferable, as three-phase induction motors with squirrel-cage rotors are cheaper, and require less maintenance than d.c. motors.

Motor-generator. To simplify starting procedure the motor must start as a series machine, and unless a permanent load (e.g. a blower) is directly connected to it, additional excitation must be provided to prevent large changes of speed with variation of the generator output. This additional excitation is usually supplied by a separately-excited shunt winding connected to the generator. The output voltage of the generator is maintained constant by an automatic regulator (both carbon-pile and vibrating-contact types being in use).

* The principle of this system is described in a paper by A. H. Maggs, "Single-phase to three-phase conversion by the Ferraris-Arno System," *Journ. I.E.E.*, Vol. 93, p. 133.

CHAPTER XV

TRAMWAYS ROLLING STOCK, TROLLEYBUSES AND COMMERCIAL BATTERY VEHICLES

I. Tramways Rolling Stock

Types of Cars. In Great Britain the double-deck car with covered upper saloon is used almost exclusively on account of its suitability for British operating conditions and its lower cost, on a per-passenger basis, in comparison with a single-deck car. Typical modern cars with closed vestibules are shown in Fig. 168.

In other countries the climate, operating conditions, system of fare collection (pay-as-you-enter), large number of standing passengers, and the requirements of rapid loading and unloading are unfavourable to double-deck cars, and the single-deck car is, therefore, in universal use. This type of car is also in use on some British tramways where the traffic conditions require rapid loading and unloading. Examples of European and British single-deck cars are shown in Figs. 169, 172, and data are given in Table III.

Features of Modern Tramcars. Modernization of tramway rolling stock throughout the world has progressed along the lines suggested by the Transit Research Corporation, U.S.A., and modern tramcars follow the design, or incorporate the principal features, of the Presidents' Conference Committee (P.C.C.) single-deck car, which was the outcome of this research and of which over 5,000 are in service in N. America alone.

The chief features are high acceleration and braking retardation, quiet and smooth running, rapid loading and unloading, light-weight design, low maintenance.

The high acceleration and braking retardation (3·5 to 4·75 m.p.h.p.s. and 4 to 6 m.p.h.p.s respectively)* are obtained by using bogie trucks with four-motor equipments, and automatic multi-notch control. Quiet and smooth running are obtained by using light-weight frame-mounted motors with cardan-shaft drive, resilient (composite steel and rubber) wheels, trucks with roller axle bearings but without hornblocks, silentbloc connexions for all links, and rubber pads to avoid metal-to-metal contact.

Car Body. The main pillars, carlines and other members of the framing consist of high-tensile steel. The plating, panelling, mouldings, etc., are usually of aluminium. The underframe is integral with the body and is built of rolled steel sections. Entrance and exit doors are usually power operated and are controlled by either the driver or the conductor.

The seating accommodation consists of either transverse seats or a combination of longitudinal and transverse seats.

* The higher values refer to American operating conditions with P.C.C. cars. The braking retardation in emergencies may reach 9 m.p.h.p.s.

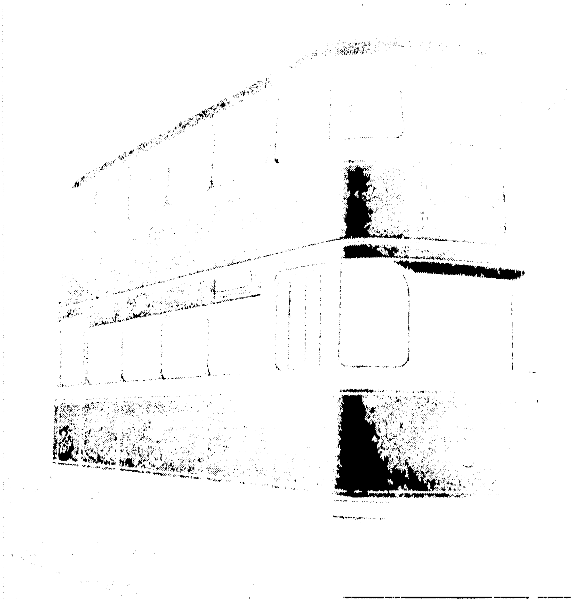


FIG. 168. MODERN BRITISH CARS WITH CLOSED VESTIBULES

Upper—Sheffield car with trolley collector single truck, 9 ft wheel base (Metropolitan-Vickers and B T -H equipment) *Lower*—Glasgow car with bow collector, bogie trucks (Metropolitan-Vickers)

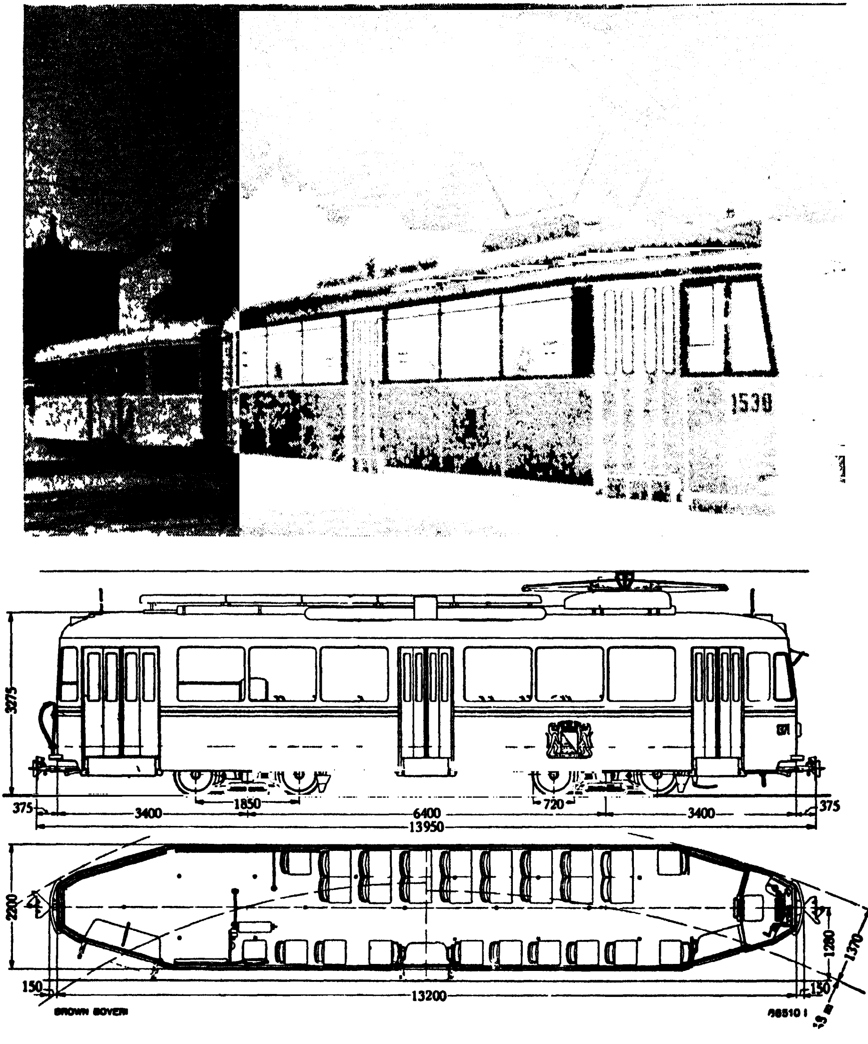


Fig. 169. Zurich Car (Brown-Boveri)

This type of car is in use on many Swiss (metre-gauge) tramways. The wedge-shaped ends reduce the overhang at curved track.

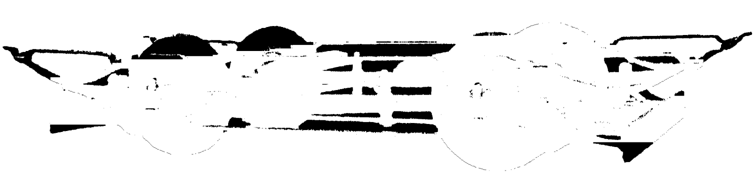


Fig. 170. Rigid-axle Single Truck (J. G. Brill Co.)

TABLE III

DATA OF TRAMCARS AND TROLLEYBUSES

	I	II	III	IV	V
Length over all	50′	32′ 7″	46′	44′ 3″	30′
Length of body (over corner posts)	43′ 6″	21′	33′ 6″	30′	22′
Width over all (maximum)	7′ 11″	6′ 11½″	8′ 4″	7′ 2″	7′ 11″
Total height	10′	15′ 9″	10′	10′ 9″	15′ 10″
Number of seated passengers (lower deck)	56	26	59	27	28
Number of seated passengers (upper deck)	—	36	—	—	38
Class of truck	Bogie	Single	Bogie	Bogie	—
Wheel base	6′	9′	6′	5′ 4″	18′ 7″
Diameter of wheels	27″	27″	25″	26″	41″
Centres of bolsters	25′	—	22′ 9″	21′	—
Motor equipment	4-45 h.p.	2–65 h.p.	4–55 h.p.	4–65 h.p.	1–120 h.p.
Weight of car equipped	14 tons	14 tons	14¾ tons	14 tons	10 tons

I, single-deck British car, 4′ 8½″ gauge: II, double-deck British car, 4′ 8½″ gauge; III, American (P.C.C.) single deck car, 4′ 8½″ gauge; IV, Swiss (Zurich) single-deck car, metre gauge, designed for max. of 73 standing passengers and for hauling two trailers; V, double-deck, 3-axle British trolleybus.

TRUCKS

Trucks for tramcars may be divided into two classes, (i) single trucks, (ii) bogie trucks.

Single trucks may be subdivided into three types, (*a*) trucks in which the axles are maintained rigidly parallel (called rigid-axle trucks); (*b*) trucks in which the axles are allowed transverse oscillatory movement (called swing-axle trucks).

Bogie trucks may be subdivided into two types, (*c*) the maximum traction or single-motor truck, in which the pivotal centre is displaced from the centre of the truck towards the driving axle; (*d*) the equal-wheel truck, in which the pivotal centre coincides with the centre of the truck.

Single Trucks. *Rigid-axle single trucks* were formerly of the type shown in Fig. 170, in which the axle boxes slide in machined guides or "horns" formed in the side frames. The latter are supported on short spiral springs which are carried in pockets on side extensions of the axle boxes. The car body is supported on the side frames by a combination of spiral and semi-elliptic laminated springs.

A modern truck is shown in Fig. 171. The chief features are (i) the roller axle-box bearings, (ii) the elimination of sliding axle-box guides (which when worn are a source of noise), (iii) the support of the truck frame on rubber pads, (iv) the slight lateral flexibility of the axles. The rubber pads are carried on side extensions of the axle-boxes at a much greater distance apart than the supporting springs in the truck of Fig. 170, thereby giving steadier riding at maximum speed. These extensions are connected to the truck frame by links of spring-tempered steel. The car body is supported by a combination of spiral and semi-elliptic springs. The truck is built with a wheel-base of 9 ft, whereas the wheel-base of the rigid-axle truck of Fig. 170 is usually 6 ft or 6 ft 6 in.

The principle of one form of *swing-axle truck* is shown in Fig. 173. The axle

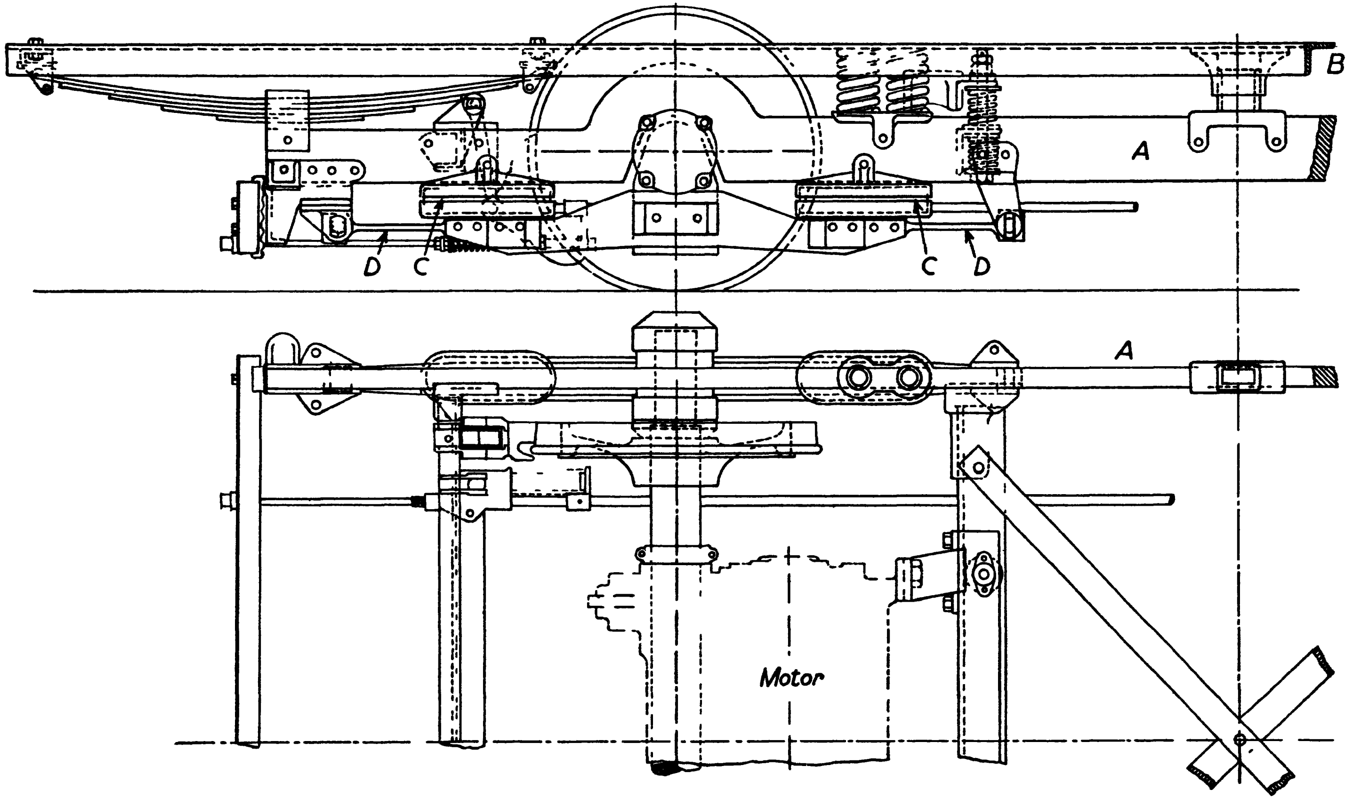


FIG 171. MALEY TAUNTON HORNLESS SINGLE TRUCK (HALF ELEVATION AND PART PLAN)
A, side frame *B* support for car body under frame, *C*, rubber pads, *D*, links of spring tempered steel

boxes have spherical-seated roller bearings and are fixed to long semi-elliptic springs which are attached to brackets on the side frames of the truck, thereby, eliminating axle-box guides or "horns" which are necessary in the rigid-axle truck of Fig. 170. The spherical roller bearings in conjunction with the long springs allow the axles to move sideways relatively to the side-frames when the wheel flanges strike the rail. Such movements cause the springs to twist slightly and produce a restoring force to return the axle to its normal position.

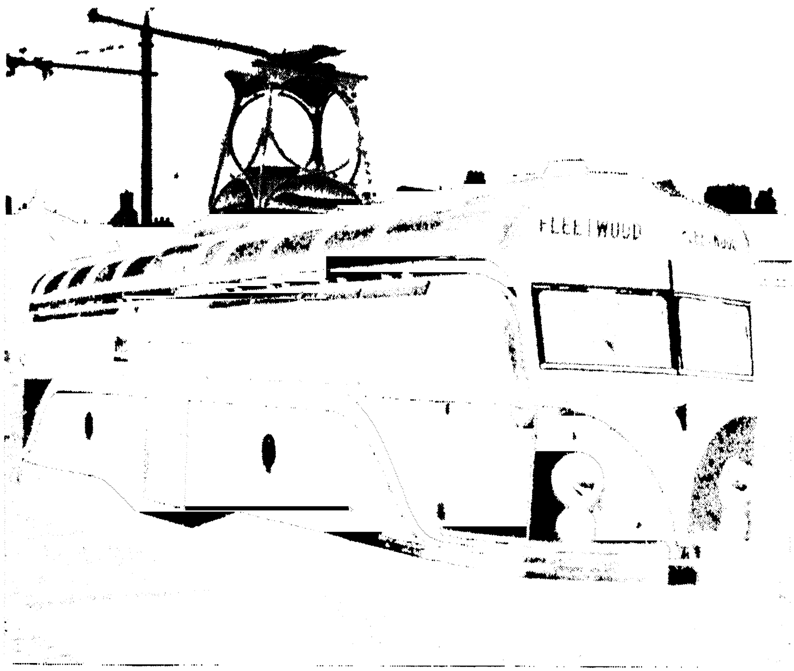


FIG. 172. BLACKPOOL TRAMCAR WITH "VAMBAC" EQUIPMENT (CROMPTON PARKINSON)

The transverse movements of the axles, due to track irregularities, are not, therefore, transmitted to the car body as would be the case with a rigid-axle truck.

Features of Modern Bogie Trucks. Innovations introduced into the design and construction of bogie trucks for modern tramcars have been towards obtaining silent running, light weight and low maintenance. At the outset the trucks were developed for frame-mounted motors and a cardan-shaft drive, the whole weight of the motors being spring supported. In the P.C.C. design the armature shaft is at right angles to the axle, and the final drive is by spiral bevel gearing with a reduction ratio of 7·2 . 1. In other designs the armature shaft is parallel to the axle and is hollow, the cardan shaft passing through it and being connected by flexible couplings to spur gearing. Examples are illustrated in Figs. 175, 177.

Although the original P.C.C. design was based on the use of both rheostatic and air-operated wheel brakes, supplemented by magnetic track brakes (battery operated) for emergencies, in recent designs the wheel brakes have been abandoned to eliminate noise therefrom and to reduce both weight and maintenance. The rheostatic brake has been extended so as to be effective at speeds as low as 2 m.p.h., and the final stopping and parking is effected by a drum-type friction brake on each armature or propellor shaft, the brake shoes being applied by a spring and released by a solenoid.*

Bogie Truck with Propellor Shaft and Spiral-bevel Drive. Fig. 174 shows a British truck incorporating the above features. The motors are supported by inverted cradles fixed to the side frames. The car body is supported on lubricated side-bearing pads fixed to a cross beam or bolster, to which is also fitted the spherical bearing for the king pin through which the tractive effort is transmitted to the underframe of the car body. The ends of the bolster are

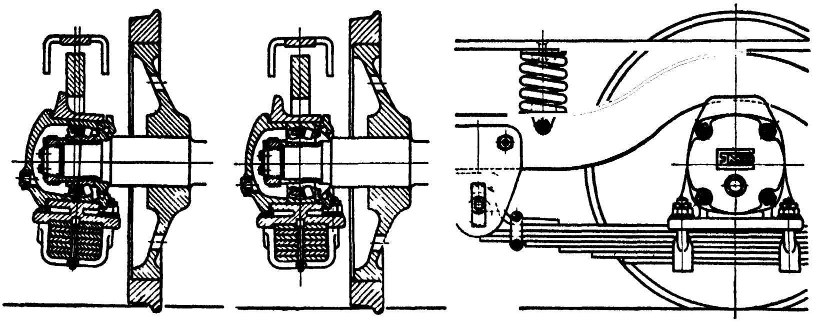


FIG. 173. ESSENTIAL FEATURES OF E.M.B. SWING-AXLE TRUCK

bolted to long semi-elliptic springs located just outside the side-frames of the truck. Bearers at the ends of these springs rest upon rubber springs or pads (eight in all) which are seated in brackets, *A*, attached to the side frames.

The truck frame is built of steel forgings and channels, and is supported by rubber pads (eight in all) carried on bearers, *B*, attached to the axle casings, *C*, the latter being maintained in their correct positions, relative to the truck frame, by links anchored to the side frames.

The wheels are of the resilient type and are described later. Separate drums are fitted for the non-ferrous brake shoes, which are operated by two air cylinders (located near the bolster) through a compensated system of levers, rods and beams.

Bogie Trucks with Cardan Shafts and Spur Gearing. Two examples, which are typical of Swiss practice, are shown in Figs. 175, 177. In each case the armature shaft is parallel to the axle and is hollow, the cardan shaft passing through it and being connected by a flexible coupling to a stub shaft carrying the pinion, which is mounted in ball or roller bearings fitted to the gear case, as shown in the sectional drawing of Fig. 178.

* For details see "Recent developments in track brakes and drum brakes for P.C.C. cars," by S. J. Vouch, *Trans. A.I.E.E.*, Vol. 66, p. 302. Also, "Some recent developments in the P.C.C. car," by S. B. Cooper, *ibid.* p. 288. For other papers see Vol. 70, pp. 222–231.

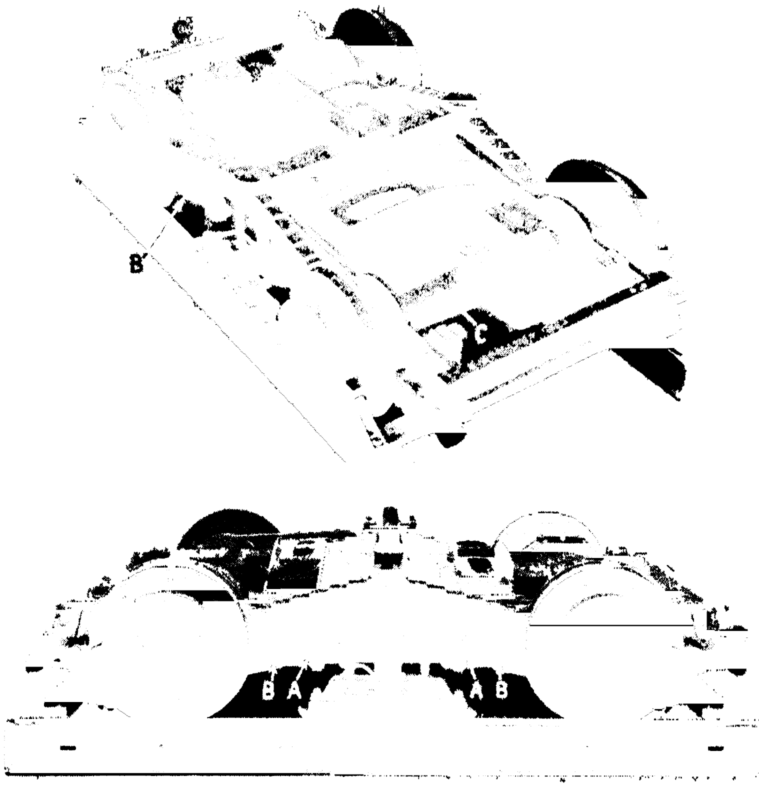


Fig. 174. Maley-Taunton Bogie Truck with Cardan shaft Drive and Magnetic Track Brakes

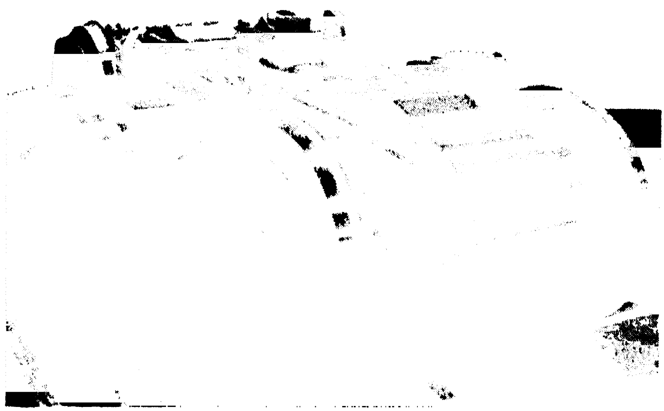


Fig. 175. Brown-Boveri "Simplex" Bogie Truck with Cardan-shaft Drive, Disc Couplings and Magnetic Track Brakes

A feature (originally due to Brown, Boveri & Co.) of both trucks is the absence of a conventional truck frame (i.e. side frames, transoms and head-stocks), The motor frames are rigidly connected to a central structure of welded steel plates and sections to form a main frame which is supported from either

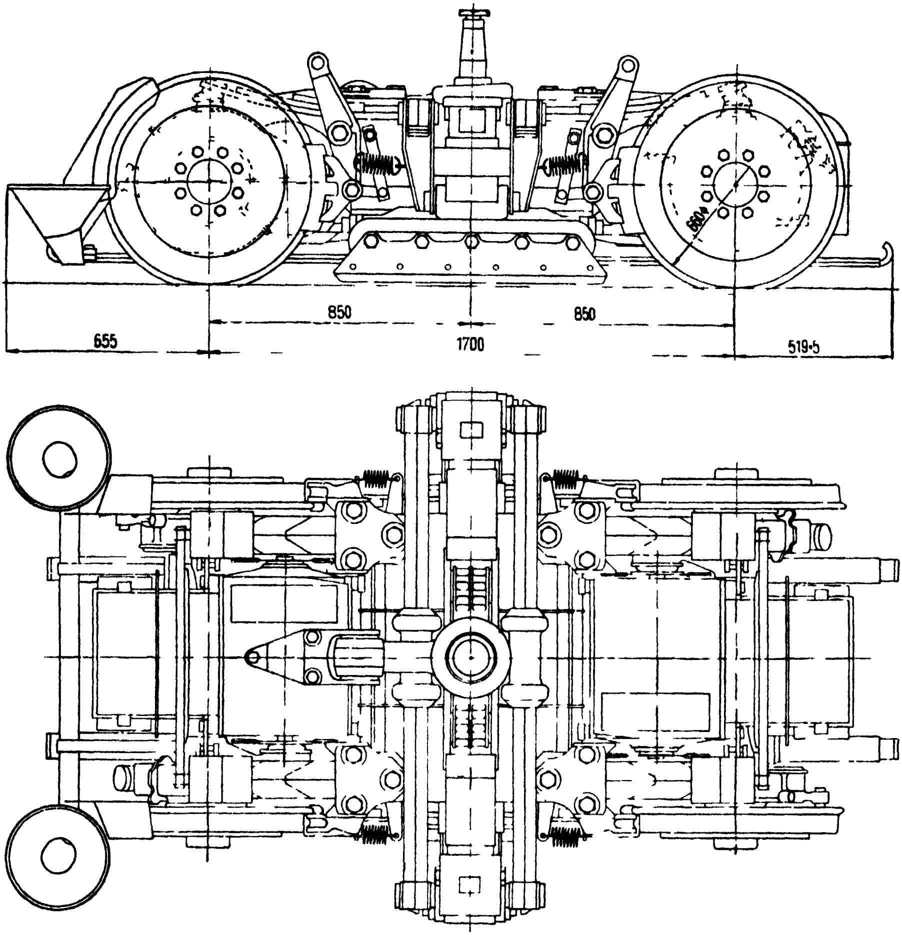


FIG. 176. ELEVATION AND PLAN VIEWS OF BROWN BOVERI 'SIMPLEX" BOGIE TRUCK

Dimensions in mm

the axle boxes (Fig. 178), or the axle casings (Fig. 176), by long semi-elliptic springs arranged longitudinally. The axle boxes, or axle casings, are maintained in their correct positions relative to the main frame by longitudinal and transverse links fitted with silentblocs.

In the truck shown in Fig. 175 the car body is supported on a bolster which is carried, by swinging links, from the ends of a pair of long transverse semi-elliptic springs fixed to the central frame.

Similar semi-elliptic springs are employed in the truck shown in Fig. 177,

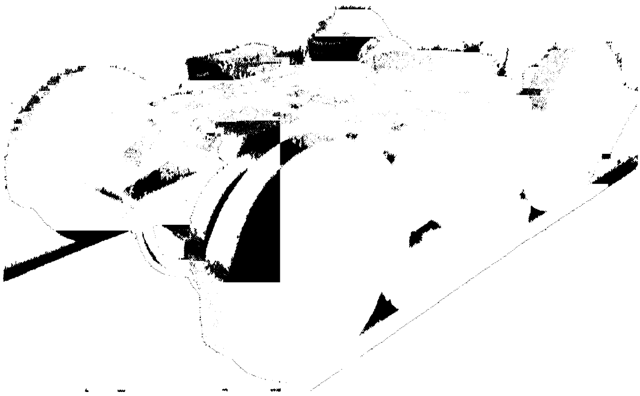


FIG. 177. BOGIE TRUCK WITH SÉCHERON CARDAN-SHAFT DRIVE AND MAGNETIC TRACK BRAKES

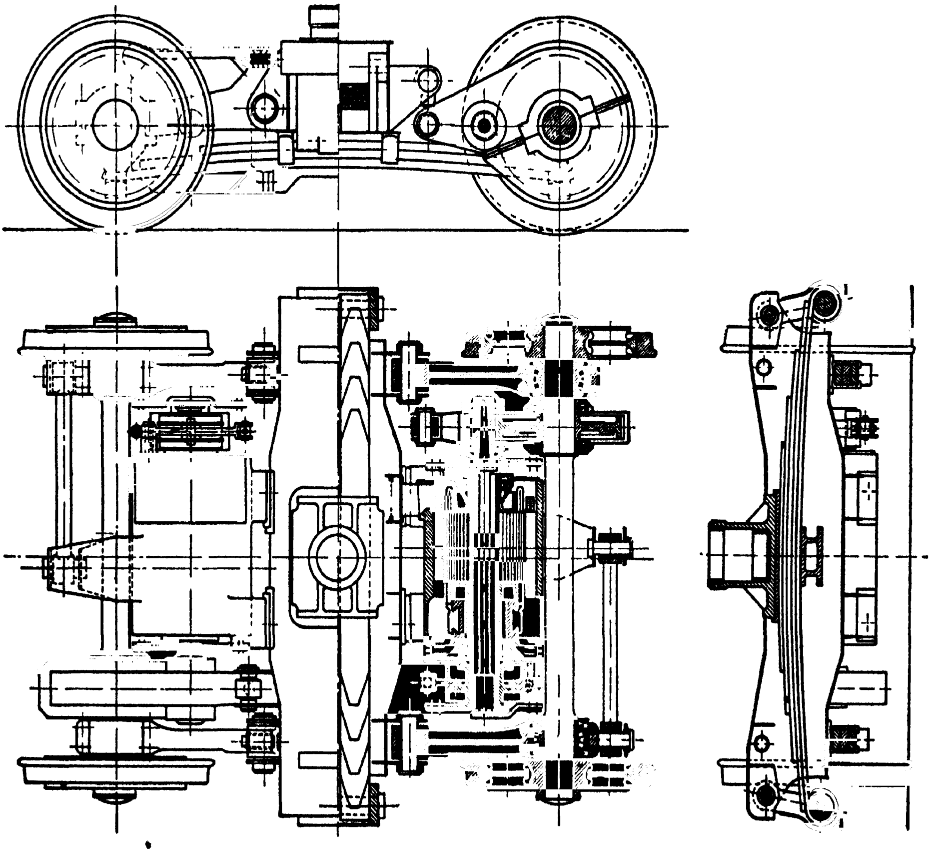


FIG. 178. ELEVATION AND SECTIONAL PLAN VIEWS OF BOGIE WITH SÉCHERON CARDAN-SHAFT DRIVE

but in this case there is no bolster and the car body is supported entirely by the centre pivotal bearing which is fixed to the springs, the ends of which are attached to swinging links pivoted to the central structure as shown in Fig. 178.

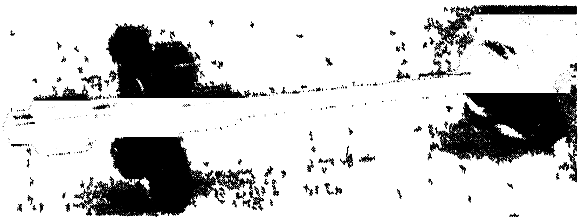


FIG. 179. SÉCHERON FLEXIBLE COUPLING

The flexible couplings employed in the truck shown in Fig. 175 consist of thin steel discs (described and illustrated in Chapter XVII), while those in the truck shown in Fig. 177 consist of links of spring steel fixed to cross arms (one

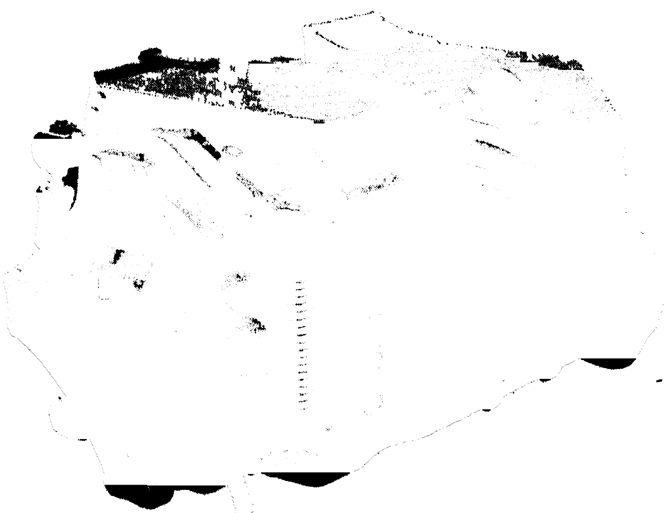


FIG. 180. MOTOR WITH DRUM BRAKE AND FLEXIBLE COUPLING (SÉCHERON)
A, *B*, Cross-arms connected to armature and cardan shaft, *C*, flexible links of spring steel.

of which is fitted to the driving shaft and the other to the driven shaft) as shown in Figs. 179, 180.

The internal cardan-shaft drive necessitates the mounting of the pinion on a stub shaft which must be carried in ball or roller bearings fitted to the gear case as shown in Fig. 178. The latter must therefore be of rigid construction and must be supported on roller bearings fitted to the axle or gear-wheel, with a link connecting the pinion end to the truck frame.

Both trucks have resilient wheels.

Mechanical braking is obtained by applying brake shoes in one case to the treads of the wheels, and in the other case to a drum fitted to the armature shaft as shown in Fig. 180.

Wheels. Both solid and resilient wheels are in use. The resilient type gives quieter running than the solid type due to the cushioning effect of the impact blows between wheel and rail.

Solid wheels have renewable tyres which are shrunk on to a steel centre and retained in position by either set screws or rings. The tyres are usually 2 in. to 2½ in. thick at the tread and have a life of about 60,000 miles, based on a radial wear of ⅛ in. per 5,000 miles.

Resilient wheels. One method of construction is shown in Fig. 181. The central or tyre portion, A, is sandwiched between rubber discs, B, about one inch thick, which are themselves sandwiched between the hub-plates, C, D. Metal dowels, E, bonded to the rubber, transmit the load from hub to tread, the rubber being loaded in shear. This arrangement gives the wheel a slight lateral flexibility. The normal wheel diameter is 27 in. and the maximum static load is approximately 7,000 lb. The life of the rubber discs is about three years. As the tyre is insulated from the axle flexible bonds are necessary between the tyre and the hub plates.

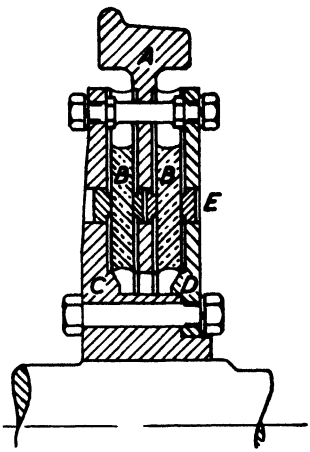


FIG. 181. CONSTRUCTION OF RESILIENT WHEEL

Service Brakes. Electric rheostatic braking (also called dynamic braking), in conjunction with mechanical braking, is employed for normal service stops, the mechanical brake being applied at low speeds when the electric brake becomes ineffective. In some cases the exciting coils of magnetic track brakes are connected in series with the loading rheostats so that a higher retardation is obtained for a given current. But in P.C.C. car equipments the magnetic track brakes are employed only in emergencies, when a retardation higher than normal is required, and the coils are excited from a battery.

Mechanical brakes are, of course, necessary for holding the car stationary. Hitherto mechanical brakes for this purpose consisted of brake shoes acting on the treads of the wheels, the application being made either by air pressure in an actuating cylinder or by hand-operated gear at the driving platform in conjunction with pull rods, levers and beams connected to the brake shoes.

Recent developments in *electro-mechanical drum brakes* (in which the brake drum is fitted to the motor shaft and the brake shoes are applied by springs and released by a solenoid excited from a battery) have made the hand-applied wheel brake redundant. Such electro-mechanical brakes are now standard on P.C.C.* and other cars in service in America and many other countries, and their use has recently been sanctioned by the Ministry of Transport for one tramway system in England.

* For details see *Trans. A.I.E.E.*, Vol. 66, p. 302.

Magnetic Track Brake. The electromagnet is of the bipolar type with elongated pole faces a short distance apart and longitudinal to the rail. The body is of cast steel and the renewable pole faces are of soft steel. The excitation is supplied by a single coil which is enclosed in a watertight metal case.

The magnetic circuit is transverse to the rail head, as shown in Fig. 182. Hence the vertical force between magnet and rail is, for a given flux density, proportional to the length of the pole faces in contact with the rail. A force of 2 tons can easily be obtained on British tramway rails with pole faces 24 in. long.

The magnet is suspended from the side-frame of the truck by spiral springs so that the pole faces normally clear the rail head by about $\frac{1}{4}$ in., and the thrust between magnet and truck is taken by a bracket on the latter engaging lugs on the side of the magnet (*see* Figs. 174, 177).

The *vertical force between magnet and rail* may be calculated from the fundamental equation for the tractive force of a magnet, i.e. force (dynes) $= B^2A/8\pi$,

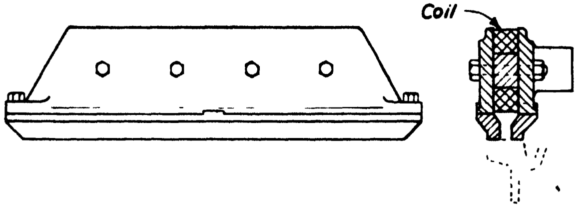


FIG. 182. SKETCH (ELEVATION AND CROSS-SECTION) OF MAGNETIC TRACK BRAKE

where B is the flux density (lines per cm²) at the pole face and A is the total contact area (in cm²) of the pole faces. Expressing the force (F) in kilogrammes we have, approximately, $F = 4B^2A \times 10^{-4}$. Whence the pressure between magnet and rail is approximately $4B^2 \times 10^{-4}$ kg per cm², or $57{\cdot}7B^2$ lb per in.² For $B = 15{,}500$ (which would correspond to a moderate saturation in the cast-steel body of the magnet) the pressure is 138 lb per in.² of pole face area. If, therefore, each pole face is 24 in. $\times$ $\frac{3}{4}$ in. the force at this flux density will be $24 \times 2 \times 0{\cdot}75 \times 138 = 4{,}970$ lb.

II. TROLLEYBUSES

General. The general design and arrangement of the body and chassis closely follows that of other mechanically propelled buses, e.g. the body (which may be of either the single-deck or double-deck type) is mounted on a chassis with either two or three axles and wheels with low-pressure pneumatic rubber tyres. Typical examples are shown in Figs. 183, 184. The front wheels are fitted with steering gear and the rear wheels are driven through differential gearing so that these wheels may rotate at different speeds when the vehicle is negotiating curves.

British Ministry of Transport Regulations. Trolleybuses for service in Great Britain must conform to Ministry of Transport regulations with respect to (i) laden weight, (ii) ground clearance, (iii) overall dimensions, length of loading platforms, overhang at rear.

The maximum laden weight of a two-axle vehicle is $12\frac{1}{2}$ tons, and that of a three-axle vehicle is $14\frac{1}{2}$ tons. The load on any axle must not exceed 8 tons.

A minimum ground clearance of 10 in. must be provided for a distance of 12 ft from the front of the vehicle and a width of not less than $\frac{2}{3}$ of the distance between the centre line of the front wheel tracks.

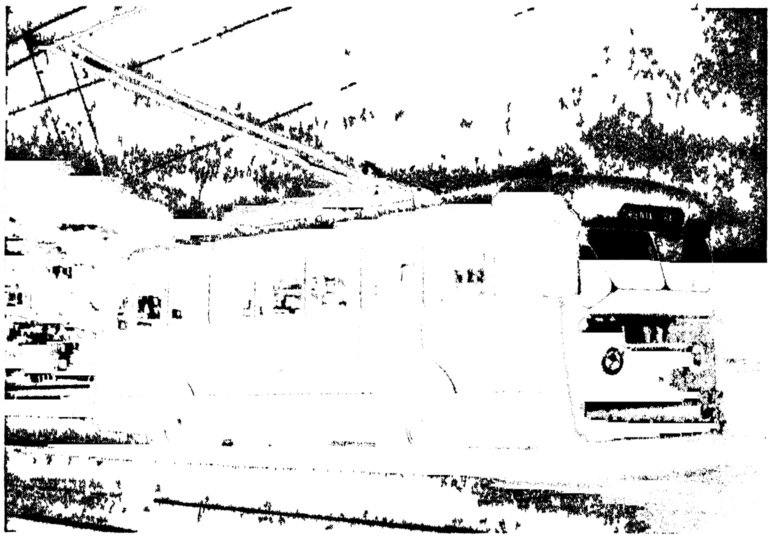


FIG. 183 BRITISH BUILT SINGLE DECK TROLLEYBUS FOR SERVICE OVERSEAS (METROPOLITAN VICKERS AND METROPOLITAN CAMMELL-WEYMAN)

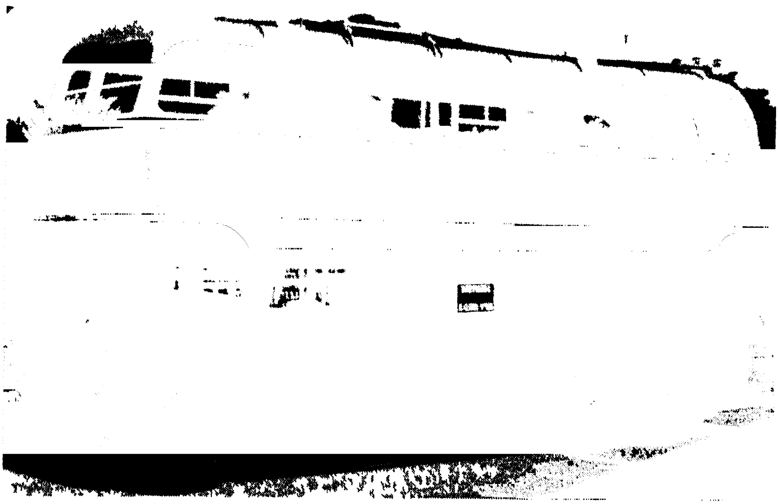


FIG 184 DOUBLE-DECK THREE AXLE TROLLEYBUS (METROPOLITAN-VICKERS AND METROPOLITAN-CAMMELL-WEYMAN)

The maximum overall dimensions are—width 8 ft, length (2 axles) 27 ft, (3 axles) 30 ft, height (unladen) to trolley standard 15 ft 10 in.

Chassis. The frame consists of main longitudinal members, of pressed steel, with cross bracings (usually tubular) and cast-steel brackets for carrying the

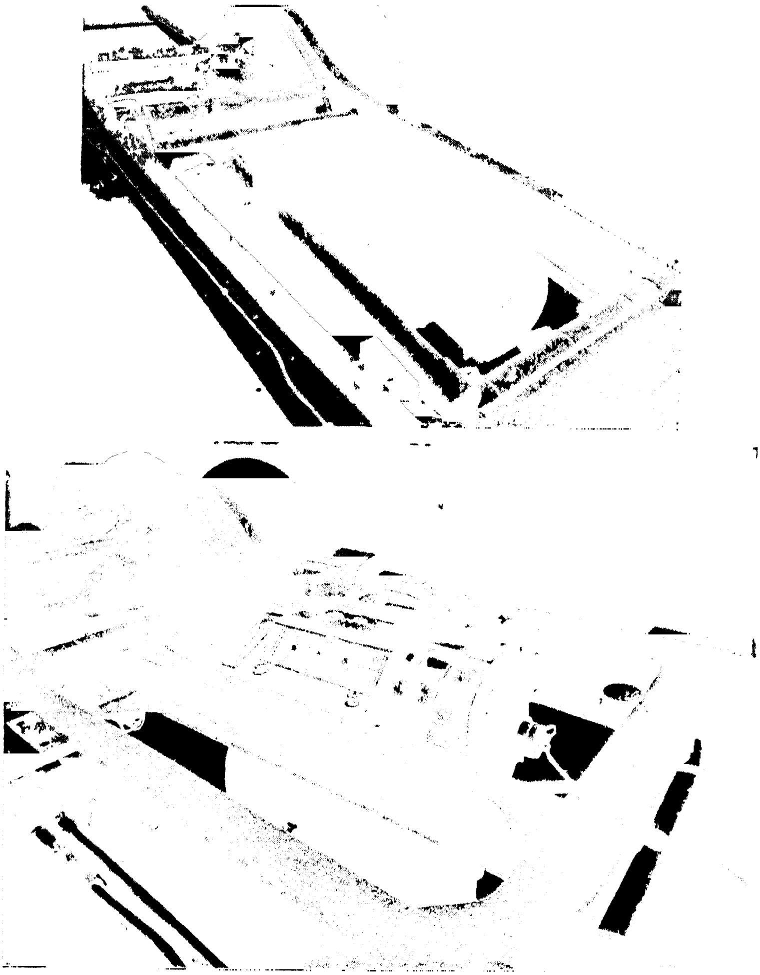


FIG. 185. METHODS OF MOUNTING DRIVING MOTOR ON TWO-AXLE AND THREE-AXLE TROLLEYBUSES (CROMPTON-PARKINSON AND G.E.C.)

[In both cases the motors are fitted with air filters, and in the upper view the motor is fitted with an overhung generator for lighting and battery charging.

springs, brake gear and other fittings. The springs at both front and rear are of the semi-elliptic type.

The motor is mounted between one of the main longitudinal members and an auxiliary member as shown in Fig. 185, and is located outside the restricted ground-clearance area so that a short horizontal propellor shaft may be employed. The speed reduction is by a worm and wormwheel, and the gear ratio is between $9\frac{1}{3}:1$ and $10\frac{1}{3}:1$.

The *front axle-beam* is a drop forging of nickel steel with forked ends which carry the swivelling stub-shafts on which the wheel hubs are mounted.

The *rear axle* is of the "full floating" type, i.e. the axle shafts are subjected only to torsion, the load being carried by a separate casing concentric with the shafts. The outer ends of these shafts are connected to the wheel hubs and the inner ends are splined to take the bevel wheels of the differential gear, the four pinions of which are mounted in a carrier fitted to the worm wheel.

With a *three-axle chassis* each driving axle is fitted with a differential gear and another differential gear (of smaller dimensions) is fitted between the two

FIG. 186. DRIVING CAB OF TROLLEYBUS (G.E.C.)

worm shafts to allow for differences in the effective diameters of the tyres. A "torque blade" of spring steel is fitted centrally between the axle casings to provide a reaction for the accelerating and braking torques and so relieve the springs of stresses due to these torques.

Wheels and Tyres. The wheels are of the disc type: they are spigoted to the hubs and secured by a number of studs and nuts. The low-pressure tyres are 20 in. by 10 in. (effective outside radius 19¼ in.) for the smaller (two-axle) vehicles and 20 in. by 10½ in. (effective outside radius, 20½ in.) for the larger (three-axle) vehicles.

Mechanical Brakes. Statutory regulations require each vehicle to be equipped with mechanical brakes which may be applied by a foot pedal or a hand lever. The brakes are of the expanding type with internal friction-lined shoes acting on steel or cast-iron-alloy drums fitted to the wheel hubs.

The foot pedal controls the electric (rheostatic) brake and the mechanical brakes acting on all wheels, the brake shoes being applied by individual air cylinders. The air supply to these cylinders is controlled by a valve actuated by the pedal, and the pressure in the cylinders is proportional to the depression of the pedal. The air is stored in a reservoir which is maintained charged (at a pressure of 85 lb per in.^{2}) by a small motor-driven compressor.

The hand lever actuates the brake shoes on the rear wheels independently of the air cylinders, this brake being intended for holding the vehicle stationary.

Location of Equipment. Vehicles for service in the United Kingdom are predominantly of the double-deck type with a bulkhead separating the driver

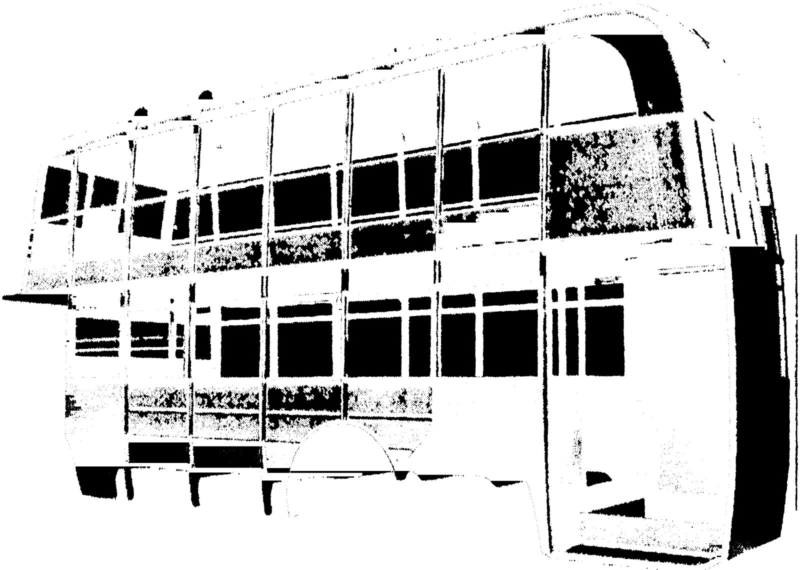


FIG. 187. BODY STRUCTURE OF THREE-AXLE DOUBLE-DECK TROLLEY BUS (METROPOLITAN-CAMMELL WEYMAN)

from the passenger compartment. Adequate space is therefore available in the cab for the control equipment. On some older vehicles, however, this equipment is mounted on one of the side-frames of the chassis, which position, or alternatively at the rear of the chassis, is necessary for single-deck vehicles. The starting rheostats are mounted between the side-frames, as in Fig. 185, and the battery, when provided, is mounted forward of the rheostats. The motor-generator, when provided, is mounted on one of the side-frames alongside the air compressor and air reservoir. The circuit breakers and other switches, instruments, etc., are located in the cab, as shown in Fig. 186.

Body. All-metal bodies have now superseded composite bodies. Lightness, together with great mechanical strength and durability, are obtained by the use of high-tensile steel for the framing, sheet steel for the inner panelling, sheet aluminium for the outer panels, and aluminium or light alloys for the

mouldings and other fittings. The body is constructed as an integral unit and is bolted to the chassis frame.

The framing consists of longitudinal, vertical and cross members of tubular, angle or channel section connected by gusset brackets. The inner panelling is of 20 S.W.G. sheet steel and is riveted to the framing thereby adding to the strength of the structure. The outer panels are screwed to timber fillets in the pillars and rails, this construction enabling any panel to be easily replaced if damaged. Fig. 187 shows the complete body structure ready for the external panelling, a portion of which is in place.

With the concealed type of trolley-base gantry (which gives reduced overall height compared with the exposed type) the roof structure is reinforced as shown in Fig. 188. Each trolley base is mounted on a block of rubber which is moulded to a steel plate bolted to the gantry. A rubber covered plate, with

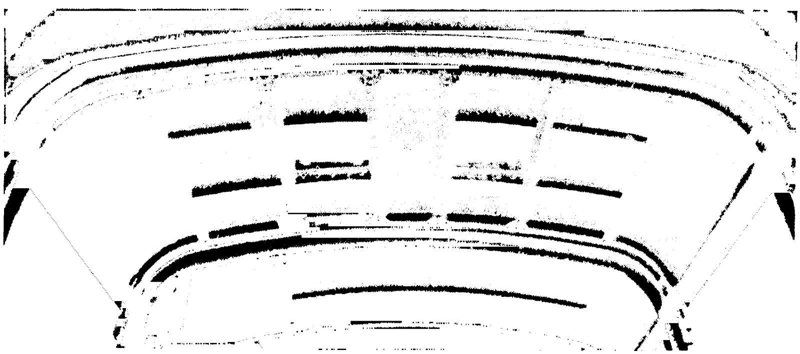


FIG. 188. REINFORCEMENT OF ROOF STRUCTURE (METROPOLITAN-CAMMELL-WEYMAN)

the front and rear edges bent upwards, is interposed between each trolley base and the rubber mounting (*see* Fig. 159) to divert rain water and prevent the formation of a leakage path between base and roof.

III. COMMERCIAL BATTERY ELECTRIC VEHICLES

These vehicles, in the form of light delivery vans, are employed for the delivery of parcels, food stuffs, etc., in large towns, where the aggregate daily run is of the order of 30 to 40 miles. Their advantages for this service over vehicles driven by internal combustion engines are low maintenance, long life of motor and control gear, absence of fumes. Data of vehicles are given in Table IV.

The first cost of the vehicle (without battery) is, with the present limited production, about 25 per cent less than that of the corresponding mass-produced petrol-driven vehicle. The cost of the battery is an appreciable item and may be 25 to 30 per cent of that of the vehicle (without battery). Replating of the battery is necessary after approximately four years of normal service. But this cost, together with that of the electrical energy required for recharging the battery, should be regarded as a "running" or fuel cost, which if averaged over the life of the battery will be actually lower than the cost of fuel for the petrol-driven vehicle.*

* Useful information is given in a paper by H. W. Heyman on "Economic Basis of Battery Electric Road Vehicle Operation and Manufacture," *Proc. I.E.E.*, Vol. 99, Pt. II, p. 457.

TABLE IV

DATA OF COMMERCIAL BATTERY ROAD VEHICLES*

Rated load (payload), cwt	14	20	40
Gross laden weight, cwt	42	60	90
Licensing weight, cwt	18	22	29
Weight of battery, cwt	10	16½	20
Overall length†	11′ 9½″	12′ 1″	15′ 10½″
Overall width†	5′ 2½″	5′ 11½″	6′ 7½″
Overall height†	6′ 9½″	6′ 11″	8′ 0″
Wheel base	6′ 3″	6′ 10″	9′ 6″
Tyres (low pressure)‡	17″ × 5¼″	15″ × 6″	13″ × 6″
Motor	1–10¼ h.p.	1–12¼ h.p.	1–12¼ h.p.
Gear ratio	12 : 1	12 : 1	15 : 1
Battery, No. of cells	30	36	36
Battery, Ah capacity	174	226	315
Speed on level with rated load, m.p.h.	16 to 18	18 to 20	14 to 16
Average range on full charge, miles	35 to 40	25 to 35	35 to 40

* Smith's Electric Vehicles.
† Standard delivery van body.
‡ The first figure denotes diameter of wheel rim to which the tyre is fitted. The second figure denotes the diameter of the cross-section of the inflated tyre cover.

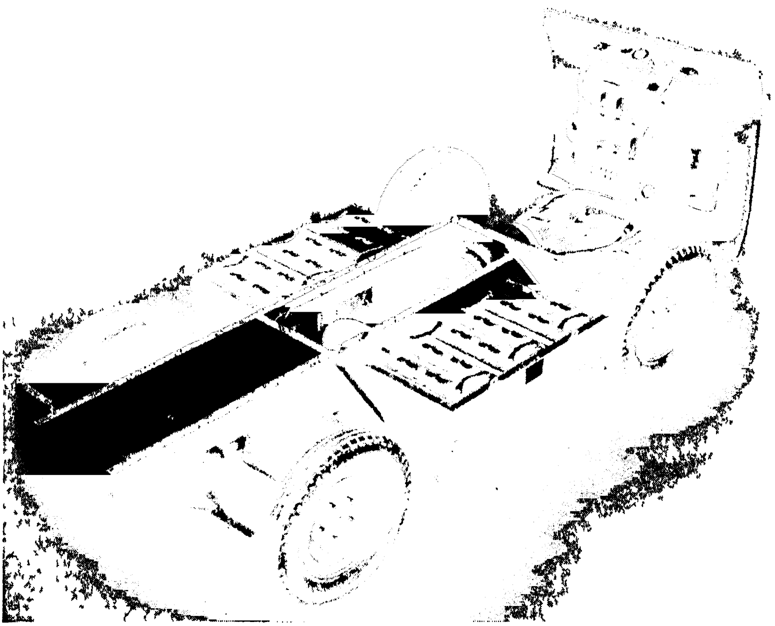


FIG. 189. CHASSIS OF BATTERY VEHICLE WITH FULL EQUIPMENT (SMITH'S ELECTRIC VEHICLES AND B.T.-H.)

Chassis and Equipment. The general arrangement of the chassis, axles, transmission and brakes follows that of a petrol-driven vehicle. The battery is carried in the manner shown in Fig. 189, the crates being easily removable (sideways) through openings in the side walls of the body.

The motor is mounted between the side-frames of the chassis just forward of the battery, and the drive to the back axle is by a propellor shaft and double-reduction gearing, helical spur gearing being employed for the first reduction and a spiral bevel gear for the final reduction. The overall gear ratio is between 12 : 1 and 15 : 1.

The contactor control gear (*see* Chapter VIII) is mounted on a panel in the cab, as shown in Fig. 99, and the starting rheostats are mounted below the floor of the cab.

Battery. This is usually of the lead-acid type, the number of cells being either 30 or 36. The capacity depends on the vehicle and may be between 175 and 315 amp-hours at the 5-hour rate.

Charging. When a fleet of vehicles operates from one depot charging may be effected from a central plant at the depot. In other cases each vehicle may be equipped with a charger of the metal-rectifier type and a relay to control the charging current. In both cases interlocking contacts are provided, as shown in Fig. 101, to open the control circuit of the contactors while charging.

Body. The main frame and underframe are constructed of hardwood, the valances are of sheet aluminium and the mudguards are of rubber. The cab is enclosed, and for a parcel delivery vehicle the side walls of the body are of aluminium-faced plywood.

For further information on battery road vehicles and their equipment see *Proc. I.E.E.* Vol. 99, Pt. II, p. 457.

CHAPTER XVI

ROLLING STOCK FOR ELECTRIC RAILWAYS

MOTOR-COACH TRAINS

Advantages of Motor-coach Trains. The traffic requirements of urban and suburban railways involve the running of frequent trains at relatively high schedule speeds (for this class of service), together with facilities for (i) varying the composition of the trains to suit variations in traffic density; (ii) getting the trains into and out of the terminal stations in the minimum time and with a minimum number of signal and train movements. These conditions can best be satisfied by the use of motor-coach trains, which possess a number of advantages over locomotive-hauled trains for urban and suburban service. Thus—

1. Since the passenger coaches are equipped with motors, the whole, or a large portion, of the train weight is available for adhesion, and a large number of driving axles is available. The loading on the permanent way due to a train is more uniform than in the case of a locomotive-hauled train (which would necessitate the use of a very heavy locomotive).

2. The trains can be made up to suit variations in traffic density with a minimum of shunting operations.* If the train weight per motor is maintained constant, the trains for light and heavy traffic will be able to maintain the same schedule speed with the same specific energy consumption.

3. The capacity of a terminus (expressed in terms of passengers per hour) is greater with motor-coach trains owing to fewer signal and train movements being necessary, and the absence of docks or platform sidings which would have to be provided for locomotives.

Adhesive Weight. In trains made up wholly of motor coaches and all axles equipped with motors, the adhesive weight is equal to the total train weight. On many railways in this country, however, it is the practice to operate one motor coach with one, two, or three trailer coaches as a *train unit*, and to run the service with trains made up of one unit, or of two or more units coupled together, as demanded by the traffic. In these cases the adhesive weight is equal to the total load on the axles which are equipped with motors. It should be at least 25 per cent of the total train weight (generally it is from 33 per cent to 50 per cent), and, to avoid slipping of the driving wheels under unfavourable conditions of weather, the total accelerating tractive effort should not exceed about 15 per cent of the adhesive weight. The adhesive weight and total train weight of some typical motor coach trains are given in Table VI. Other particulars of the motor coaches are given in Table V.

* A striking example of the rapidity with which electric trains can be made up at a terminus is given by J. Shaw in a paper entitled "The Equipment and Working Results of the Mersey Railway under Steam and Electric Traction" (*Min. of Proc. I.C.E.* Vol. 179, p. 19).

In the paper it is stated that the trains are allowed three minutes at the termini, during which interval any alteration in the making-up of the train has to be done. The average time for making up a train is two minutes, which includes shunting and coupling of brakes and electric connexions.

TABLE V

DATA OF D.C. MOTOR-COACHES -URBAN AND SUBURBAN LINES, LONDON TRANSPORT AND BRITISH RAILWAYS—4 FT 8½ IN. GAUGE

	I	II	III	IV	V	VI
Type of coach	Saloon (Al-alloy body)	Saloon	Saloon	Saloon	Saloon	Saloon
Length of body	51 ft 1 in.	52 ft 4 in.	51 ft 1 in.	58 ft	62 ft 6 in.	60 ft
Width of body	8 ft 11 in.	8 ft 8 in.	8 ft 11 in.	8 ft 3 in.	9 ft 0 in.	9 ft 3 in.
Number of seats	40	42	40	58	82	52
Distance between bogie centres	35 ft	33 ft 6 in.	35 ft	41 ft	44 ft	41 ft 3 in.
Wheel base	7 ft 10 in.	6 ft 3 in.	7 ft 10 in.	7 ft 6 in.	8 ft 9 in.*	8 ft 6 in.
Diameter of wheels	36 in.	31 in.	36 in.	36 in.	42 in.	43 in.
Number of motors per coach	2	2	2	4	2	4
Number of motors per bogie	1	1	1	2	2	2
Rated h.p. of each motor	125	168	152	135	180	210
Gear ratio	74/17	59/18	65/16	63/16	65/17	71/17
Weight of coach in service order (tons)	28	27·5	35·7	36	43	50·9
System of control	Electro-pneumatic cam-shaft		Metadyne	Electro-pneumatic		contactor

* Motor truck; wheel base of trailer truck 8 ft.

I, II, III, London Transport Executive (I, District Line; II, Tubes (Northern line); III, Metropolitan-District line); IV, London Midland Region, Wirral lines; V, Southern Region; VI, Eastern Region (Liverpool St.–Shenfield, 1,500 V).

NOTES. All motor-coaches have a driving compartment at one end only. All except V have air-operated sliding doors; V has passenger-operated hinged doors (8 each side). Motor-coaches IV, V, VI have a luggage compartment adjacent to the driving compartment.

COACHES

Types. Three types are in use, namely (i) the compartment type with side doors, (ii) the compartment side-corridor type, and (iii) the saloon type with end and intermediate doors. Saloon stock is the only type permissible for deep-level underground (or tube) railways. Although there exists a considerable difference of opinion among railway engineers as to the advantages of saloon and compartment stock, nevertheless, the saloon type, by virtue of its better facilities for the distribution of the passengers when entering the train, is more suitable for urban service with dense traffic than the compartment type.

Limitations to Dimensions. The maximum length and width of a coach which can be used on a given line is determined by the layout of the track (which affects the clearance between passing trains), the loading gauge, and the size of the tunnels. When sliding or inwardly opening doors are adopted—as in the saloon type of coach—the clearance between passing trains can be made smaller than that when outwardly opening side doors are used.

The length of trailer coaches for express service on the principal steam railways in this country varies from 50 ft to 75 ft while the width varies from 8 ft to 9 ft 3 in.

The largest electric coaches in this country (i.e. those in service on the Liverpool-Southport section of the London Midland Region have a length of

TABLE VI

Adhesive Weights of Typical Motor Coach Trains on British Railways

Region	Class of Stock	Weight of one Motor coach	Number of Motors per Motor-coach	Weight of one Trailer-coach or non-driving motor-coach	Number of Fixed Seats for Passengers: Motor-coach	Number of Fixed Seats for Passengers: Trailer-coach	Composition of Train or Train-unit	Total Weight of Train: Without Passengers	Total Weight of Train: With Full Number of Passengers	Adhesive Weight of Train: Without Passengers	Adhesive Weight of Train: With Full Number of Passengers	Ratio—Adhesive Weight to Total Weight: Without Passengers	Ratio—Adhesive Weight to Total Weight: With Full Number of Passengers
		Tons		Tons				Tons	Tons	Tons	Tons		
L.T.E. (District lines).	Saloon (steel)	34·2	2	33 3†	40	40	2M, 4N	201·6	216·6	110	119	0·55	0·55
L.T.E. (District lines).	Saloon (alloy)[1]	29	2	28 1†	40	40	2M, 4N	170·4	185·4	95·7	104	0·55	0·55
L.T.E. (Tubes) .	Saloon	27·5	2	20 7	42	40	4M, 1N, 2T	178	196	75	82·3	0·421	0·42
Southern .	Saloon[2]	43	2	27 5	82	102, 120	2M, 2T	141	165	56·4	61·9	0·4	0·375
Southern .	Express[3]	59	4	36 9‡	52	204§	2M, 4T	265 5	284·7	118	124·5	0·445	0·438
Southern .	Pullman[4]	62·4	4	41·7‡	48	96§	2M. 3T	250·5	262·5	124·8	129·8	0·5	0·494
London Midland .	Compartment[5]	56	4	28, 30*	84	100, 96*	1M, 1T, 1T*	114	131	56	61·25	0·491	0·467
London Midland .	Saloon[6]	54	4	28, 30*	48	55, 60*	1M, 1T, 1T*	112	123	54	57	0·452	0·463
London Midland .	Saloon[7]	36	4	20, 21*	58	55, 68*	1M, 1T, 1T*	77	88·3	36	39·63	0·467	0·45
Eastern .	Saloon[8]	50 9	4	26·4, 27·5*	52	64, 60*	1M, 1T, 1T*	104 8	115·8	50·9	54·15	0·486	0·468

NOTES—M, motor-coach with one driving compartment; N, motor-coach without driving compartment; T, trailer coach; * Trailer coach with driving compartment. † Non-driving motor-coach. ‡ Average values for trailer-coaches. § Total number of seats in trailer coaches.

[1] Body framing, control-gear cases and plating of Al-Mg-Si alloy (Kynal M39 2). [2] London suburban stock with side doors; motor-coaches and one trailer coach are of saloon type (8 and 10 doors, respectively, on each side): one trailer coach is of compartment type (10 compartments, each with 12 seats). [3] Motor-coaches of saloon type, 3 trailer-coaches of compartment side-corridor type (1st and 3rd class), one trailer coach of Pullman saloon type with kitchen. [4] All coaches of Pullman saloon type with restaurant service. [5,6] London suburban stock. [7] Wirral lines, one trailer coach 1st and 3rd class (15 and 40 seats). [8] Liverpool St.–Shenfield lines. All London suburban coaches have accommodation for one class (3rd) only. Luggage compartments are provided in motor-coaches except those on L.T.E. lines and tubes.

60 ft with a width of 10 ft. They are of the end-door saloon type, with transverse seats, a central corridor (2 ft wide), and seating accommodation for 100 passengers. Each of the transverse seats on one side of the corridor is arranged to accommodate three passengers, while on the other side of the corridor two passengers are accommodated on each seat.

A similar seating arrangement is adopted, with a slightly narrower corridor, on the steel saloon stock on some of the suburban lines of the Southern Region, British Railways, but in this case the overall width is 9 ft 0 in. This stock has side doors (10 on each side of a coach, i.e. one for each group of seats), and the seating capacity is 102 for a coach 62 ft over the body.

Construction. In the design of rolling stock for urban and suburban railways it is important to reduce the weight by the use of suitable materials, as unnecessary weight not only increases the energy consumption but also leads to increased maintenance costs when the whole of the equipment is considered. Of course, if the stock is built too lightly it will not be sufficiently strong to withstand the stresses due to high acceleration and braking, and in this case the maintenance costs will be high. But the use of aluminium and steel of high tensile strength will enable the weight to be reduced without sacrificing strength. With saloon coaches constructed of steel sections and sheets the sides of the body may be combined with the sole-bars of the underframe, thereby allowing the latter members to be reduced in section.*

The design of the *underframe for a motor coach* will be influenced by the type of coach, the method of control, and the disposition and weight of the control apparatus, auxiliary apparatus, etc. In multiple-unit trains the control and auxiliary apparatus may be located either in a compartment at one end of the coach or under the coach between the bogies. As the total weight of the control apparatus for a coach equipped with four d.c. motors may be of the order of 3¼ tons, the disposition of this apparatus must be carefully considered in the design of the underframe.

In the majority of cases the underframe is constructed of steel sections. The principal longitudinal members are of channel section, and are connected together by cross-bars and end frames (the latter being called *head stocks*) to which the buffing and draw-gear is attached. The outer longitudinal members (called *sole-bars*) are fitted with adjustable truss-rods,† which carry the tension component of the stress produced by the bending moment, while the compression component of the stress is carried by the channels. The centre pins, centre bearings, and side-bearing plates are fitted to cross channels, which, for a motor coach, have to be specially reinforced and braced to the sole-bars and head stocks. In some types of underframes these channels are replaced by steel castings (called *body bolsters*). When the control apparatus is located in the

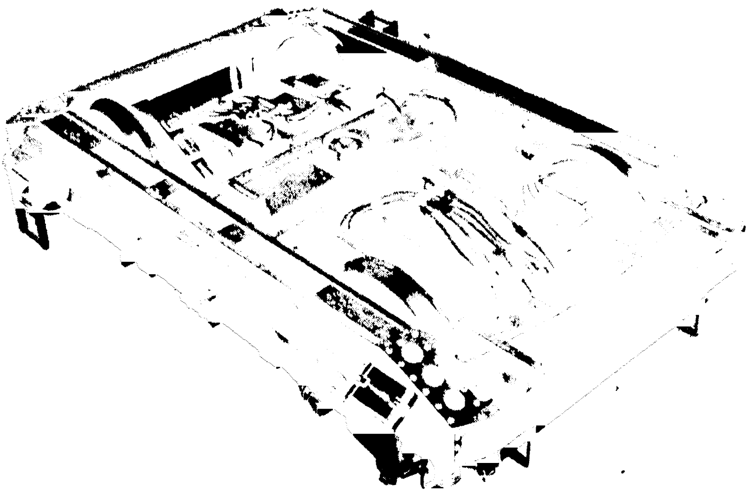
* See "The Design of Rolling Stock for Electric Railways," by H. E. O'Brien, *Journ. I.E.E.*, Vol. 52, p. 455; "The Mechanical Design of Modern Light-weight Motor-and trailer-coaches," by F. W. Sinclair and S. C. Lyon, *Journ. I.E.E.*, Vol. 97, Pt. IA, p. 205.

The results of energy consumption tests carried out on the District Line, London Transport, with steel and aluminium-alloy coaches are given in *Engineer*, Vol. 195, p. 630, and show that for this service (i.e. half-mile runs at a schedule speed of 18 m.p.h.) the energy consumption of a train with aluminium-alloy coaches is about 13 per cent lower than that of an equivalent train with steel coaches. The savings are estimated to exceed the extra capital cost of the coaches.

† The truss-rods are adjusted to give a slight upward deflection, or camber, to the sole-bars when the coach body is unloaded and in position on the trucks. Truss-rods are not required on steel cars in which the sides and sole-bars are designed to form plate-girders.

coach body, the sole-bars and longitudinal members are braced by diagonal bracings, but these have to be omitted when the control apparatus is fixed to the underframe. The cross-bars must then be supplemented by gusset plates.

(a)



(b)

FIG. 190. TYPES OF BRITISH NON-EQUALIZED MOTOR-TRUCKS WITH MOTORS IN POSITION

(a) District Railway (London Transport Executive) bogie, 4 ft 8½ in gauge, with one driving motor and individual brake cylinders.
(b) Estoril Railway bogie (British built, 5 ft 6 in. gauge, with two driving motors (G.E.C. equipment)).

The above types of underframes are adopted for rolling stock running on suburban railways, when the dimensions do not have to conform to a restricted loading gauge. In the case of *tube railways*, however, the loading gauge is much below that of surface railways on account of the limited diameter of the tunnel. For example, in the majority of the London tube railways the internal diameter of the tunnel (for straight single track) is 11 ft 8¼ in., while the height

from the track rails to the top of the tunnel is 9 ft 11½ in. The rolling stock on these lines is of the saloon type, and the underframe is incorporated with the body.

Trucks

The trucks in use on the passenger rolling stock of railways are usually of the four-wheel bogie type with a central bolster, although in some cases a six-wheel bogie is adopted. The trucks for motor-coaches are, in general, of similar design to those for trailer coaches but are of heavier construction to withstand the greater stresses to which they are subjected.* The wheel base is from 6 ft to 10 ft.

Motor-trucks can be divided broadly into two classes, according to the spring system adopted between the truck frame and the axles. Thus (i) the truck

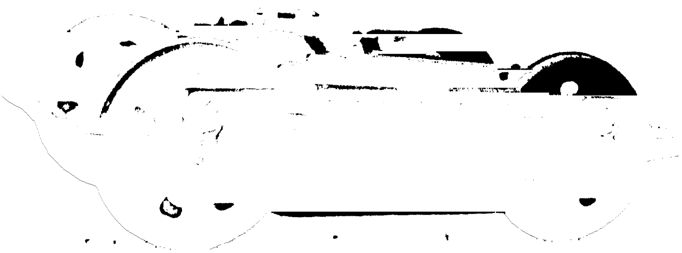


Fig. 191. Brill "27 M.C.B." Equalized Motor-truck

frame may be supported on the axle boxes through laminated springs (Fig. 190), or (ii) the truck frame may be supported on spiral springs carried on equalizing bars, the ends of which are supported directly on the boxes (Fig. 191). In each type of truck the bolster is supported on springs carried by the spring plank, which may be of either the swinging or the rigid type, the former being the more general for railways operating at moderate speeds.

Trucks of the first class (which may be called *non-equalized trucks*) are practically standard for all British railways, while trucks of the second class (which are called *equalized trucks*) are largely used in America and are only used to a limited extent in this country. This type of truck has superior riding qualities to the non-equalized type on poor track, but with the excellent track construction on our larger railways the riding qualities of the non-equalized truck are quite satisfactory. Moreover, the equalized truck is not only heavier and more costly to maintain than a non-equalized truck of equal wheel base, but it is subjected to a tilting action during braking—the forward end of truck being depressed and the rear end raised—this action being greatest when outside-hung brake shoes are used.

In view of the large forces to which a motor-truck is subjected (the truck having to perform the work of a tractor in addition to carrying and guiding the car body), the frame must be very rigid and must be sufficiently braced to

* Much useful information is given in a paper by W. S. Graff-Baker on "Considerations on Bogie Design, with Particular Reference to Electric Railways", *Proc. I.M.E.*, Vol. 162, p. 217.

maintain its "squareness" under all conditions. As the only means of bracing the side frames is by the transoms and the end frames, these members must be liberally designed, and must be reinforced by gusset plates at the connexions to the side frames.

Wheels and Axles. The standard wheel diameter for rolling stock on steam railways in this country is approximately 43 in., and wheels of this diameter have been adopted on most of the suburban electrifications in this country, as the trailer coaches of the electric stock can be coupled to standard steam stock. With underground and other railways having a restricted loading gauge, wheels of smaller diameter (31 to 36 in.) must be employed.

The diameter of the axle at the motor bearings varies from about 6 in. to 7½ in., according to the size of the motor, while the diameter at the journals varies from 4½ in. to 6 in., this dimension being influenced by the weight of the coach.

Brake Rigging

It is the general practice on steam railways to fit two brake shoes to each wheel for all passenger rolling stock. With motor-trucks having a wheel base of from 6 ft to 7 ft there is considerable difficulty in finding room for the operating gear, since the position of the motors prevents the use of brake beams for the inner set of brake shoes. Consequently many motor-trucks of this wheel base are provided with only one brake shoe to each wheel. These difficulties may be overcome by the use of individual brake cylinders, as in the trucks shown in Figs. 190 (*a*), 197. The cylinders are of relatively small diameter (3½ to 4½ in.) and can, therefore, be easily accommodated in the truck frame. Moreover, such an arrangement eliminates a lot of levers, rods, etc., which are necessary when a single brake cylinder is employed.

When the wheel base of the truck is of the order of 8 ft to 10 ft, however, it is generally possible to provide two brake shoes to each wheel.

The use of outside-hung brake shoes enables brake beams to be adopted with a convenient arrangement of levers, but these advantages result in a tilting of the truck* when the brakes are applied, the front end of the truck being depressed and the rear end raised. This tilting action is more pronounced in trucks of the equalized type, due to the short spring-base and the method of mounting the truck frame on equalizer bars. When trucks of this type (with outside-hung brake shoes) are braked to give a high retardation, the tilting action compresses the front springs (on the equalizer bars) and removes a portion of the load from the rear springs. Thus when the car comes to rest a reaction is produced which results in a sudden backward jerk of the car body.

Brake Rigging for Inside-hung Brake Shoes. Fig. 192A shows a typical arrangement. As brake beams cannot be employed, the brake levers must be pivoted to the brake shoes and the force must be transmitted in line with brake shoes. In the present case divided brake rods *C* (Fig. 192A) are employed in order to clear the wheels, and are attached, at one end, to a radius beam *B*, and at the other end to the brake levers *D*. The latter are pivoted to the brake

* The tilting action with outside and inside brake shoes is discussed fully in a paper by R. A. Parke, on "Railroad Car Braking," *Trans. A.I.E.E.*, Vol. 20, p. 235. See also paper by H. R. Broadbent on "Weight Transfer in Multiple-unit Electric Stock," *Railway Gazette*, Vol. 92, p. 69.

shoes, and the lower ends are connected to the brake levers on the outer wheels by the equalizing rods E.

The brake shoes of the inner wheels are suspended, by links, G, from brackets fixed to the side frames of the truck. The links G also support the weight of the brake levers D and a portion of the weight of the brake rods and equalizing rods.

The brake levers D' (for the outer wheels) are pivoted to their respective shoes, and the upper end of each lever is provided with a fulcrum F, formed by

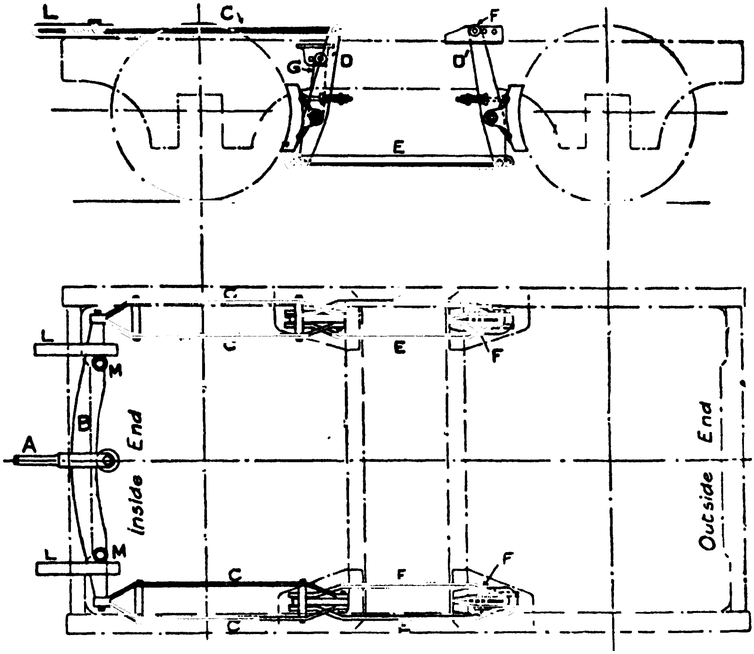


FIG. 192A. DIAGRAM OF BRAKE-RIGGING FOR INSIDE-HUNG BRAKE SHOES

angle brackets fixed to the side frames. This fulcrum is adjustable in order to allow for the wear of the brake shoes, and the effective length of the equalizing rods is adjustable for the same reason.

The radius beam B is usually supported by guides L attached to the inner head stock, and the transverse movement of the beam is limited by the rollers M engaging with these guides.

When a tension is applied to the pull rod A, the brake shoes of the inner wheels come into operation first, and the thrust is then transmitted through the equalizing rods to the brake shoes on the outer wheels.

Brake Rigging for Two Brake Shoes per Wheel. Fig. 192B shows the arrangement for a motor-truck. The outside brake shoes for both wheels are pivoted to the brake levers D, which are suspended from brackets H on the head stocks of the trucks. The brake levers D_1 of the inside shoes of the outer wheels are suspended in a similar manner from brackets F, fixed to the transom, but for the inner wheels of the trucks the inside brake shoes (and the brake levers D_2

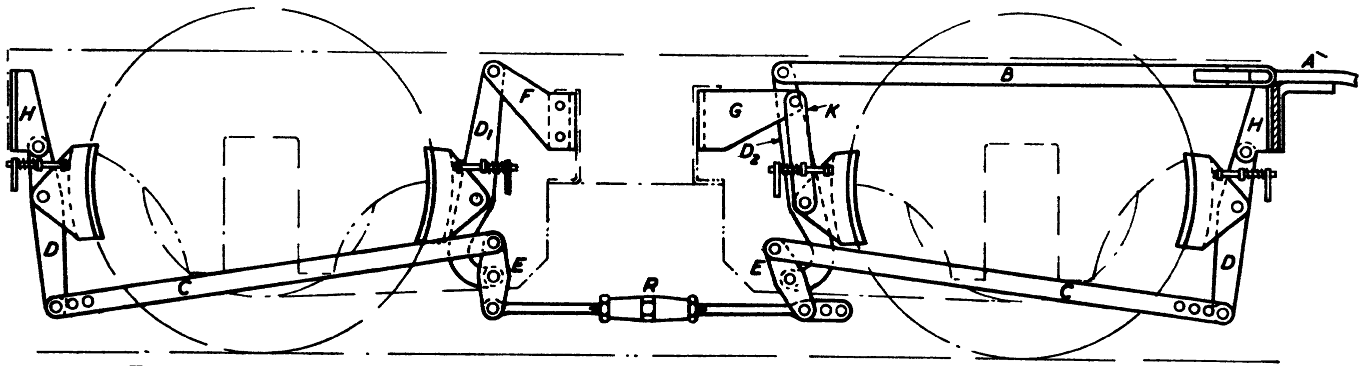


FIG. 192B. DIAGRAM OF BRAKE-RIGGING FOR TWO BRAKE SHOES (WITHOUT BRAKE-BEAMS) TO EACH WHEEL

connected to them) are suspended, from brackets G fixed to the opposite transom, by the links K. The inside brake levers D_1, D_2 are of a special "L" shape, and their lower ends are connected to the lower ends of the outside brake levers D through the equalizing levers E and the brake rods C, the other (lower) ends of the equalizing levers being connected together by the adjustable rod R. The upper ends of the inside brake levers D_2 (for the inner wheels of the truck)

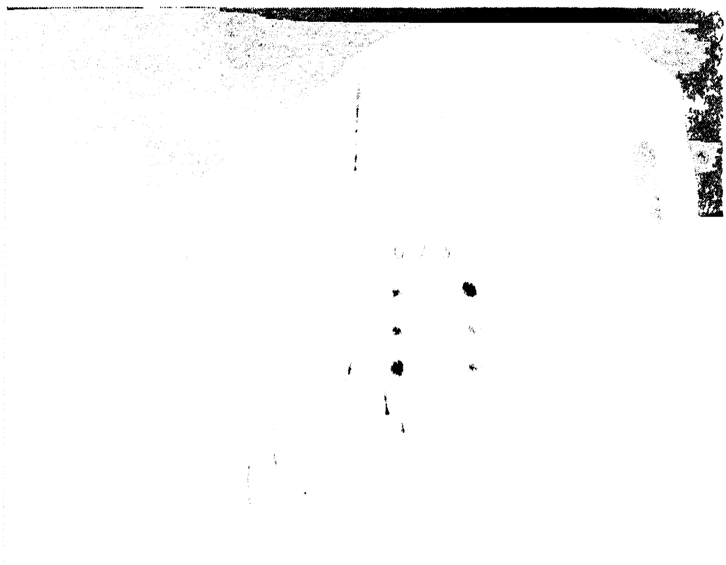


FIG. 193. STEEL MOTOR-COACH OF THE DISTRICT RAILWAY (LONDON TRANSPORT EXECUTIVE)

are connected to a radius beam by dividing brake rods B, and the radius beam is operated by the pull rod A as explained above.

When the brakes are applied the inside shoes of the inner wheels come into operation first, and provide the brake levers D_2 with fulcrums. The thrust is then transmitted to the other brake shoes by means of the equalizing levers and brake rods.

EXAMPLES OF MOTOR-COACHES*

Motor-coaches for Low-voltage, d.c. Railways. A *District Railway* (London Transport Executive)† motor-coach attached to a train is shown in Fig. 193. The coach is of the saloon type, with sliding doors (pneumatically operated and controlled by the guard of the train).‡

In the latest stock aluminium and light alloys have largely replaced steel in the construction of the body and the frames of the control gear.

The present practice on this rolling stock, and that on the tube railways,

* Dimensions and data are given in Table VI.

† Some details of the London Underground railway system are given in a paper, by T. S. Pick and R. Dell on "Review of British Underground Railway Practice," *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 27.

‡ For details see paper by R. J. D. Arthurton on "Power-operated Doors on Multiple-unit Electric Railway Stock," *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 246.

is to fit only one motor to each bogie of a motor-coach, and to employ electro-pneumatic cam-shaft control gear slung from the underframe. Each bogie is also equipped with positive and negative collector shoes, all shoes of like polarity on a train being connected in parallel by "bus-line" cables.

An interior view of a coach is shown in Fig. 194. Fluorescent lighting is installed the lamps (2 ft, 20 W) being arranged along the ceiling in two rows, each having 12 lamps, as shown at A. Secondary lighting, supplied by a 50 V,

FIG. 194. INTERIOR OF DISTRICT RAILWAY COACH (LONDON TRANSPORT EXECUTIVE)

57-Ah battery, takes the form of two filament lamps enclosed in a fitting, B, at the end of each coach.

The fluorescent lamps are supplied with alternating current at 110 V, 850 c/s, which is obtained from a special winding* in the pole faces of the 4-kW, 50-V, d.c. generator supplying the control and auxiliary circuits. The lamps are operated in parallel and are started by means of a resonant circuit in series with each lamp.

As the successful operation of the lamps depends on the maintenance of the correct frequency the speed of the motor driving the generator must be controlled within narrow limits notwithstanding the large variations of voltage (e.g. 550 to 700 V) at the terminals of this machine under certain operating conditions (regenerative braking). This close control of the speed is effected by means of an automatic regulator of the vibrating-contact type in conjunction with a separately excited field winding on the motor (in addition to the series starting and stabilizing winding) and flat-compounding of the generator when operating at a constant speed.

A typical motor-car in service on the London tube railways is shown in

* See "Control Gear and Auxiliary Machines for d.c. Multiple-unit Electric Trains" by E. T. Hippisley and F. E. Butler, *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 244.

Fig. 195, and one of the bogies is shown in Fig. 196. The car is of the saloon type with sliding doors pneumatically operated and controlled by the guard of the train. On account of the low overall height and the necesssity for a level

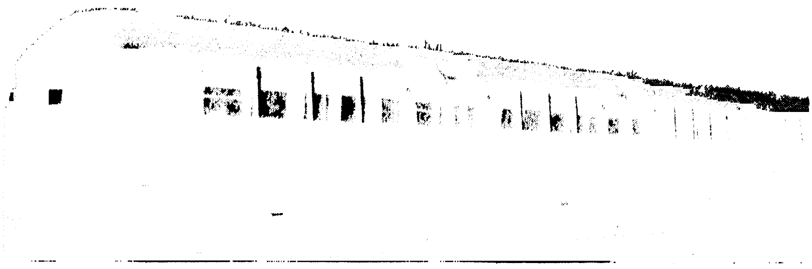


FIG 195 MOTOR-CAR OF THE LONDON TUBE RAILWAYS (LONDON TRANSPORT EXECUTIVE)

floor throughout the train the bogie is designed with a low overall height and the wheels are 31 in. in diameter. In consequence only a single motor, of 168 h.p., can be accommodated on each bogie, and to obtain the maximum

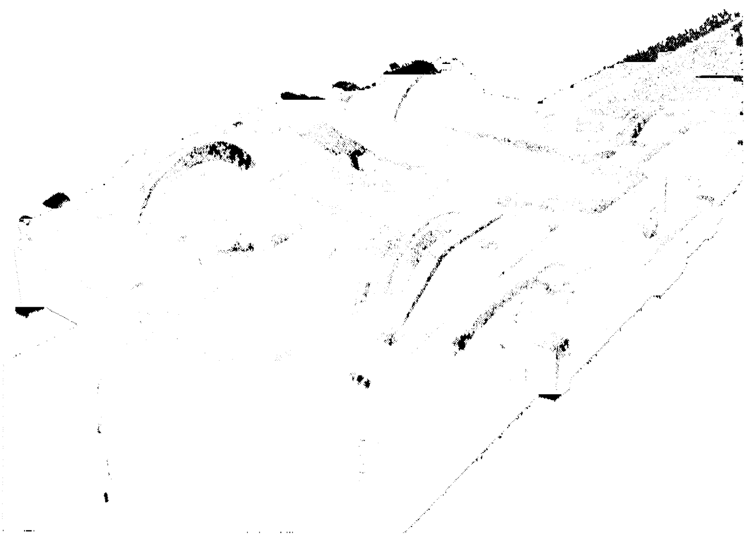


FIG. 196 LOW-HEIGHT MOTOR TRUCK OF LONDON TUBE RAILWAYS (LONDON TRANSPORT EXECUTIVE) WITH MOTOR AND COLLECTOR SHOES IN POSITION

(Crompton-Parkinson Equipment)

weight for adhesion the pivotal centre is displaced, from the centre line of the truck, towards the non-driving axle. Each bogie is equipped with positive and negative collector shoes, those of like polarity being interconnected on each car, but not throughout the train, as a "bus-line" is not permitted.

The motor-coaches in service on the extensive 660-V system of the Southern *Region are of several types* (e.g. compartment, saloon, corridor, Pullman)*. Originally, all motor-coaches were of the compartment type, with a luggage compartment adjacent to a spacious driving cab, and the control gear, of the electromagnetic contactor type, was housed in an intermediate compartment. Subsequently, with the extension of the electrification to main lines, a large number of additional motor-coaches were acquired, all of which were equipped with electro-pneumatic control gear slung from the underframe. This type of control gear and mounting is also installed on rebuilt stock. The control

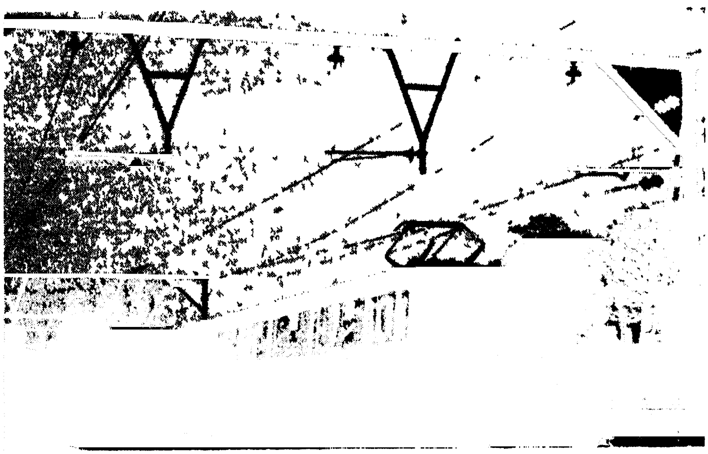


FIG. 197 TRAIN OF THREE "TRAIN UNITS" ON THE LIVERPOOL STREET SHENFIELD LINE
(English-Electric Equipment)

circuit is supplied at about 70 volts from a potentiometer-type rheostat connected across the traction circuit.

A four-coach train-unit (two motor coaches and two trailers is standard for suburban services, each motor-coach being equipped with two motors (on one bogie) of ratings between 180 h p. (ventilated, new stock) and 275 h.p. (non-ventilated, old stock).

The non-stop expresses on the London-Brighton service consist of six-coach units (two motor-coaches and four trailers), each motor-coach being equipped with four 225 h.p. non-ventilated motors. These, and other main-line express trains have motor generator sets for supplying the low-voltage lighting, control, and other auxiliary circuits in the kitchen and buffet cars.

Motor-coaches for High-voltage (1,500-V) d.c. Railways. The motor-coaches on the Liverpool Street (London)–Shenfield lines are representative of modern design and equipment.† They are of the saloon type, with driving and luggage compartments at one end, and two pairs of doors, pneumatically

* Details of this electrification are given in a paper, by C. M. Cock, on "Electric Traction on the Southern Railway," *Proc. I.E.E.*, Vol. 95, Pt. II, p. 115.

† Details of this electrification are given in a paper, by H. H. Swift, on "The Electrification of the Liverpool Street–Shenfield Lines, Eastern Region, British Railways," *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 42

operated, in the passenger compartment. Fig. 197 shows a train consisting of three "train units" each comprising one motor-coach and two trailers.

Each bogie is equipped with two 210-h.p., 750-V motors, connected

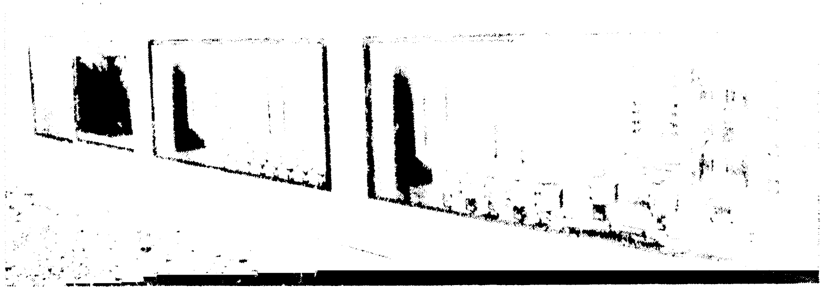


FIG. 198. ENGLISH-ELECTRIC 1,500-V CONTROL EQUIPMENT ON MOTOR COACH

The compartments (left to right) contain —line contactors and fuses, rheostatic contactors, field-shunting contactors, motor cut-out switch and reverser.

permanently in series, which are self-ventilated, the air being drawn through ducts from openings near the coach roof.

The control gear is of the electro-pneumatic individual-contactor type and is fitted into cases slung from the underframe of the coach, as shown in Fig. 198.

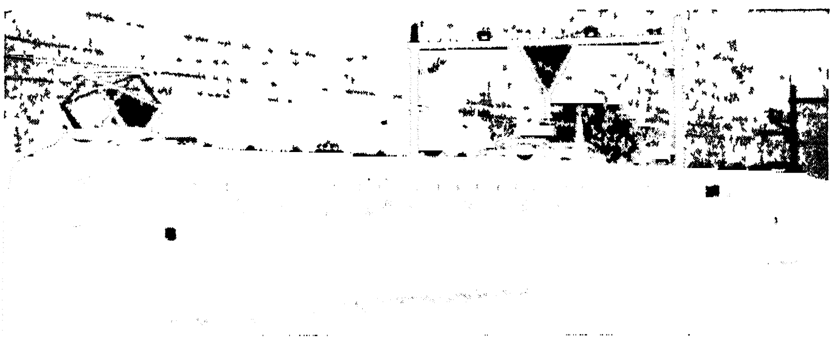


FIG. 199. TRAIN UNIT FOR MAIN-LINE SERVICE ON NEDERLANDS RAILWAYS

A motor-coach train-unit (four coaches) for main-line service on the *Nederlands Railways* is shown in Fig. 199. The coaches are of the saloon type with communicating passages between the end and intermediate coaches. The bodies of the two coaches at each end of the train are close-coupled, and the ends are carried on a single bogie (not equipped with motors) so that only six bogies are necessary for the four coach bodies. Such an arrangement has given satisfactory results in practice at speeds up to 125 km.p.h. (78 m.p.h.).

The motor equipment consists of eight 270-h.p., 750-V motors (two permanently in series). The control equipment is of the individual-contactor electro-pneumatic type and is slung from the underframe.*

* Details of the Nederlands railways electrification are given in a paper by H. J. VAN Lessen on "Electric and Diesel-electric traction on the Nederlands Railways," *Proc. I.E.E.*, Vol. 97, Pt. I, p. 63.

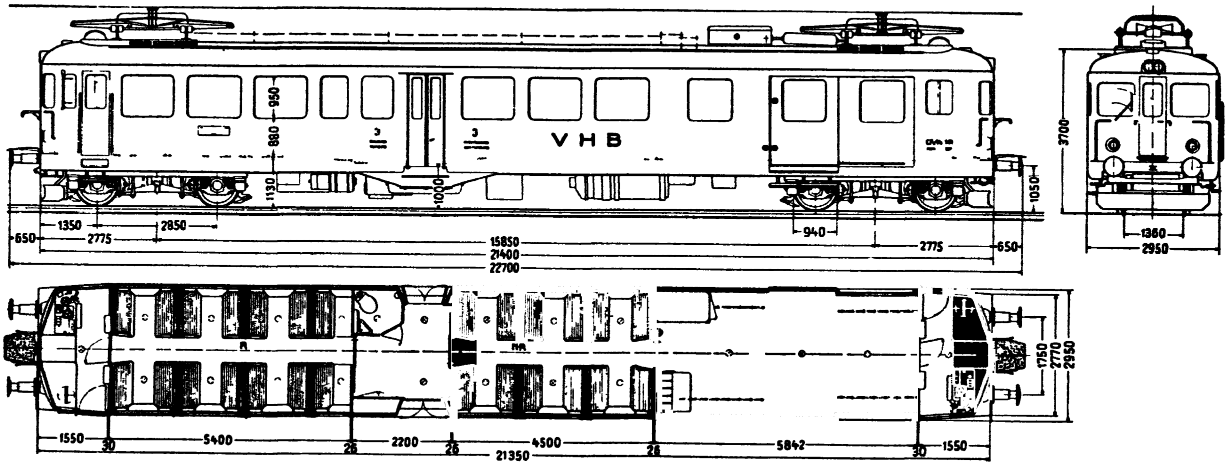


FIG. 200. OUTLINE OF 1,200-H.P. SINGLE-PHASE MOTOR-COACH, UNITED HUTTWILL RAILWAYS, SWITZERLAND (OERLIKON)

The control and lighting circuits are supplied at 52 volts from a 6·1-kW generator operating in parallel with a 125-Ah alkaline storage battery. Both the generator and the 1,500-V driving motor have shunt and series windings, the shunt winding of the generator being self-excited and that of the motor being separately excited from the generator terminals. The series windings of both machines are connected in the motor circuit, the (light) series winding on the generator being for the purpose of ensuring excitation of correct polarity. The voltage of the generator is controlled automatically to a constant value by a carbon-pile regulator.

Each generator supplies the lighting circuits of a train-unit, i.e. three coaches, the number of 60-W lamps totalling 78.

Train heating, approximately 8 kW per coach, is supplied directly from the traction circuit.

Motor-coaches for Single-phase Railways. Fig. 200 shows the general design and principal dimensions of a modern motor-coach for a 15-kV, $16\frac{2}{3}$ c/s system. The electrical equipment consists of four frame-mounted motors, having an aggregate 1-hour rating of 1,200 h.p., and is designed to allow trailer coaches aggregating 300 tons to be hauled on the level, or 120 tons on a 2·5 per cent gradient at a speed of 34·8 m.p.h. Provision is made for regenerative braking using the exciter-motor connexion, Fig. 154, one machine supplying excitation to the remaining three machines which are all connected in series. For motor operation the two motors on each bogie are connected in series, and the two pairs are operated in parallel, tap-changing being effected by a group of 18 contactors according to the scheme of Fig. 131.

The transformer, Fig. 201, is oil-cooled with forced oil circulation and air-blast cooling for the tubular oil-cooler. It is constructed with low overall height for mounting below the underframe of the coach, together with the regenerative braking equipment, Fig. 202, and other auxiliary equipment (e.g. compressor, motor-generator, battery). Nine tappings, for voltage steps from 175 V to 865 V, are provided for starting and speed control, two—at 820 V and 1,050 V—for train heating (200 kW) and one at 220 V for the auxiliary services. The contactor group (18 contactors) is mounted in the luggage compartment.

The electrical equipment of motor-coaches, with 50-c/s series motors, for a 50-c/s system is in general similar to that of those for a low-frequency system except that (i) provision is made for shunting the main field winding at starting and at low speeds (up to about 6 m.p.h.) to reduce the circulating currents at the brushes, (ii) forced cooling is necessary for the non-inductive shunts for the commutating-pole windings, (iii) the auxiliary motors are of the induction (capacitor) type.

On account of the steeper speed/torque characteristic of the 50-c/s motor, fewer tappings are necessary on the transformer (e.g. 7 instead of 9 for an equipment of about 1,600 h.p.).

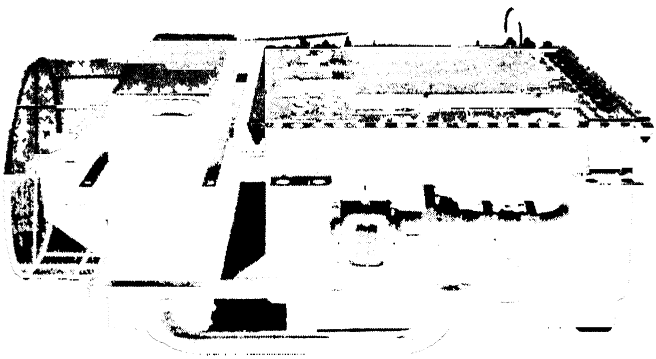
The weight of the electrical equipment for a motor-coach with four motors of an aggregate output of 1,600 h.p. (1 hour) is about 18 to 20 tons, which is approximately the same as that for a motor-coach with $16\frac{2}{3}$-c/s motors of the same output.

Brakes

The brakes on railway passenger rolling stock are always operated by power, since it would be impossible to obtain sufficient braking force with hand-operated

brakes. *Hand brakes, however*, are fitted to motor-coaches and guards' compartments for operation by the motorman or guard in cases of emergency.

Two *types of power brakes* have been developed, the compressed air (or Westinghouse) brake, and the vacuum brake. The former brake is largely used on electric railways, and is also extensively adopted in America on steam



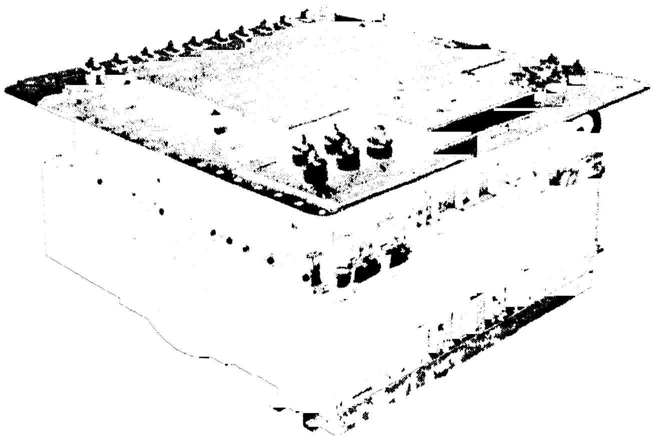


FIG. 201. OIL-COOLED TRANSFORMER FOR UNDERFRAME MOUNTING (OERLIKON)
Upper view—Transformer complete with circulating pump and air-blast oil cooler
Lower view—Transformer removed from tank showing the core and the three centre-tapped reactors (preventive coils) employed with the tap-changing scheme of Fig 131

railways. The vacuum brake is standard on all our large steam railways, and has been adopted on some electrified lines (e.g. the Lancashire sections of the London Midland Region and other railways abroad).

For electric railways the compressed-air brake possesses some advantages over the vacuum brake, as compressed air can be stored so that a quick release of the brakes can be obtained; whereas, with the vacuum brake, the vacuum must be created by means of a pump or exhauster. This disadvantage, however, can be overcome by the use of either vacuum reservoirs and equalizing valves, or a two-speed exhauster.

The Vacuum Brake. In its simplest form this brake consists of a vertical cylinder (called the brake cylinder) fitted with a piston and piston rod, the latter operating the brake rigging through suitable levers. A vacuum is maintained continuously on the top of the piston, while air—at atmospheric pressure—can be admitted to, or exhausted from, the underside of the piston. Under normal conditions (i.e. brakes off) a vacuum is maintained on both sides of the piston, and the latter rests against the lower cylinder cover. When an application of the brakes is required, the vacuum is broken on the underside of the piston and the latter is forced upwards, thereby applying the brakes. The

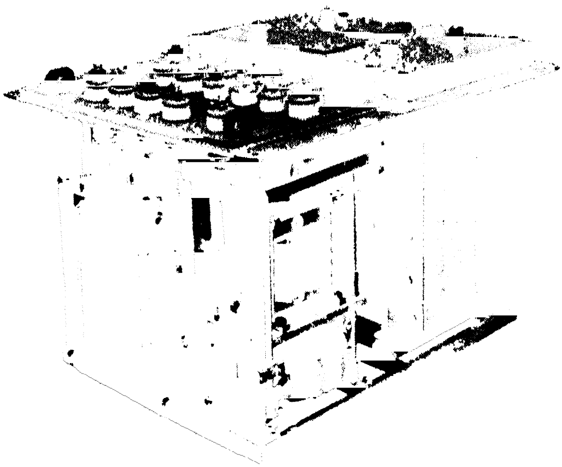


Fig. 202. Regenerative Equipment Removed from Case (Oerlikon)
This consists of an auxiliary transformer and reactors (*D* and *E*, Fig 153)

brakes are released either by re-creating the vacuum, or by equalizing the pressure on each side of the piston.

In practice each coach is equipped with one or two brake cylinders (according to the nature of the brake rigging on the bogies) which are connected to the "train pipe," as shown in Fig. 203. The latter is continuous throughout the train, and is connected to the operating (or driver's) valve on the locomotive or motor-coach. On steam trains this valve is a combination of an air valve and two steam ejectors, one large and one small. Under normal conditions the small ejector maintains the vacuum in the train pipe, while the large ejector is only operated for releasing the brakes. On electric trains these ejectors are replaced by a motor-driven exhauster (or vacuum pump) which is run at two speeds—one being double the other—the higher speed being used only for releasing the brakes. With d.c. equipments the lower speed is obtained by inserting a rheostat in series with the motor, and the rheostat is cut out when the driver's valve is moved to the "off" or "release" position. With single-phase, a.c. equipments two or more operating speeds for the exhauster motor can readily be obtained from tappings on the transformer.

One type of vertical *brake cylinder* (manufactured by the Vacuum Brake Co.) is shown in Fig. 203. The cylinder *A* is combined with the vacuum chamber *B*, which is provided with trunnions for mounting in a vertical position under

the coach. The piston *C* is an easy fit in the cylinder, and is provided with a rolling rubber ring *D*, while the piston rod *E* is provided with a packing gland in the lower cylinder cover. The sides of the piston near the top are provided with three small holes and ball valves (one of which is shown at *F*), by means of which communication can be established between the vacuum chamber and the underside of the piston when the pressure in the former exceeds that in the lower portion of the cylinder. This portion of the cylinder may be connected directly to the train pipe, or the connexion may be made through an

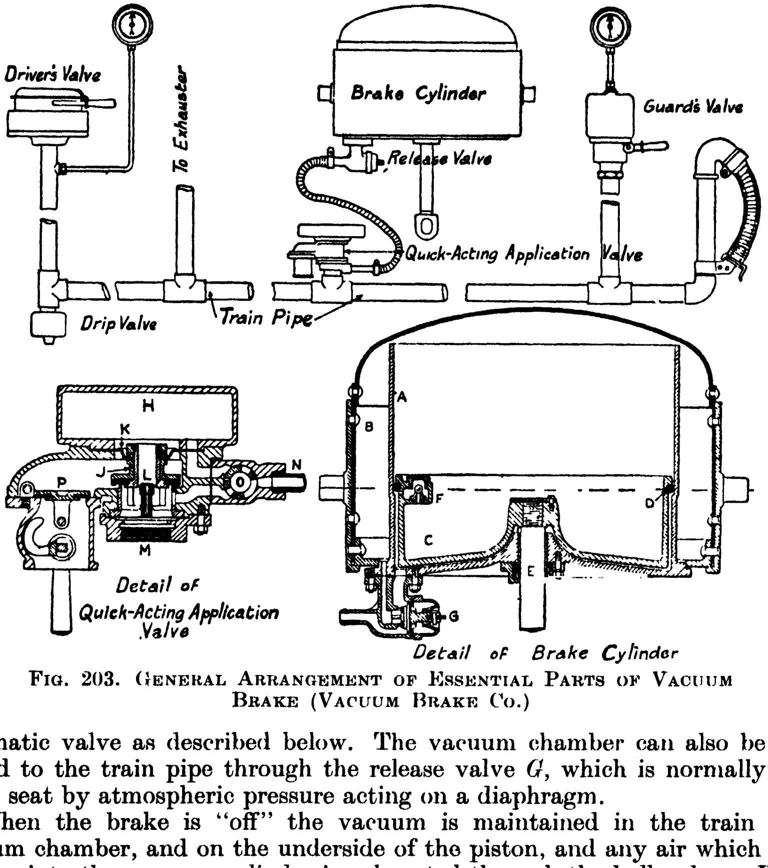


FIG. 203. GENERAL ARRANGEMENT OF ESSENTIAL PARTS OF VACUUM BRAKE (VACUUM BRAKE CO.)

automatic valve as described below. The vacuum chamber can also be connected to the train pipe through the release valve *G*, which is normally held on its seat by atmospheric pressure acting on a diaphragm.

When the brake is "off" the vacuum is maintained in the train pipe, vacuum chamber, and on the underside of the piston, and any air which finds its way into the vacuum cylinder is exhausted through the ball valves. When an application of the brakes is required, the train pipe is opened to the atmosphere, and air is admitted to the underside of the piston, which is moved upwards. The force with which the brake shoes are applied to the wheels depends on the rapidity with which the vacuum is destroyed, while the brakes may be partially released by partially restoring the vacuum.

The air in the train pipe is prevented from reaching the vacuum chamber (and the top of the piston) by the ball valves and the packing ring. The function of the release valve is to allow the brakes to be released on a coach which is disconnected from the locomotive. This is done by lifting the valve from its

seat, which equalizes the pressure on both sides of the piston, thereby allowing the latter to return to the bottom of the cylinder.

When a *quick-acting brake* is required on a long train, the brake cylinders are not connected directly to the train pipe but to auxiliary valves which are connected to the train pipe and to the atmosphere. A cross-section of one type of auxiliary valve (called a "quick-acting application valve") is shown in Fig. 203. A rubber-seated valve *J* is provided with a diaphragm *K*, over which is placed a small air chamber *H*. This air chamber communicates with the train pipe through a small annular space formed by the fixed stem *L* and a central hole in the body of the valve *J*. The underside of the diaphragm and the top of the valve *J* can be placed in communication with the atmosphere by means of the hinged clap valve *P*. Under normal conditions valve *J* is maintained on its seat by the excess of pressure on the upper part of the valve, the underside of the latter and the top of the diaphragm being connected to the train pipe (at *M*). When an application of the brake is required, the vacuum in the train pipe is destroyed by operating the driver's valve. This unseats the valves *J* and *P*, and air is admitted through the latter to the train pipe and brake cylinder, thereby producing a rapid application of the brakes throughout the train.

The Compressed-air Brake. In its simplest form the compressed-air brake consists of a reservoir (in which compressed air is stored), a brake cylinder, an operating valve, and a train pipe. The brake cylinder is of the single-acting type, and is shown in Fig. 204.

The brakes are kept "off" by springs in the brake cylinders acting against the pistons, while they are applied by admitting compressed air, from the reservoir, to the back of the pistons, the force of the application depending upon the quantity of air admitted to the brake cylinders. The brakes are released by exhausting the air from the brake cylinders.

When this brake is used on trains consisting of several coaches, additional features are required in order to obtain quick action of the brake throughout the train. These features are shown in Fig. 204. Each coach is equipped with a brake cylinder, an auxiliary reservoir, a "triple valve," and a "train pipe" (to which the triple valve is connected); while the motor coaches are equipped in addition, with a compressor, a main reservoir, and the operating (or driver's) valve.

The train pipe is continuous throughout the train, and is connected to the driver's valve, to which is also connected a pipe from the main reservoir, and a connexion to the atmosphere.

The main reservoir pipe is also continuous throughout the train, and it connects the main reservoirs to the compressors. Each coach of a motor-coach train must, therefore, be provided with two hose couplings at each end.

The driver's valve is of the rotary type, and can connect (i) the train pipe to the main reservoir, (ii) the train pipe to atmosphere; while it can also cut off the connexion between the train pipe, the main reservoir, and atmosphere.

The function of the triple valve is to admit air from the auxiliary reservoir to the brake cylinder, this operation depending upon the *difference* of pressure between the auxiliary reservoir and the train pipe. When the pressure in the auxiliary reservoir exceeds that in the train pipe (such as would happen if the driver's valve connected the latter to atmosphere) the triple valve admits air to the brake cylinder and the brakes are applied. On restoring the pressure

in the train pipe (e.g. by connecting it to the main reservoir), the triple valve releases the air in the brake cylinder and charges the auxiliary reservoir.

In order to secure a quick release after an application of the brakes, the pressure in the auxiliary reservoirs is about 20 lb per in.^{2} lower than the pressure in the main reservoir (which is between 80 and 90 lb per in.^{2}). With normal applications of the brake it is necessary to discharge the air gradually from the train pipe and to stop the exhaust gently. This is accomplished by means of an equalizing piston in the driver's valve, in conjunction with a small reservoir (called the equalizing or brake valve reservoir). The equalizing piston controls

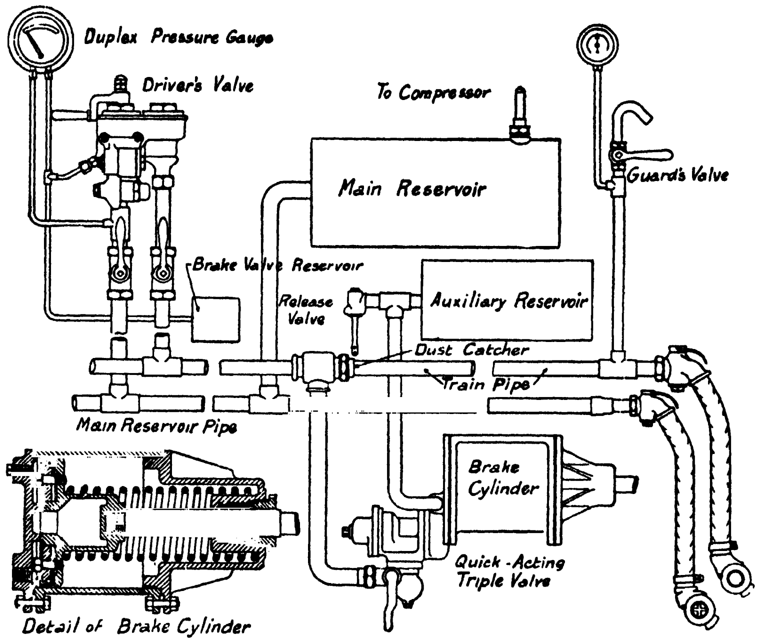


FIG. 204. GENERAL ARRANGEMENT OF ESSENTIAL PARTS OF WESTINGHOUSE QUICK-ACTING COMPRESSED-AIR BRAKE (WESTINGHOUSE BRAKE CO.)

the exhaust valve (in the driver's valve) for the train pipe, and is operated by the difference of pressure between the brake valve reservoir and the train pipe. The driver's valve is constructed so that for *normal applications* of the brake, the air is exhausted from the brake valve reservoir, and by means of the equalizing piston a corresponding reduction of pressure is obtained in the train pipe.

The *gradual* reduction of pressure in the train pipe causes the triple valves to admit air gradually from the auxiliary reservoirs to the brake cylinders, so that the brakes are applied gradually throughout the train.

When a *rapid application* of the brakes is required in cases of emergency the driver's valve is arranged to exhaust air directly from the train pipe, thereby producing a *sudden* reduction of pressure. Under these conditions the triple valves admit air from *both* the train pipe (through a check valve) and the auxiliary reservoirs to the brake cylinders, so that a rapid application of the brakes occurs throughout the train.

With the above forms of the compressed-air brake a reduction in the braking effort (after the brakes have been applied) can only be obtained by *fully releasing* the brakes and re-applying them. The disadvantage has been overcome by the addition to the brake system of electrically-operated valves, by means of which air may be released, under the control of the driver, from the brake cylinders. Such a system is called the *electro-pneumatic brake*;* it is adopted on the latest rolling stock of the London Transport Executive, the Liverpool Street–Shenfield line, etc.

* A comprehensive description is given in a paper, by H. R. Broadbent, on "The Electro-pneumatic Brake," *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 250.

CHAPTER XVII

ELECTRIC LOCOMOTIVES

General Considerations

General Classification. Long-distance main-line traffic involves high operating speeds and the running of various types of rolling stock, such as restaurant and dining cars, sleeping saloons, corridor and non-corridor coaches, brake vans, etc. Such traffic is best handled by locomotives. Moreover, on a large railway system there are two other classes of traffic for which locomotives are necessary, (i) local passenger traffic, intermediate between suburban and long-distance traffic; (ii) freight or goods traffic (with which must be included the marshalling and shunting operations).

Thus a large railway system will require at least three classes of locomotives (i) "express passenger" locomotives, capable of giving large outputs (1,000–3,000 h.p.) at high speeds (60–90 m.p.h.); (ii) "local passenger" locomotives of moderate output; (iii) "goods" locomotives, giving large outputs at slow speeds. In addition, a few "shunting" locomotives will be required for working the marshalling yards and large sorting sidings.

Mechanical Design. The mechanical design of an electric locomotive involves many considerations, some of which are external to the locomotive and others are concerned with the operating requirements. The former include the loading and track gauges, the strength of the permanent way, and the strength of the couplings and draw-gear of the rolling stock. Moreover, the wheel arrangement and the disposition of the various parts of the locomotive must be considered with reference to the wear and the cost of maintenance of the permanent way.

The operating requirements concern the draw-bar pull and speed, the weight to be carried on the driving axles, the running qualities, and the cost of maintenance of the locomotive itself.

External Considerations. 1. The *loading gauge* affects both the overall width and height of the locomotive. When overhead collectors are employed the lowest operating position of the collectors must be within the loading gauge, and the height of the roof must be arranged accordingly.

The *track gauge* affects the overall length of the motor, and the effect of this limitation on the output is discussed in Chapter IV.

2. The *strength of the permanent way* limits the maximum load which can be carried on each driving axle.

The strength of bridges limits the number of driving axles carrying limiting loads and the total wheel-base over which these loads are distributed. In this country, under steam locomotive conditions, the limiting axle load (25 tons) may be carried on each of three driving axles having an overall wheel-base of 14 ft.

3. The *strength of the couplings and draw-gear* limits the maximum draw-bar pull. With the passenger-rolling stock in use on our main-line railways the draw-bar pull at starting must be limited to about 16 tons. But with goods trains in this country the draw-bar pull must be limited to about 12 to 14 tons,

on account of the large number of privately-owned wagons in use.* When specially built wagons are employed, the limiting value of the draw-bar pull can be raised to about 30 tons.

4. The *wear and maintenance of the permanent way* is affected by a number of features, such as traffic density, wheel arrangement, size of driving wheels, height of centre of gravity of locomotive, etc. Generally, for a given traffic, the track wear and maintenance will be reduced if the locomotive has a leading bogie, driving wheels of medium to large diameter, a high centre of gravity, and a minimum of dead, or non-spring-borne, weight on the axles.

Operating Requirements. 1. The *draw-bar pull* required in a given case can be estimated when the weight of the train, the acceleration, the gradient, and the train resistance are known.

2. The *total weight to be carried on the driving wheels* (called the *adhesive weight*) is determined by the condition that the locomotive shall exert the required tractive effort without slipping the wheels.

The ratio of the tractive effort to slip the wheels and the adhesive weight is called the *coefficient of adhesion*, the value of which is influenced by the condition of the rails and also by the speed.

Average values of the coefficient of adhesion at various speeds are—

Speed m.p.h.	0	10	20	30	40	60
Coefficient of adhesion	0·25	0·18	0·14	0·12	0·1	0·09

Under normal starting conditions, with clean, dry rails, a value of 0·25 may be assumed for the coefficient, and a maximum value of 0·3 when sand is used. If the rails are wet or greasy, the coefficient of adhesion will be much lower; for example, with a thoroughly wet rail a value of 0·18 to 0·2 is usually assumed, while for a moist or greasy rail the coefficient is of the order of 0·15. But with the application of sand these values may be increased to about 0·25.

If the tractive effort fluctuates during starting, the maximum value of the tractive effort must not exceed (adhesive weight × coefficient of adhesion). Hence the more uniform the tractive effort during the starting and initial accelerating periods the heavier will be the train which can be operated by a locomotive of given adhesive weight. This point is of considerable importance for goods traffic, where the weight of the train usually approaches the maximum weight which can be handled by the locomotive.

3. The *running qualities* are influenced by the wheel arrangement, the height of the centre of gravity, and the disposition of the masses of the various parts with respect to the longitudinal and transverse axes of the locomotive. Good running qualities are characterized by freedom from vibration and oscillations at all speeds. These qualities are obtained by a symmetrical disposition of the masses about the longitudinal and transverse axes of the locomotive, a moderately high centre of gravity, and dynamic balancing of the rotating parts.

Speed-torque Characteristics. In all electric motors operating at *constant voltage* (and constant frequency in the case of a.c. motors) the speed and torque are intimately connected with each other, as has been discussed in Chapters IV, V, VI. With series motors the speed increases as the torque decreases,

* The usual type of coupling on wagons is tested to 50 tons, and a factor of safety of 4 is usually allowed.

and large variations of torque produce large variations in speed. *On the other hand, with shunt and induction motors the speed is almost unaffected* by variations of torque.

Such characteristics are unsuitable for traffic requirements and do not compare favourably with those of steam locomotives, in which, by regulation of the cut-off, the speed and tractive effort are almost inversely proportional to each other (i.e. the output is constant) over a wide range.

More favourable characteristics, however, can be obtained with electric locomotives having series motor equipments by voltage and field control. For example, with single-phase locomotives, regulation of speed over the whole range is effected by voltage control. In this case the speed-output characteristic is governed by limitations of heating and commutation, the permissible output increasing with the speed until the maximum value (as governed by the prescribed limitations) is reached.

Again, with d.c. locomotives dynamical characteristics somewhat similar to those of steam locomotives can be obtained by the combination of double series-parallel and field control when three or more weakened field steps are provided.

Power Plant. In general, the main power plant of a locomotive may consist of either one or two motors of large output or a number of motors of moderate output. The number of motors to be employed in a given case depends largely upon the power required, the method of speed control and the system of power transmission. For example, with single-phase equipments duplication of the motors for purposes of speed control is unnecessary, and a single motor may be employed if the operating conditions are suitable. Similarly, with three-phase equipments, a single motor could be employed under suitable conditions, as four speeds could be obtained by two pole-changing stator windings. But two motors are necessary if cascade control is to be employed. On the other hand, with d.c. equipments, in which the motors are wound for the line voltage, two motors are necessary for series-parallel control, and four motors for double series-parallel control. With high-voltage equipments, in which each motor receives a fraction of the line voltage, a corresponding increase in the number of motors is necessary.

Power Transmission. The axles may be driven either individually or collectively, the latter drive involving the coupling of the axles by side rods, and the transmission of power thereto by cranks and connecting rods. The individual-axle drive is preferred—except in locomotives having a single motor or a pair of motors—because of the lower first cost and low maintenance.

With the individual-axle drive, however, wheel slip may occur, under unfavourable conditions of adhesion, on any of the driving axles during starting and running.* Indication of such slipping may be given to the driver by special indicators, the action of which depends on a comparison of either the speeds of the axles or the inputs to the motors. Correction of wheel slip may be effected by either a light application of the mechanical brake to the slipping wheels, or

* Wheel slip and associated problems are discussed in the following papers—"Locomotive wheel-slip and wheel-lock protection," by R. M. Smith, *Trans. A.I.E.E.*, Vol. 69, p. 1,154; "Skidding of wheels: influence of series characteristics of traction motors driving individual axles," by M. Royer, *Bulletin of the International Rly. Congress Assoc.*, Vol. 2, p. 203.

a reduction of the input to all the motors. The former method is preferred and can be effected automatically by means of suitable relays when individual brake cylinders are fitted to each bogie or driving axle.

As the application of this drive to axle-mounted motors has been discussed in Chapter IV, the present discussion will be confined to methods of applying the drive to frame-mounted motors.

Individual-axle Drive with Frame-mounted Motors. Due to the motors being mounted on the spring-supported frame of the truck or locomotive, the distance between a given armature shaft and the corresponding driving axle may not be a fixed quantity, and the parallelism of their axes may not always be maintained. Hence a flexible connexion is necessary between either the armature shaft and the pinion, or the gear-wheel and the driving axle. In

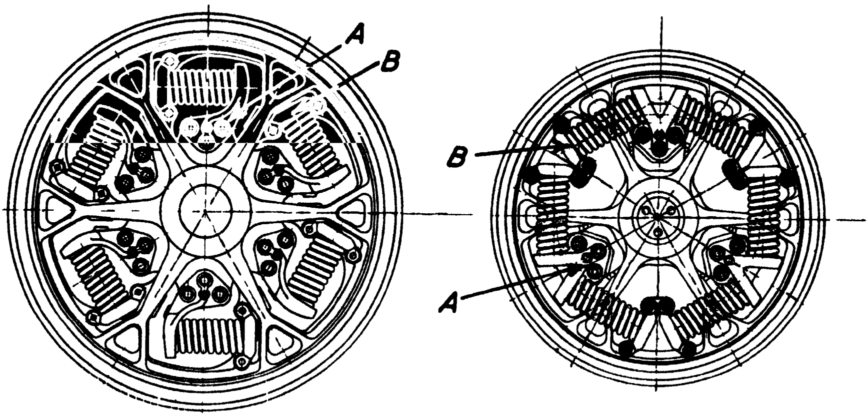


FIG. 205. (*left*) WESTINGHOUSE SPRING DRIVE, (*right*) SÉCHERON SPRING DRIVE
A—Spring-cup on quill. *B*—Spring-cup on wheel.

the former case a cardan shaft is usually necessary, and in the latter case the gear-wheel is usually mounted on a quill and the latter is flexibly coupled to the axle or the driving wheels. Numerous schemes, involving these and other principles, have been devised, but many have been abandoned due to considerations of cost and maintenance. Those now manufactured are characterized by their simplicity and the relatively few parts subjected to wear. Some typical examples will be considered.

Quill Drive with Spiral Springs. The original (Westinghouse) form is shown in Fig. 205 (*left*) and a modification (Sécheron) (*right*). Although both possess the merit of simplicity and cheapness, in comparison with later forms, they have the disadvantages that the springs are exposed to dirt and wet, and have no lateral support to resist the effects of centrifugal force. Nevertheless many applications have been made to locomotives and motor-coaches in America and Europe, and the Sécheron form is employed on the latest locomotives of the Austrian railways.

The modern (*Brown-Boveri*) form is shown in Fig. 206. The springs are enclosed in the gear-wheel which is carried by roller bearings mounted on a short quill which is clamped to the motor frame. Since the quill is stationary

and has to provide only for vertical displacements of the axle, the hole in the quill to accommodate the axle may be oval in shape to enable the motor frame to be located nearer to the axle than would be possible with a rotating quill. The torque is transmitted by spiral springs to the projecting arms of a spider,

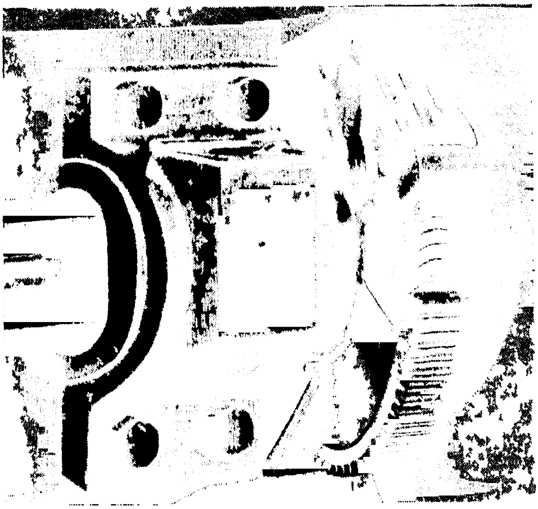


FIG. 206 MODERN FORM OF SPRING DRIVE (BROWN BOVERI)

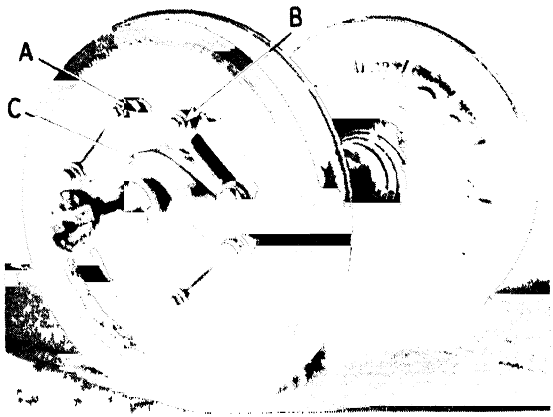


FIG 207. QUILL DRIVE WITH LINK COUPLING (ALSTHOM)

S, which is pressed on the axle. These arms bear against the spring cups in the centre plane of the gear-wheel. The springs are relieved of stresses due to centrifugal force by the lateral support of the spring cups. This drive is used on many passenger locomotives and motor-coaches in Switzerland, and also on the Brown-Boveri gas turbine locomotive (No. 18000) of the British Railways.

Quill Drive with Link Coupling. The *Alsthom* drive is shown in Fig. 207, and is employed on many locomotives in service on the French Railways. The gear-wheel is fixed to the quill which extends the whole distance between the wheel hubs. Each end of the quill is coupled to the corresponding driving wheel by two pairs of links which connect pins, *A*, on the driving wheel with

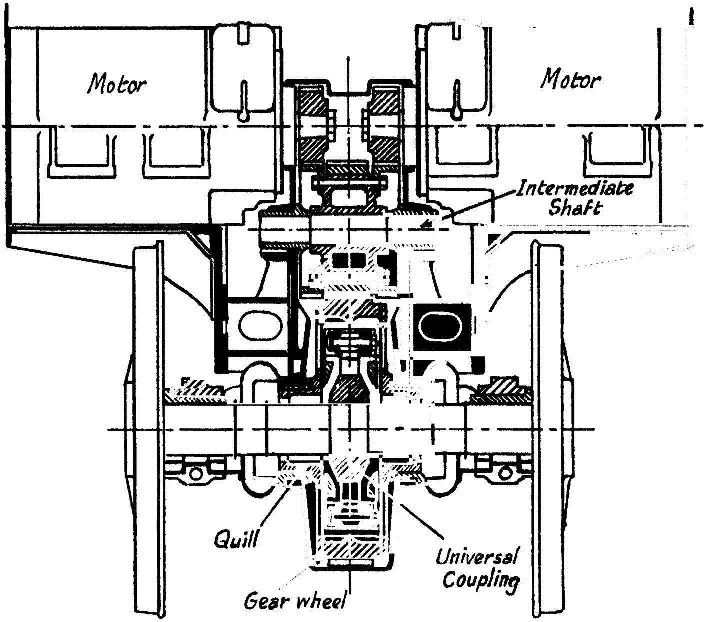


FIG. 208. SWISS LOCOMOTIVE WORKS' INDIVIDUAL-AXLE DRIVE WITH TANDEM MOTORS

pins, *B*, on the quill through the intermediate lozenge-shaped plate *C*. All links and pins have silentbloc bushes, and no lubrication is necessary. The cost of maintenance is very low.

Quill Drive with Universal Coupling. This drive (due to the *Swiss Locomotive and Machine Works*) was developed for tandem motors with outputs of 1,000 h.p. or more per axle. The general arrangement is shown in Fig. 208, and the gear-wheel, with Oldham-type coupling, is shown in Fig. 209. The gear-wheel is mounted on stub quills (one on each side) and is arranged centrally between the driving wheels. The coupling is located inside the gear-wheel. The motors are mounted on the locomotive frame above the driving wheels and their pinions mesh with an idle or intermediate gear.

Cardan-shaft Drives. These drives for locomotives have been developed in Switzerland. The axis of the motor is parallel to the driving axle and the cardan shaft is located inside the hollow armature shaft. The gear-wheel is fixed to the axle and the pinion is mounted on roller bearings in the rigid gear-case, which is carried by roller bearings on the hub of the gear-wheel. The pinion is

coupled to the cardan shaft by a flexible coupling and a similar coupling connects the other end of this shaft to the hollow armature shaft.

Flexibility is obtained by either steel discs (Brown Boveri) or links of spring steel (Sécheron), both forms occupying very little space longitudinally.

The arrangement of the disc drive is shown in Fig. 210. The sleeve *A*, keyed to the armature spider, is connected to one end of the cardan shaft, *B*, by means of the disc *C* and the two cross-arms *D*, *E*. The other end of the cardan shaft is similarly connected to the stub-shaft, *F*, to which the pinion is

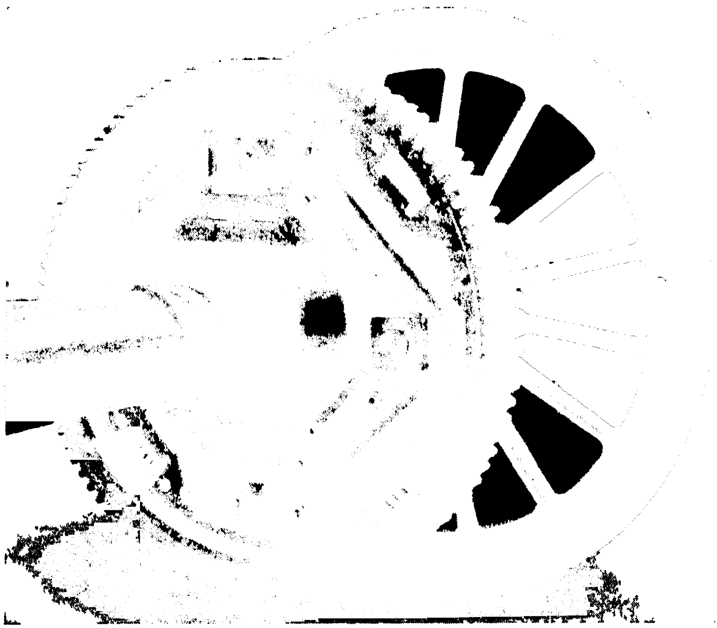


FIG. 209. GEAR-WHEEL AND FLEXIBLE COUPLING FOR TANDEM MOTOR DRIVE (METROPOLITAN VICKERS AND SWISS LOCO. WORKS)

fitted. This shaft is carried in roller bearings in the gear-case, *G*, which is centred, by roller bearings, on the hub of the gear-wheel. The pinion end of the gear-case is supported from the truck frame.

The Sécheron drive is arranged similarly to the tramcar form (Fig. 178). A pinion-end coupling is shown in Fig. 211.

Gearless Collective Drives. In these drives the armature shaft of each motor is extended at both ends, and "quartered" cranks (i.e. cranks set at right angles to each other) are fitted to the shaft extensions. The cranks drive coupled wheels on each side of the locomotive by means of connecting rods. In the application of the connecting-rod drive, however, due consideration must be given to the fact that the driving axles have vertical play relatively to the armature shafts. Hence the crank pins of the armatures and driving wheels may be rigidly coupled only if the armature shafts are practically on a level with the driving axles and long connecting rods are employed. In this

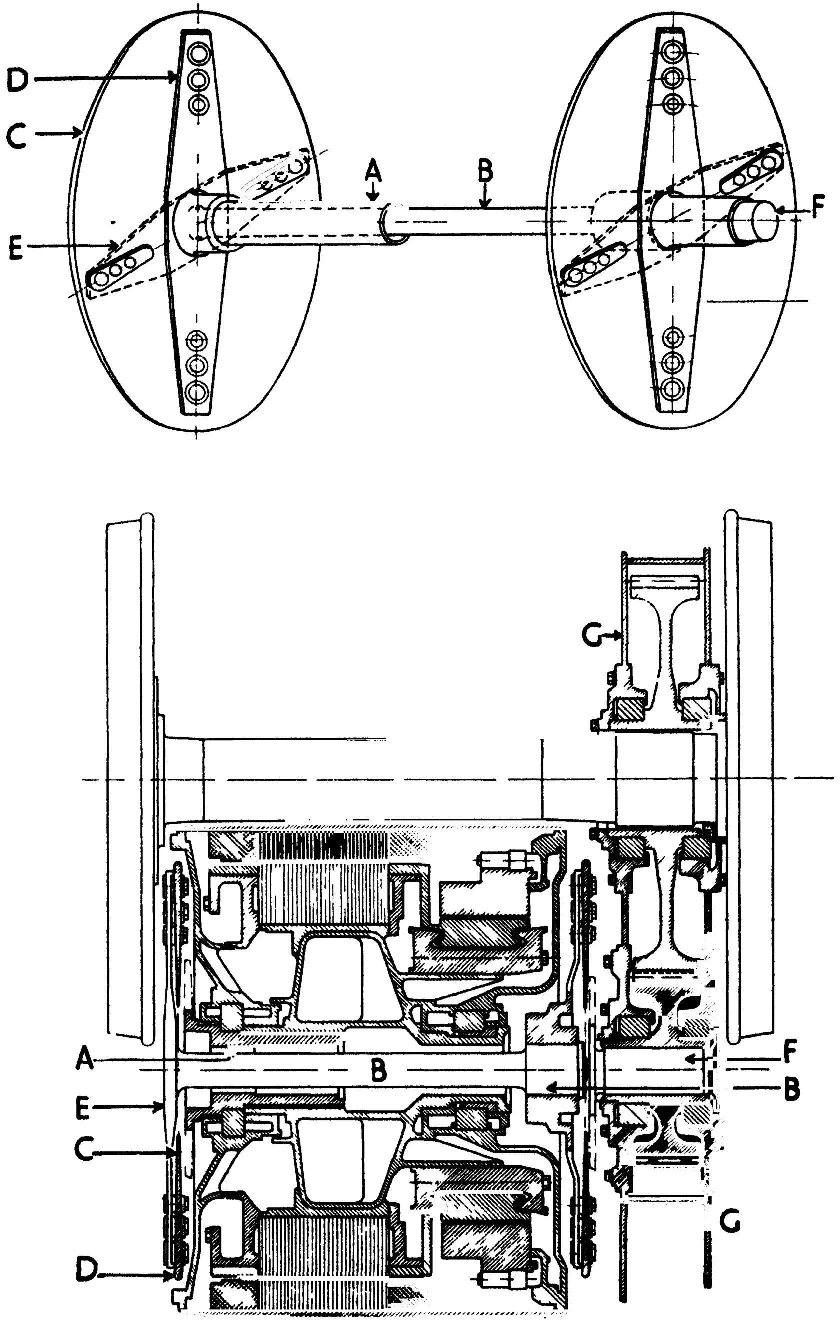


Fig. 210. Brown-Boveri Disc Drive

Upper—Cardan-shaft and discs. *Lower*—Actual arrangement of cardan-shaft, discs, etc.

case the variation in the obliquity of the connecting rods, due to the vertical motion of the driving axle, is small enough to be provided for by the clearances between the axle boxes and guides, so that no additional stresses are produced in the connecting rods. In other cases, either slotted crank-pin bearings must be provided at the driving axle, or an intermediate crank-shaft (called a "jack-shaft") must be employed, from which a horizontal drive can be obtained for the driving wheels.

Alternatively, when the power equipment consists of a pair of large motors (1,000 h.p. or over), as on three-phase locomotives, satisfactory operation may be obtained with the system of articulated links shown in Fig. 212, which has been applied to many three-phase locomotives on the Italian State Railways.

The inclined rods S_2, S_3 have their upper ends connected to a horizontal coupling rod S_1 (which couples together the crank-pins of the motors), and their lower ends connected to levers Z and W. These levers are fulcrumed on the coupling rods K, and are connected by a link L, so as to form a closed linkwork. The complete mechanism is so dimensioned that (i) the principal members S_1, S_2, S_3 form the sides of an isosceles triangle (of which S_1 is the base), the apex of which is, normally, at the centre of the crank pin of the driving wheel; (ii) the axis of the link L passes through this point; (iii) the arms *ec*, *eg*, of lever Z are equal to the arms *fd*, *fh*, respectively, of lever W.

FIG. 211. SPRING-STEEL LINK COUPLING BETWEEN CARDAN-SHAFT AND PINION (SÉCHERON)

In consequence, only the horizontal component of the torque of the motors is transmitted to the driving wheels. Moreover, vertical displacements of the driving axle relatively to the motors cause the inclined rods S_2, S_3 to make equal small angular movements in opposite directions, and to maintain their isosceles arrangement.

Disadvantages of Gearless Collective Drive. Direct drives having inclined connecting rods with or without jack shafts require more maintenance (e.g. adjustment of bearing clearances) than the connecting-rod drive of a steam locomotive.

This feature, which is common to all side-rod drives with rigid transmission rods, is due to several fundamental differences between the electric locomotive side-rod drive and the steam locomotive side-rod drive. Thus there is no "free end" corresponding to the piston, and consequently the centres between all crank pins must be rigidly maintained. Any wear in the crank-pin bearings or the bearings of the jack-shaft will, therefore, result in excessive stresses and bearing pressures.* In order to avoid excessive vibrations from these causes means must be provided for the accurate adjustment of the bearings. The parts will also have to be designed to withstand the additional stresses due to any incorrect adjustment of the bearings.

* See an article on "The Crank Drive of Electric Locomotives," by J. Buchli (*The Electrician*, Vol. 73, p. 992; *The Engineer*, Vol. 120, p. 287).

Again, the conversion of the uniform torque of the motors into a reciprocating motion produces severe stresses in the motor shaft, jack-shaft, and locomotive framing. For example, since the cranks on the armature shaft are set at right angles to each other, there will be four positions in each revolution where the full torque of the motor is transmitted through one crank. Therefore each connecting rod is subjected to an alternating force having a maximum value represented by maximum value of torque/radius of crank.* Now if the forces acting on each crank pin are resolved in two directions at right angles—one direction being along the line of centres—it will be found that the motor shaft is subjected to reciprocating forces and alternating couples in these directions, the reciprocating forces and couples changing their direction four times in each revolution.† These forces act on the motor bearings and frame, and must be taken into account in the design of the machine and the locomotive framing.

The jack-shaft will be subjected to greater stresses than the motor shaft, for, in addition to the alternating forces and couples, there is the large twisting moment, due to the transmission of the full power through alternate cranks four times in each revolution. The forces and couples resulting from the connecting rods and coupling rods will depend on the angle between the former and the latter, the maximum values occurring when this angle is 90 degrees, i.e. when the connecting rods are at right angles to the coupling rods. Moreover, the forces at each crank pin can be resolved into a couple and a force, the axis of the couple and the direction of the force both rotating with the jack-shaft.

It is apparent, therefore, that the jack-shaft, the cranks, and the connecting rods will have to be exceptionally strong, while the bearings for the jack-shaft must be liberally designed.

Geared Collective Drives. In these drives the motors are geared to jack-shafts, from which the wheels are driven by connecting rods.

The gearing enables more favourable armature speeds to be chosen, together with large driving wheels, and its use is particularly desirable with single-phase motors in order to obtain an economical design of motor together with high efficiency and power factor. Moreover, springs can be fitted to the gears for the purpose of damping pulsations and sudden fluctuations in the torque at the motor shaft, and thereby improving the conditions at the jack-shaft.

Discussion on the Limitations of Geared and Gearless Methods of Power Transmission. With *individual axle drives* from *geared motors* the size of motor which can be accommodated is limited by both the diameter of the wheels and the track gauge, as explained in Chapter IV. Except in the case of narrow track gauges, however, the limitation imposed by the wheel diameter is not serious, provided that unduly small wheels are not required, as d.c. axle-mounted motors of 500 h.p. can be accommodated with 51-in. wheels.

When the size of driving wheel and the maximum speed of the locomotive are fixed, the maximum diameter of the pitch circle of the gear-wheel can be obtained by assuming an appropriate value for the limiting gear velocity. This diameter, however, must provide sufficient clearance between the bottom

* With single-phase motors the torque is pulsating, and consequently the maximum or crest value must be used in the above expression.

† An analytical treatment is given in a paper on "The Electric Locomotive," by Dr. F. W. Carter. *Proc. Inst. C.E.*, Vol. cci, p. 231.

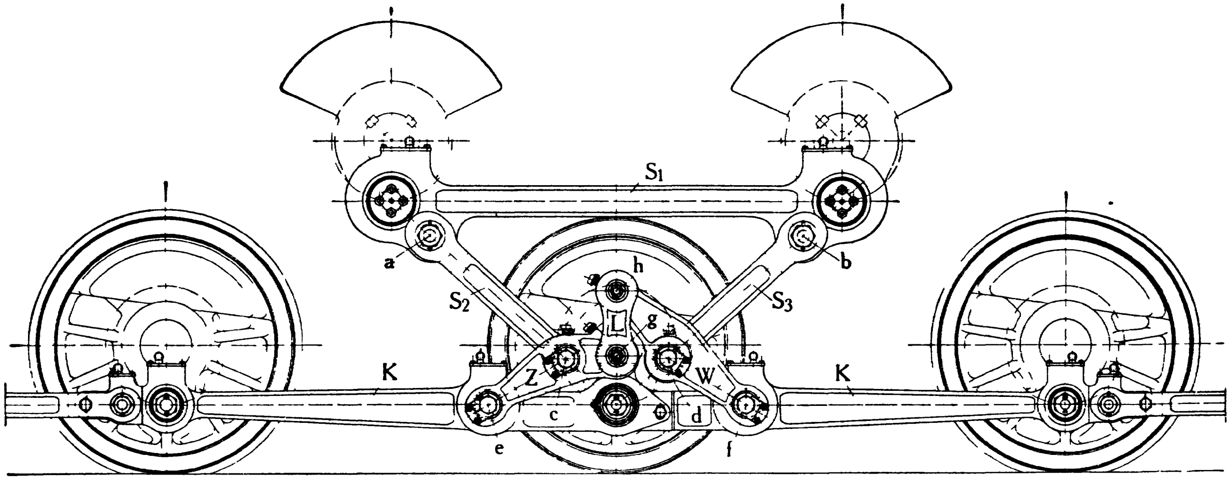


FIG. 212. BIANCHI'S JOINTED-LINK DRIVE FOR THREE-PHASE LOCOMOTIVE (BROWN BOVERI)

of the gear-case and the track,* which is generally the limiting feature in slow and moderate speed locomotives. The maximum gear ratio and the peripheral speed of the armature can then be obtained when the distance between the centres of axle and armature shaft is known.

For an economical design of motor the peripheral speed of the armature at the maximum speed of the locomotive should approach the limiting value, i.e. 10,000 to 13,000 ft per minute. But the gear velocity must also be within the prescribed limits, which are about 4,000 ft per minute at the pitch circle for ordinary lubrication and about 6,000 ft per minute for forced lubrication. The relationship between the peripheral speeds of armature and gearing depends upon the gear ratio, and can be expressed in general terms.

Thus if D_a, D_w denote the diameters of armature and driving wheel respectively, C the distance between centres of armature and axle, V_m the maximum speed of the locomotive in m.p.h., V_a, V_g the corresponding peripheral speeds, in feet per minute, of armature and gearing for the gear ratio employed, we have

$$V_g = 2CV_a/(D_a + D_wV_a/88V_m)\dagger$$

Hence the gear velocity corresponding to the maximum speed of the locomotive depends upon: maximum peripheral speed of the armature, diameter of armature, size of driving wheel, and distance between centres of armature and axle. By proper choice of the size of driving wheel the peripheral speed of the armature can reach the limiting value at the maximum speed of the locomotive, and the gear velocity under these conditions can be kept within the prescribed limits. The geared drive, therefore, enables an economical design of motor to be obtained under all operating conditions.

EXAMPLES. 1. Consider an *axle-mounted motor*, for which $D_a = 29{\cdot}5$ in., $C = 25$ in., to be used with 52 in. wheels for freight service, the maximum locomotive speed being 35 m.p.h. If the armature is to run at a peripheral speed of 8,000 ft per minute at the maximum locomotive speed, the corresponding gear velocity is

$$V_g = 2 \times 25 \times 8{,}000/(29{\cdot}5 + 52 \times 8{,}000/88 \times 35)$$
$$= 2{,}430 \text{ ft per minute.}$$

The pitch-circle diameter of the gear wheel $= D_wV_g/88V_m = 52 \times 2{,}430/88 \times 35 = 41$ in., which is just within the limiting value, i.e. $0{\cdot}8 \times 52 = 41{\cdot}6$ in. The gear ratio $= V_aD_w/88V_mD_a = 4{\cdot}58$, and the diameter of pinion at pitch circle $= D_wV_g/88V_m\gamma = 41/4{\cdot}58 = 8{\cdot}96$ in.‡

* For approximate purposes the maximum diameter of the gear wheel may be assumed at about 80 per cent of the diameter of the driving wheel. This value, however, must be considered with reference to the corresponding diameter of the pinion, which may be the limited feature in some cases.

† This expression is obtained as follows—

Peripheral speed, V_a, of armature (ft per min) at locomotive speed V_m (m.p.h.) and gear ratio $\gamma = (5{,}280/3{,}600)\, V_m \times \gamma \times D_a/D_w = 88V_m \times \gamma \times D_a/D_w$.

Diameter of pitch circle of gear wheel $= D_wV_g/88V_m$.

Diameter of pitch circle of pinion $= 2C -$ diameter of pitch circle of gear wheel $= 2C - D_wV_g/88V_m$.

Whence
$$V_a = 88V_m\frac{D_a}{D_w} \times \frac{D_wV_g}{88V_m(2C - D_wV_g/88V_m)}$$
$$= D_aV_g/(2C - D_wV_g/88V_m)$$

or
$$V_g = 2CV_a/(D_a + D_wV_a/88V_m)$$

‡ The values obtained for the diameter of pinion, gear ratio, and gear velocity may require slight modification in practice when the number of teeth in the gearing and the pitch of the teeth are considered.

But if the maximum speed of the locomotive were reduced a corresponding reduction would have to be made in the maximum peripheral speed of the armature, in order to keep the diameter of gear wheel within the limiting value (i.e. $0{\cdot}8 \times$ diameter of driving wheel), as the given diameter of driving wheel is the minimum for this motor.

If the gear ratio is changed to give a maximum locomotive speed of 55 m.p.h., and the limiting peripheral speed of the armature is to remain at 8,000 ft per minute, then

$$V_g = 2 \times 25 \times 8{,}000/(29{\cdot}5 + 52 \times 8{,}000/88 \times 55)$$
$$= 3{,}480 \text{ ft per minute,}$$

which is well within the prescribed limit for ordinary lubrication. In this case the ratio $V_g/88V_m = 3{,}480/4{,}860 = 0{\cdot}716$. The gear ratio

$$V_aD_w/88V_mD_a = 2{\cdot}9.$$

Alternatively, if a lower gear velocity were desired, the size of driving wheel could be increased. Thus, with 60-in. wheels, a locomotive speed of 55 m.p.h., and an armature speed of 8,000 ft per minute,

$$V_g = 2 \times 25 \times 8{,}000/(29{\cdot}5 + 60 \times 8{,}000/88 \times 55)$$
$$= 3{,}120 \text{ ft per minute}$$
$$V_g/88V_m = 3{,}120/4{,}860 = 0{\cdot}642. \text{ Gear ratio} = 3{\cdot}35.$$

2. In the case of a *frame-mounted motor*, for which $D_a = 24{\cdot}5$ in., $C = 25{\cdot}7$ in., used with 66-in. wheels for a passenger locomotive, for which the maximum speed is 75 m.p.h.; if the limiting armature speed is 10,000 ft per minute, then

$$V_g = 2 \times 25{\cdot}7 \times 10{,}000/(24{\cdot}5 + 66 \times 10{,}000/88 \times 75)$$
$$= 4{,}130 \text{ ft per minute.}$$
$$V_g/88V_m = 4{,}130/88 \times 75 = 0{\cdot}626. \text{ Gear ratio} = 4{\cdot}08.$$

If this motor were used, with smaller wheels, on a freight locomotive for which the maximum speed was 40 m.p.h., the minimum diameter of driving wheel is determined by the condition that the maximum permissible diameter of the gear wheel $= 0{\cdot}8 \times$ diameter of driving wheel. The corresponding value of the gear velocity is, therefore, $0{\cdot}8 \times 88V_m$, and if this value is substituted in the above equation for V_g, we obtain

$$D_w = 2{\cdot}5C - 88V_mD_a/V_a$$

Hence, in the present case,

$$D_w = 2{\cdot}5 \times 25{\cdot}7 - 88 \times 40 \times 24{\cdot}5/10{,}000$$
$$= 55{\cdot}7 \text{ in.}$$

$$V_g = 0{\cdot}8 \times 88V_m = 2{,}820 \text{ ft per min. Gear ratio} = V_aD_w/88V_mD_a = 6{\cdot}45.$$

Diameter of pinion at pitch circle $= 0{\cdot}8 \times 55{\cdot}7/6{\cdot}45 = 6{\cdot}9$ in. If this diameter of pinion is too small, a lower limiting armature peripheral speed must be adopted.

With *gearless collective drives* the maximum permissible angular velocity of the crank shafts is about 500 r.p.m. This value may be considered as the limiting value of the angular velocity of the armature, and since the diameter of the armature is not restricted by that of the driving wheels, the limiting peripheral velocity may be made to coincide with the limiting angular velocity. Thus, if the limiting angular and peripheral velocities be assumed as 475 r.p.m. and 7,500 ft per minute respectively, then, for a maximum locomotive speed of 80 m.p.h., the diameter of the driving wheels will be 56·5 in., and the maximum diameter of the armature will be 60 in. If the maximum locomotive speed is of the order of 50 m.p.h., a lower value for the limiting angular velocity must be adopted, as the above value will lead to small driving wheels. Assuming

the minimum diameter of the driving wheels to be 42 in., then for a speed of 50 m.p.h., the angular velocity will be 400 r.p.m., and the diameter of the

FIG. 213. CONTROL APPARATUS AND RHEOSTAT COMPARTMENTS OF 1,500-VOLT LOCOMOTIVE (METROPOLITAN-VICKERS)

armature, corresponding to a limiting peripheral speed of 7,500 ft per minute, will be 71·5 in.

These considerations show that the sphere of usefulness of the direct drive

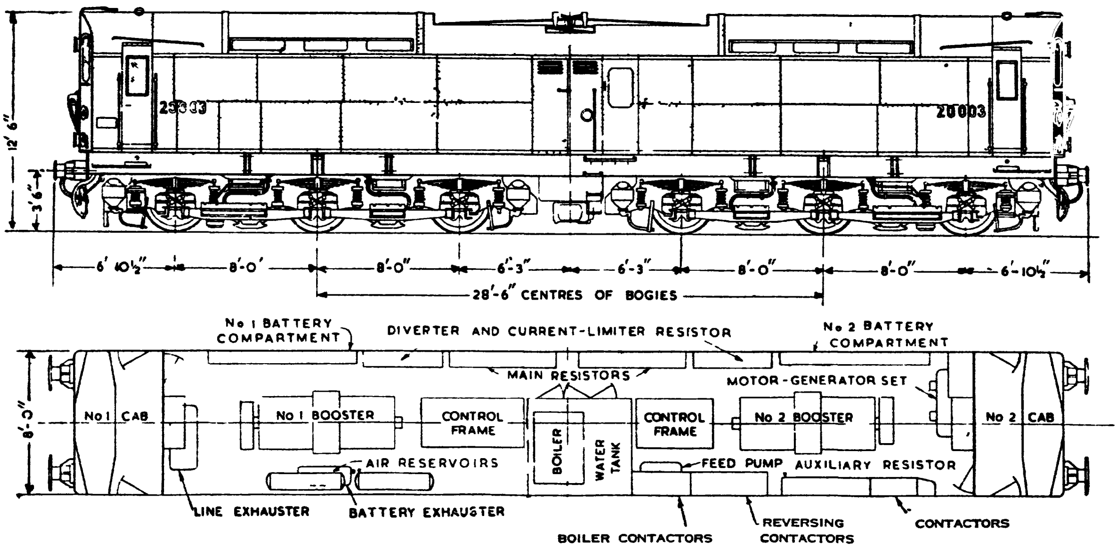


FIG. 214. OUTLINE OF 660 V, 105-TON LOCOMOTIVE AND LAYOUT OF EQUIPMENT, BRITISH RAILWAYS (ENGLISH-ELECTRIC EQUIPMENT)

is limited, and for an economical design of the motor, it will be necessary to adopt gearing between the armature and the jack-shaft when the operating speeds are of the order of 50 m.p.h. and below.

Control and Auxiliary Equipment—Location and General Arrangement. In the general layout of this apparatus on a locomotive it is necessary to arrange that (i) the master controller, auxiliary control switches, brake valves, and instruments are arranged in convenient positions and do not interfere with the driver's look-out; (ii) the contactors and auxiliary machines which require attention are placed in accessible positions; (iii) all high-tension apparatus is located in locked compartments which cannot be opened until the circuits are "dead."

With *high-voltage, d.c. locomotives* the box, or coach, type body is usually employed, and the high-voltage control equipment (contactors, reversers, etc.) is located in compartments along each side of the body. In some cases a central gangway is provided, and in other cases either one or two side-gangways. Fig. 213 shows the contactor and rheostat compartments of a 1,500-V locomotive.

The roof is removable in two or more sections, those portions carrying pantograph collectors being removable with the collectors as separate units.

Single-phase locomotives, in many cases, also have box-type bodies. The tap-changer (contactors or tapping switch) is usually mounted on, or adjacent to, the main transformer, and the reversers are mounted on the motor frames in order to reduce the length of the heavy-current connexions. Moreover, with large frame-mounted motors the blowers are usually mounted on the motor frames, so that no air trunks are necessary.

EXAMPLES OF ELECTRIC LOCOMOTIVES

The examples which follow have been selected to be representative of modern locomotives. Detailed descriptions of the motors, control and auxiliary apparatus are not included, as examples of this equipment have already been given.

I. Direct-Current Locomotives*

Bogie Locomotive with Axle-mounted Motors. Fig. 214 shows the general arrangement and the layout of the equipment of a mixed-traffic locomotive—class C_0C_0, 105 tons, reference No. 1—in service on the conductor-rail sections of the Southern Region, British Railways. A special feature is the use of fly-wheel-booster sets for starting and speed control, and for providing energy for propulsion across gaps in the conductor rails which cannot be spanned by the collector shoes.

The three 245-h.p., 400-V traction motors on each bogie are permanently connected in series and are supplied from the collector shoes through a $\pm$ 600-V booster, which forms part of a motor-flywheel-booster set, the flywheel weighing 2,000 lb.

Starting and speed control of each group of traction motors is effected, in 26 steps, entirely by field control of the appropriate booster (except for three weak-field steps which may be employed when maximum voltage is applied to the motors). Initially the voltage of the booster opposes the line voltage,

* The principal dimensions, weights, and other data of the locomotives discussed are given in Tables VIIA and VIIB, p. 307.

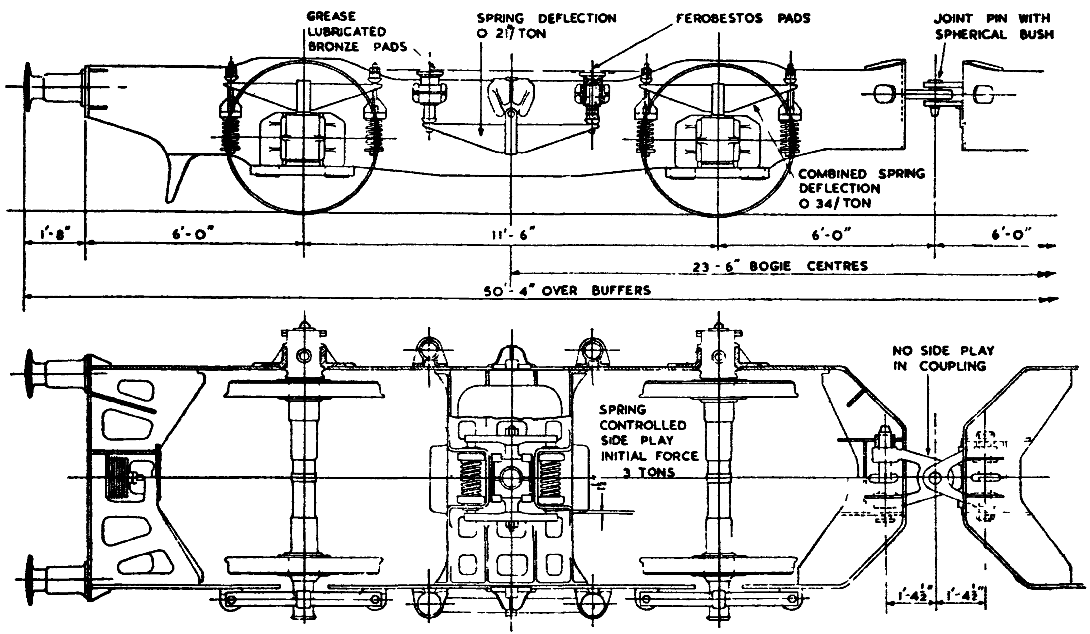


FIG. 215. OUTLINE OF ARTICULATED BOGIE OF BRITISH RAILWAYS' LOCOMOTIVE

it is reduced progressively to zero, reversed and increased progressively to a maximum.*

When the collector shoes lose power the traction motors and boosters are

FIG. 216. HEAVY PASSENGER LOCOMOTIVE (PAULISTA RAILWAY) WITH GEARED AXLE-MOUNTED MOTORS (GENERAL ELECTRIC CO. SCHENECTADY)

supplied by the motors of the booster sets acting as generators, the energy being supplied by the flywheels.

Articulated Bogie Locomotives with Axle-mounted Motors. Locomotives of this type were originally developed in America for heavy passenger and freight

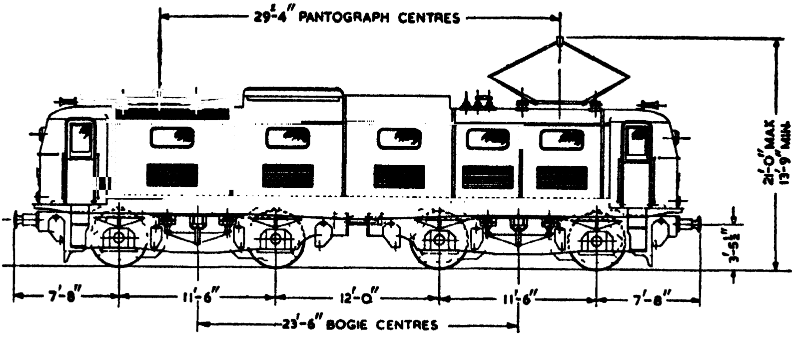


FIG. 217. OUTLINE OF 1,500-V, 89-TON LOCOMOTIVE, BRITISH RAILWAYS (METROPOLITAN-VICKERS)

traffic. The drawgear is an integral part of the truck frame, and the tractive effort is transmitted from one bogie to the other by a hinged or articulated coupling between the trucks.

The chief advantage of this construction is that a relatively light underframe may be employed. Another advantage is that the design is readily adaptable to low heights of drawgear without costly end-frame castings.

Fig. 216 shows a typical *American* locomotive of this type.

* Details of the control system are given in a paper on "Electric locomotives of the British Railways" by W. J. A. Sykes, *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 178.

The general arrangement of a *British Railways*' locomotive—class $B_0 + B_0$, 89 tons, reference No. 2—is shown in Fig. 217, and details of one of the bogies are shown in Fig. 215.

The body is spring supported at four points on each bogie, steel facings on the underframe resting on pads carried on cylindrical guides at the ends of laminated equalizing springs pivoted to the side frames. The guide bearings

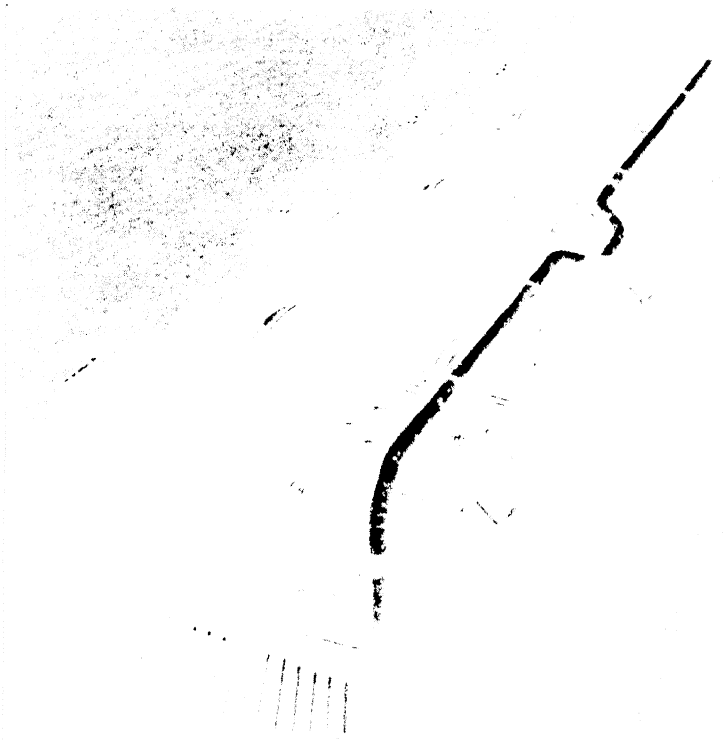


FIG. 218. THREE-AXLE BOGIE OF SOUTH AFRICAN RAILWAYS' LOCOMOTIVE (METROPOLITAN-VICKERS)

for the king pins are allowed a small lateral movement under the control of springs, and the bearing on one bogie has also a longitudinal movement of $\frac{1}{2}$ in. to allow for the foreshortening of the distance between centres when running on curved track.

Each bogie is equipped with two 465-h.p. 750-V motors which are permanently connected in series for 1,500-V operation. The motors are nose suspended and the gear-wheels are of the resilient type.

Although originally designed for mixed traffic the locomotives handle only freight traffic between Manchester and Sheffield. On this line there are some heavy gradients including a 2-mile stretch of 1 in 40. The equipments are therefore arranged for regenerative braking, the scheme shown in Fig. 148 (*b*) being employed.

FIG 219 72 TON, CLASS B_0B_0, 3,000 V LOCOMOTIVE, ITALIAN STATE RAILWAYS

Other British-built articulated mixed-traffic locomotives ($B_0 + B_0$ and $C_0 + C_0$ types) are in service on the *South African Railways* on which the track gauge is 3 ft 6 in. The $B_0 + B_0$ locomotives weigh 66 tons, the 1-hour rating is 1,200 h.p. and the maximum speed (limited by track conditions) is

FIG. 220. 100 TON, CLASS $B_0B_0B_0$, 3,000-V LOCOMOTIVE, ITALIAN STATE RAILWAYS

45 m.p.h. The $C_0 + C_0$ locomotives weigh 110 tons, the 1-hour rating is 2,700 h.p. and the maximum speed is 60 m.p.h. Both types are arranged for multiple unit working (two or more locomotives being controlled by one driver) for which the articulated design is advantageous, as large tractive efforts can be transmitted direct to the drawgear without stressing the underframes.*

One of the three-axle bogies is shown in Fig. 218. The centre bearing has

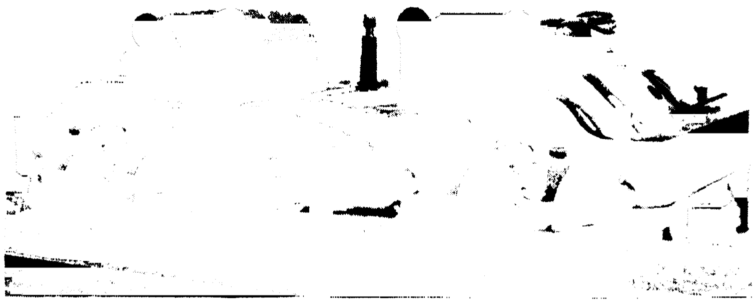


FIG. 221. SWING-BOLSTER TYPE BOGIE, ITALIAN STATE RAILWAYS

a flat face and is designed to carry a portion of the weight of the body. The side and end bearers are therefore adjustable to enable the clearances to be set correctly. The intermediate axle is displaced from the centre of the bogie to accommodate the motor, the wheel bases being 6 ft 6 in. (outer) and 7 ft 6 in. (inner). Equality of the axle loadings is obtained by equalizing levers fitted to the suspension springs.

Each bogie is equipped with three 450-h.p., 1,500-V motors (insulated for 3,000 V) and single gearing with resilient gear-wheels.

* Later locomotives—class $1C_0 + C_01$, 155 tons (129 tons adhesive)—for a maximum speed of 60 m.p.h. have a 1-hour rating of 3,150 h.p. *See G.E.C. Journal*, Vol. 20, p. 109.

Bogie Locomotives with Frame-mounted Motors. Figs. 219, 220 show locomotives recently developed for general services on the 3,000-V lines of the *Italian State Railways*. The B_0B_0 locomotive—reference No. 3—shown in Fig. 219 has a weight of 72 tons and a 1-hour rating of 2,130 h.p.; the $B_0B_0B_0$ locomotive shown in Fig. 220 has a weight of 100 tons and a 1-hour rating of 3,200 h.p.: both locomotives are designed for a maximum speed of about 90 m.p.h.

A view of one of the bogies with the motors in position is shown in Fig. 221: it is of the swing-bolster type, with the bolster suspended from laminated springs carried on swinging links outside the side-frames.

Each motor is rated at 530 h.p., 1,500 V, and is insulated for 3,000 V, two

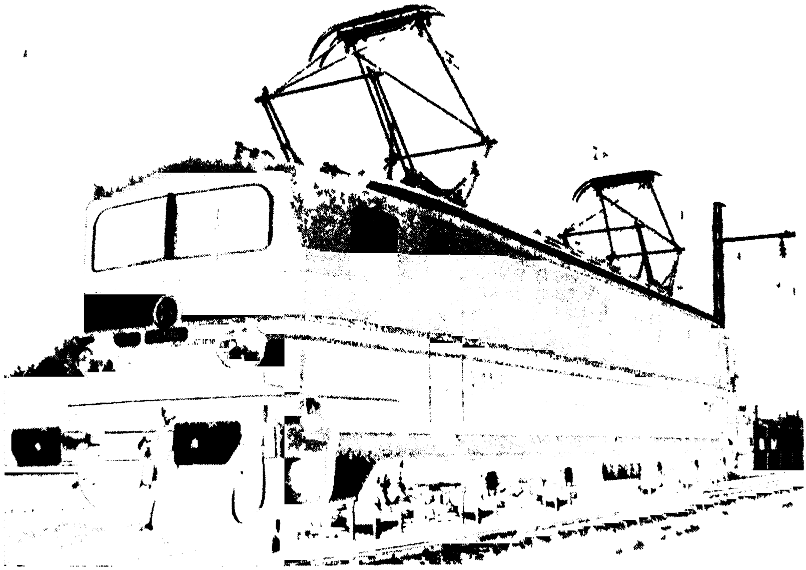


Fig. 222. Express Passenger Service Locomotive of the French National Railways (Alsthom)

motors being permanently connected in series. A quill drive with laminated springs in the driving wheels is employed, and the gear ratio is either 65 : 16 or 65 : 21 depending on the class of service. As the distance between the centres of armature shaft and quill would not allow the 65 . 16 ratio to be obtained with the conventional arrangement of pinion and gear-wheel, an intermediate or idler pinion is employed. This pinion runs on a double row of roller bearings which are arranged internally and are carried on a flange fitted to the motor frame, thereby giving great compactness. An advantage of this arrangement is that a change of gear ratio involves only changing the pinions.

The motors of some of the B_0B_0 locomotives have compound excitation (series and separate) in order to give a uniform speed, together with some recuperation of energy, when operating over undulating routes. The separately-excited windings of all the four motors are connected in series and supplied from a 24-V generator, which, with a floating battery, supplies also the control and lighting circuits.

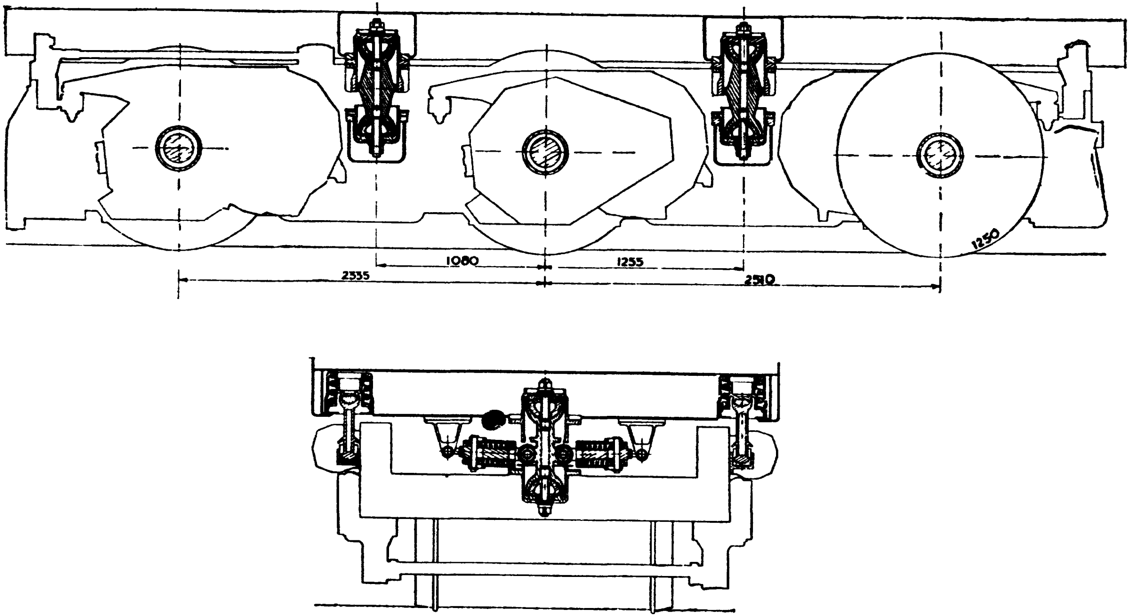


FIG. 223. ARRANGEMENT OF SWINGING PIVOTS CONNECTING BOGIE BOLSTERS TO UNDERFRAME (ALSTHOM)

Note—In later designs the spherical bearings have been replaced by rubber cones.

High Speed Bogie Locomotive with Frame-mounted Motors. Fig. 222 shows the type of locomotive which is now employed for express passenger service on the 1,500-V main lines of the *French National Railways* (S.W. region). The locomotive (reference No. 4) weighs 102 tons and has a 1-hour rating of 4,600 h.p. The maximum speed in normal service is 110 m.p.h. and a speed of 125 m.p.h. has been obtained in test runs.*

The mechanical design embodies several novel and new features to ensure smooth running and stability at high speeds. Thus (i) the body is connected to each bogie by a pair of swinging pivots with transverse (spring) restraint, (ii) the axle boxes are articulated to the bogie frames, (iii) the central axle has a limited amount of end-play under the control of springs.

The arrangement of the swinging pivots on a bogie is shown in Fig. 223 and

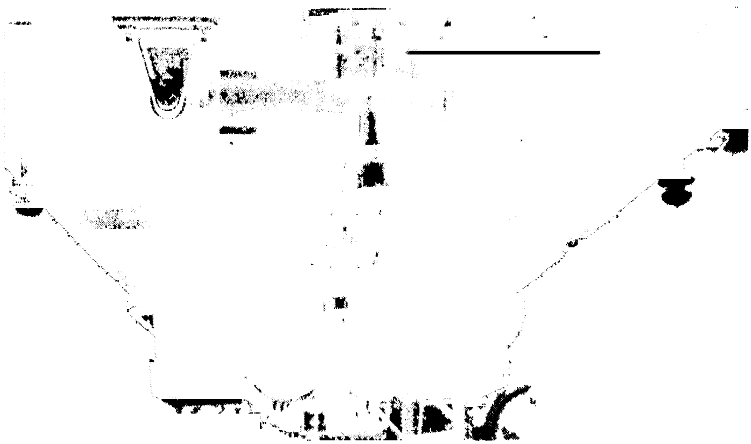


FIG. 224 VIEW OF UNDERFRAME (REMOVED FROM BOGIE) SHOWING SWINGING PIVOTS (ALSTHOM)

a view of the pivots and transverse telescopic links (in which the control springs are fitted) is shown in Fig. 224.

The pivots are carried in cross bearers between the traction motors. This arrangement not only simplifies the design of the bogie compared with that having a centre bearing but also gives support to the body at a lower height than would otherwise be possible.

Lateral support of the body is provided at two points on each bogie by links and springs, as shown in Fig. 223.

When transverse, rotary or longitudinal forces occur between the body and the bogies the pivots are displaced from their normal (vertical) position, transverse movements being controlled by the springs in the telescopic links and longitudinal movements being restricted by the side-play in these links and connexions. Rotary movement of a bogie relative to the body causes the pivots to take up equal inclinations in opposite directions, and the control springs produce a strong restoring couple. This feature, together with the articulation of the axle-boxes, has given the locomotive remarkable riding

* Performance data are given in *Engineering*, Vol. 174, p. 654. See also *Bulletin of the International Railway Congress Association*, *Vol.* 1, p. 173.

qualities at high speeds, and tests have shown that the lateral forces on the track rails are of relatively small magnitude.

The suspension springs are underslung and, with rubber springs in series, are connected by equalizing levers. At the extreme ends spiral springs, in addition to the rubber springs, are fitted.

Each axle is driven by a 765-h.p., 750-V series motor with compensating windings, the drive being through twin resilient gearing to a quill and link coupling of the type illustrated in Fig. 207.

The motor equipment is arranged for double series-parallel control with a total of 40 starting steps (18 series, 12 series-parallel, 10 parallel). In addition five weak-field steps (by inductive shunting) are provided, so that a total of

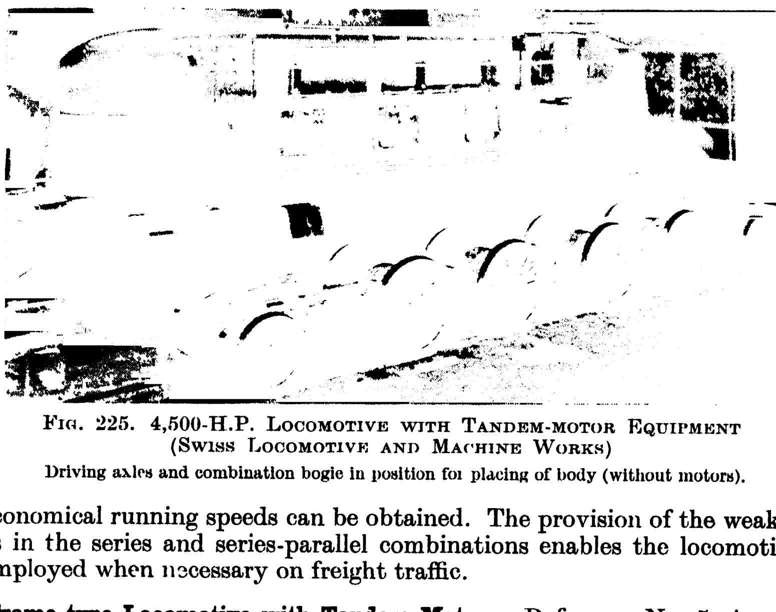


FIG. 225. 4,500-H.P. LOCOMOTIVE WITH TANDEM-MOTOR EQUIPMENT (SWISS LOCOMOTIVE AND MACHINE WORKS)

Driving axles and combination bogie in position for placing of body (without motors).

18 economical running speeds can be obtained. The provision of the weak-field steps in the series and series-parallel combinations enables the locomotive to be employed when necessary on freight traffic.

Frame-type Locomotive with Tandem Motors. Reference No. 5 gives data of a mixed-traffic locomotive (class $1D_01$) in service on the *Nederlands Railways.* The 1-hour rating is 4,500 h.p. and the maximum speed is 100 m.p.h.

The mechanical design incorporates the combination bogie (with a driving axle and a guiding axle as shown in Fig. 225) developed by the Swiss Locomotive Works. The two central driving axles have a rigid wheel-base in the main frame, and the bogies are connected thereto by swivelling centres, the buffing- and draw-gear being fitted to the main frame. Such an arrangement gives easier running on curves and less wear on the rails than the alternative arrangement with all driving axles in the main frame and a guiding bogie at each end.

The main frame is built up of plates and rolled sections, and the sides of the body are rigidly fixed to it to give additional stiffness. This structure is supported on the two central axles by laminated springs with equalizing levers, and on the bogies by circular pads resting on spiral springs.

Provision is made, as shown diagrammatically in Fig. 226, for counteracting the unloading effect on the leading driving axle when starting with high

tractive effort; the front end of the bogie being pulled upwards by a system of levers actuated by compressed air in a cylinder.

The eight motors (which are arranged in pairs above each driving axle) are mounted in the body of the locomotive, and are inserted and removed through openings in the side walls, a portable platform being employed for these operations. Each pair of motors drive, through double-reduction gearing, a gear-wheel centrally located on a quill between the driving wheels as shown in Fig. 208, the final transmission from gear-wheel to axle being by springs and a flexible coupling.

II. Single-phase Locomotives

($16\frac{2}{3}$ c/s, 25 c/s, 50 c/s)

Modern designs of single-phase locomotives for passenger and freight services

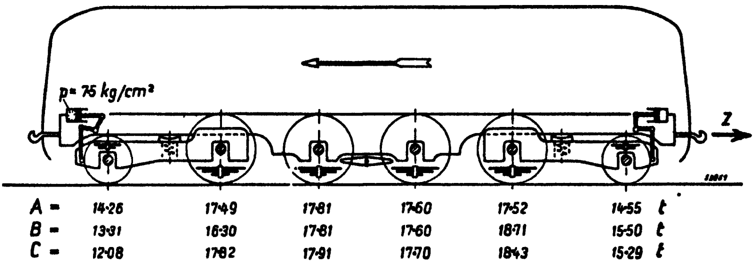


Fig. 226. Axle Loadings with and without Equalizing Device

A, normal axle loadings (metric tons), locomotive stationary; *B*, axle loadings with draw-bar pull of 18,260 kg *without* equalizing device; *C*, axle loadings with draw-bar pull of 18,260 kg *with* equalizing device.

are invariably of the individual-axle type, but many locomotives of older design with collective drives are in operation.

Compared with high-voltage d.c. locomotives a single-phase locomotive has much simpler control and auxiliary equipment, and the interior of the body shows much less congestion and complication, due to the low voltage of the motors and the absence of rheostats.

Bogie Locomotives with Frame-mounted Motors. A good example of a passenger locomotive of medium power is the present class B_0B_0 56-ton locomotive (series Re 4/4) of the *Swiss Federal Railways*, of which an outline of the general arrangement and the layout of the interior is shown in Fig. 227. This locomotive, reference No. 6, has a 1-hour rating of 2,300 h.p.* and is designed for a maximum speed of about 80 m.p.h. The three leading Swiss electrical works (Brown-Boveri, Oerlikon, Sécheron) and the Swiss Locomotive Works collaborated in the design and shared the constructional work of the many locomotives now in service.

The bogie is shown in Fig. 228. The bolster is supported by laminated springs suspended by links on the outside of the frame, the links having a limited freedom of both lateral and longitudinal movement. The bogie frame is supported on side extensions of the axle boxes by spiral springs, inside of which

* When comparing the h.p. ratings of single-phase and d.c. locomotives it is important to realize that in the former case the 1-hour rating may correspond to a speed of about 60 per cent of the maximum, whereas in the latter case the speed corresponding to the 1-hour rating may be only 40 per cent of the maximum service speed.

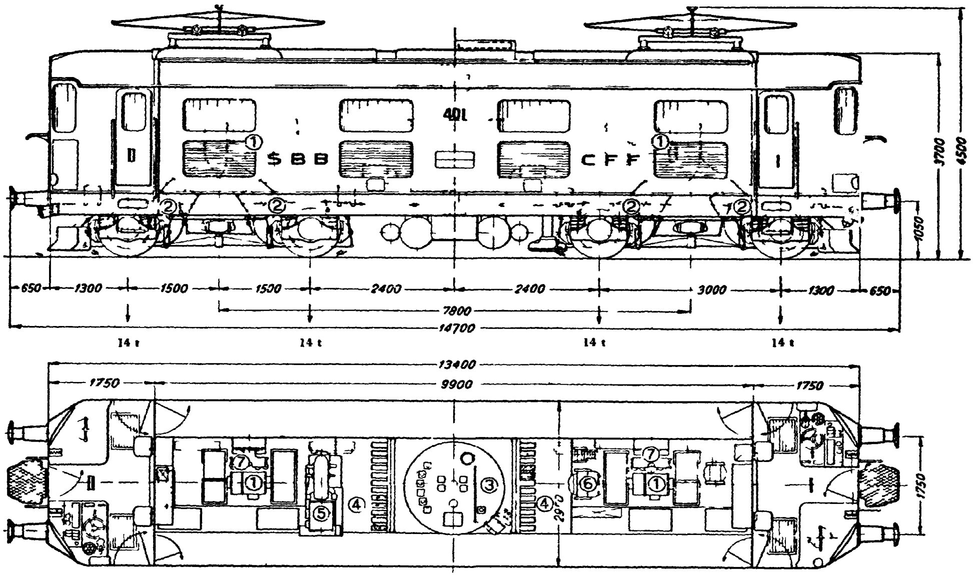


FIG 227. GENERAL ARRANGEMENT OF THE SERIES RE 4/4 LOCOMOTIVE OF THE SWISS FEDERAL RAILWAYS (15 kV, $16\frac{2}{3}$ C S)

Index to equipment—1, blowers for traction motors 2 traction motors 3 transformers 4 contactors 5, air compressor 6 reactor for regenerative braking 7 reversers

are oil-filled cylindrical guides, which also act as dashpots to control vertical oscillations. Provision is made for counteracting the unloading effect on the

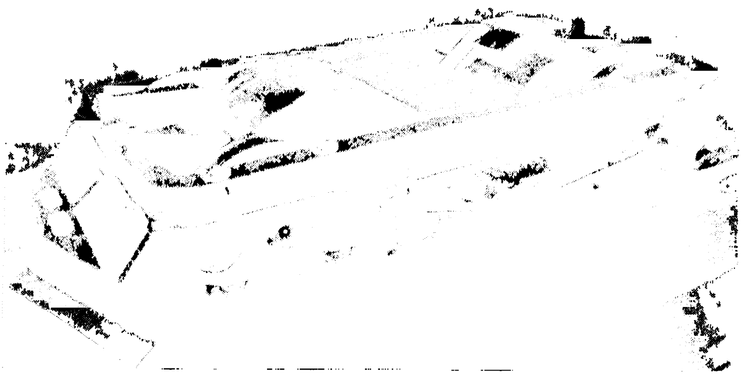


Fig 228 Bogie of Re 4/4 Locomotive with Motors in Position (Brown Boveri)

Fig 229. Connexions between Transformer and One Group of Contactors on Re 4/4 Locomotive (Oerlikon)

leading axle when starting with high tractive-effort, pressure being applied between the underframe and bogie by an air cylinder.

The motors have a 1-hour rating of 575 h p. at 415 V, 16⅔ c/s, and the drive is by springs enclosed in the gear-wheel, Fig. 206. The four motors are permanently connected in parallel and supplied from a 1,660-kVA transformer which has 12 tappings on the secondary winding for control purposes, and additional tappings for the auxiliary supplies and train heating. Provision is made for regenerative braking according to the exciter-motor scheme of Fig. 153.

Individual electro-pneumatic contactors (Fig. 134) are employed for control.

FIG 230. PARTIALLY-ASSEMBLED BOGIE SHOWING DISCS, GEAR CASE, ROLLER BEARINGS, ETC (BROWN BOVERI)

The 24 contactors for starting and speed control (according to the scheme of Fig. 131) are arranged in two equal groups and mounted on each side of the transformer, so that only short connexions are necessary between the tappings and contactors (Fig. 229). The transformer is shown in Fig. 142.

The main circuit-breaker is of the air-blast type.

Heavier 16⅔-c/s locomotives (class B_0B_0, 80 tons, 4,000 h p. and class C_0C_0, 120 tons, 6,000 h.p.) have recently been developed by Brown-Boveri, in conjunction with the Swiss Locomotive Works, for mountain routes (*Lotschberg–Simplon* and *Gotthard*). The B_0B_0 locomotives have disc drives, Fig. 210, and the C_0C_0 locomotives have spring drives, Fig. 206. In both cases high-voltage tap-changers are employed for speed control. Data of the B_0B_0 locomotives are given in reference No. 7.

Fig. 230 shows a partially assembled bogie with disc drive in which one of the discs, C, and the driving arm, B, attached to the pinion stub-shaft, are visible. The roller bearings supporting the gear case on the axle are shown at A.

Heavy Freight Multiple-unit Bogie Locomotive. This locomotive [class B_0B_0, 107 tons, 2,500 h.p. (one unit), reference No. 8] is in service on the *Pennsylvania Railroad*, and is operated either singly or in combination with other similar units up to a maximum of four units.

The trucks are of the swing-bolster M.C.B. type (Fig. 191) and each is equipped with two nose-suspended motors, with single gearing and resilient gear wheels, each motor being rated at 625 h.p. (cont.), 230 volts, 25 c/s, the locomotive speed being 26·5 m.p.h. High short-time overload ratings are obtainable at higher voltages (e.g. a peak of 1,300 h.p. at 33 m.p.h. at the end of the accelerating period, and a short-time output of 1,060 h.p. at 41·5 m.p.h. for hauling heavy trains over ruling gradients). The motors are of the normal compensated-series type without resistive connexions in the armature.

To reduce the circulating currents at starting and at low speeds up to 7 m.p.h. two steps of field shunting are employed the initial step (speed range 0–3 m.p.h.) reducing the field current to 40 per cent of the full-field current. Three values of commutating-pole shunting are employed to obtain good commutation over the range of operating speeds, the initial shunt being non-inductive and the subsequent shunts being inductive (combination of resistance and reactance).

The transformer is rated at 2,655 kVA: it is immersed in Pyranol and cooled by forced ventilation. Tappings on the secondary winding, in conjunction with contactors connected according to the scheme of Fig. 131, give 21 operating voltages from 66 V to 696 V for the two groups of motors, each group consisting of two motors connected in series.

Rheostatic braking (for controlling the speed of trains descending gradients) is obtained by operating the traction motors as separately-excited d.c. generators loaded with fixed resistors, the braking torque being controlled, in 17 steps, by adjustment of the excitation of the exciter. Each traction motor can absorb 750 h.p. within the limits prescribed by heating and commutation.

The auxiliary motors, except those driving the blowers, are of the single-phase induction type with split-phase starting. The blower motors are of the two-speed, three-phase, induction type and are supplied from an auxiliary three-phase alternator.*

Locomotives with Collective Drive. Fig. 231 shows the general arrangement, and the layout of the equipment, of a class 1C1, 78-ton, 2,000-h.p., $16\frac{2}{3}$ c/s locomotive employed for mixed traffic on the *Swedish State Railways*.† The horizontal connecting rods are driven by a jack-shaft which is geared to a twin motor, twin gearing with resilient pinions being employed. The control gear is of the electro-magnetic contactor type, the contactors and reverser operating with single-phase current.

The main transformer (which also supplies 200-kVA for train heating) is oil cooled with forced oil circulation, the oil being cooled in a separate multi-tubular cooler.

Fig. 232 shows the general arrangement of a class 1C, 48-ton, 700-h.p., $16\frac{2}{3}$ c/s *shunting locomotive* with collective drive. The single motor is geared to the jack-shaft by twin gearing with resilient pinions in which slipping couplings

*Further details are given in a paper, by Messrs. Gowans, Widell, Bredenberg, on "A New Electric Locomotive for the Pennsylvania Railroad," *Trans. A.I.E.E.*, Vol. 71. p. 27.

†Locomotives with individual-axle drives are also in service for high-speed passenger and low- to medium-speed freight traffic.

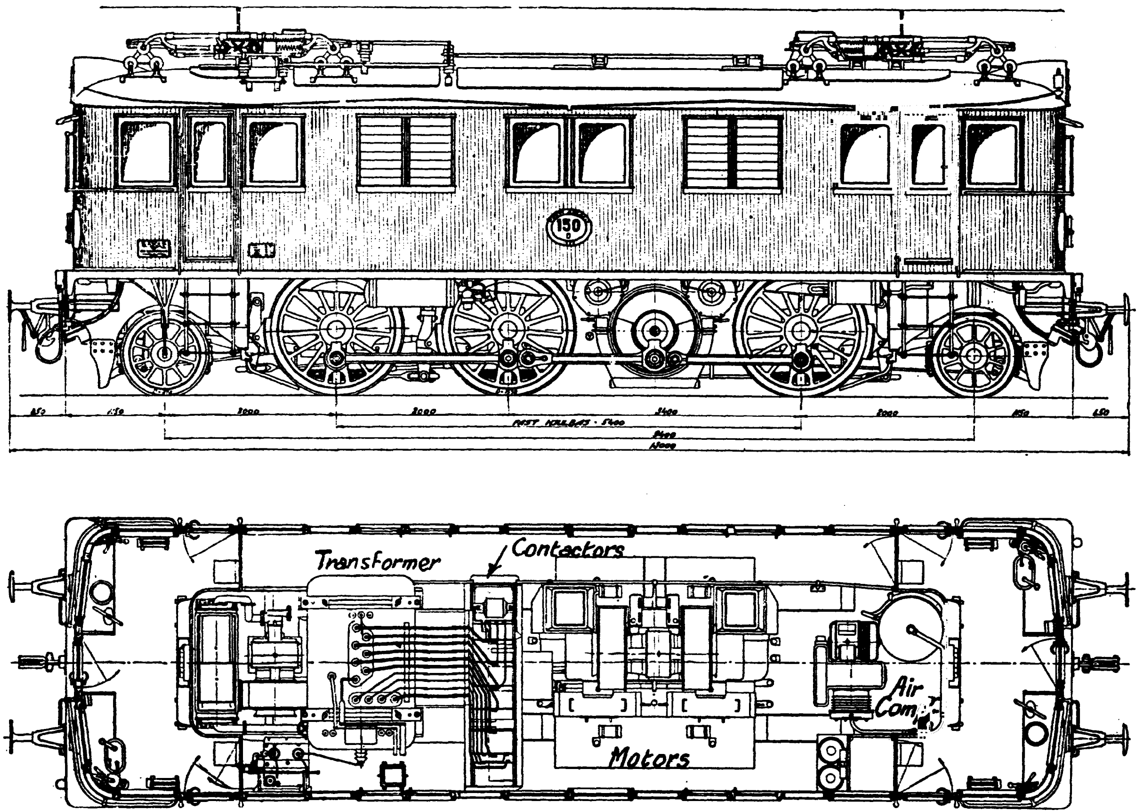


FIG. 231. FREIGHT AND PASSENGER LOCOMOTIVE (SWEDISH STATE RAILWAYS) WITH COLLECTIVE DRIVE (A.S.E.A.)

are incorporated to relieve the armature from impacts incidental to shunting service.

The motor is controlled by a 13-step motor-driven tapping switch which is

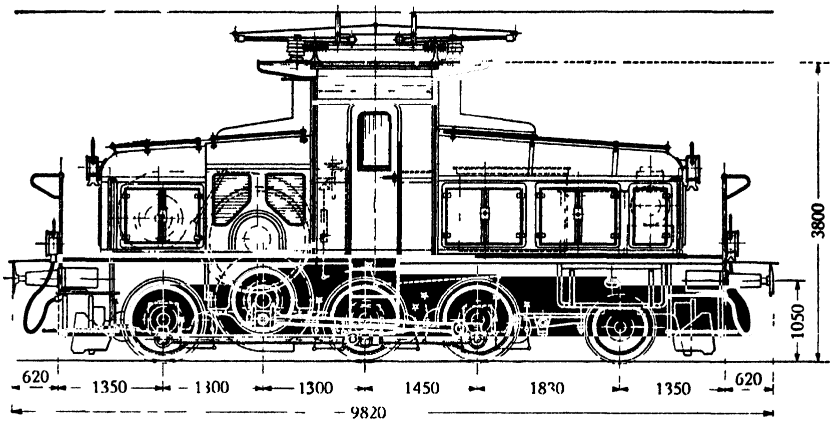


FIG. 232. SHUNTING LOCOMOTIVE (SWISS FEDERAL RAILWAYS) WITH COLLECTIVE DRIVE (BROWN BOVERI)

built on the oil-cooled transformer. The cab has two driving positions, and all controls are mechanically coupled so that the driver may commence a switching operation at one position and, if necessary, complete it at the other position.

Locomotives with Distributed Collective Drive. This drive is employed when the overall wheel-base required for the coupled driving axles is greater than

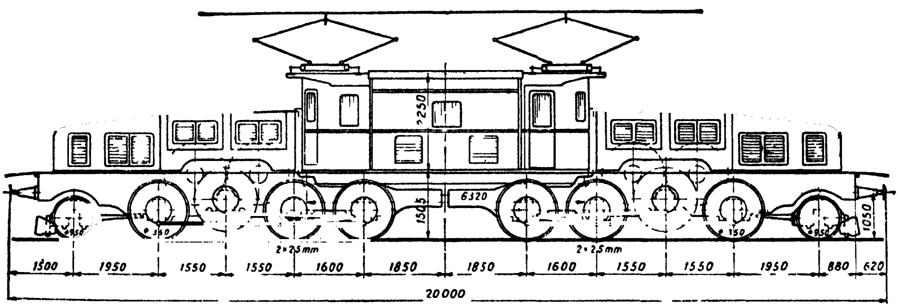


FIG. 233. HEAVY FREIGHT LOCOMOTIVE (SWISS FEDERAL RAILWAYS) (OERLIKON)

that which can be accommodated in a single frame. An example is shown in Fig. 233 which refers to a *heavy freight locomotive* (class 1C + C1, 130 tons, 3,060 h.p., 16$\frac{2}{3}$ c/s). The two articulated trucks each have three coupled driving axles and each jack-shaft is geared to a twin motor, a resilient pinion being fitted to each end of the armature shafts.

Bogie Locomotives with 50-c/s Series Motors. A locomotive (class C_0C_0, 108 tons, 4,300 h.p., reference No. 9) was built for the *French National Railways* to test the capabilites of an equipment with 50-c/s series motors for hauling passenger and freight trains (up to 600 and 1,350 tons respectively) over a hilly route with a maximum gradient of $2\frac{1}{2}$ per cent. Provision is made for regenerative braking (according to the scheme of Fig. 153) and for operation at low speeds over 1,500-V d.c. tracks at junction stations for the transfer of trains from the a.c. to the d.c. (1,500-V) system and vice versa. The simplest solution to the latter problem was the conversion of the 1,500-V d.c. input to 50-c/s single-phase

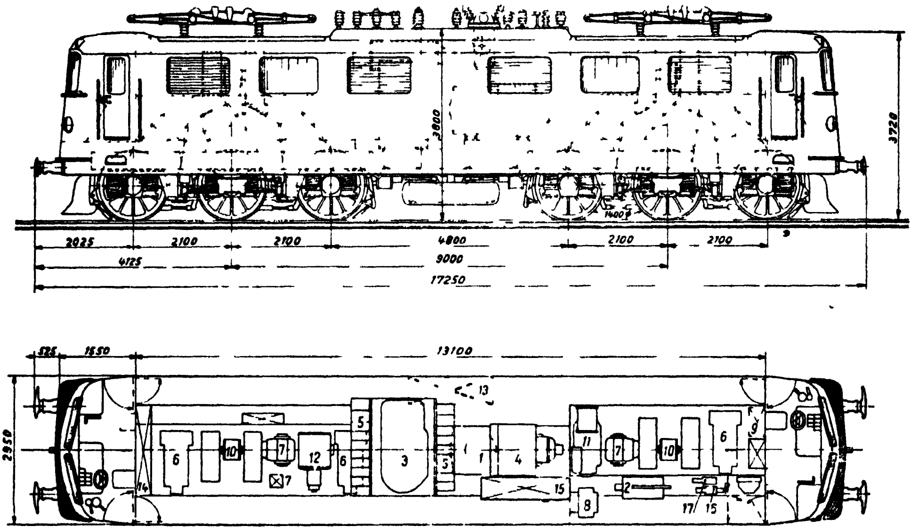


FIG. 234 GENERAL ARRANGEMENT AND LAYOUT OF EQUIPMENT ON 4,300 H.P., 50-C/S LOCOMOTIVE (OERLIKON)

Index to equipment—1, Main (h.v., a.c.) circuit breaker 2, d.c. circuit breaker, 3, transformer, 4, motor-alternator set, 5, main contactors, 6, reversers, 7, equipment for regenerative braking, 8, change-over switch (a.c. to d.c.), 9, 16, 17, auxiliary contactors, 10, blowers, 11, air compressor, 12, Arno converter 13, battery 14, relays, 15, non-inductive shunts for commutating-pole windings

power in a motor-alternator set, a rating of 400 kVA sufficing for the shunting duties of the locomotive. The general layout of the equipment is shown in Fig. 234.

The motors have been described in Chapter V. For normal running two motors are permanently connected in series, and the three pairs are connected in parallel and supplied from the tapped secondary winding of the transformer. Sixteen voltage steps are obtained with eight tappings, using the scheme shown in Fig. 131. Tap changing on the low-voltage side was employed to enable the output of the motor-alternator, when operating from the 1,500-V d.c. system, to be fed into a suitable tapping on the secondary winding. Without such dual operation tap changing on the high-voltage side would have been adopted.

The auxiliary motors are of the three-phase induction type and are supplied from a phase converter (Arno system).

A number of bogie locomotives (class B_0B_0, 73 tons, 1,660 h.p.) equipped with

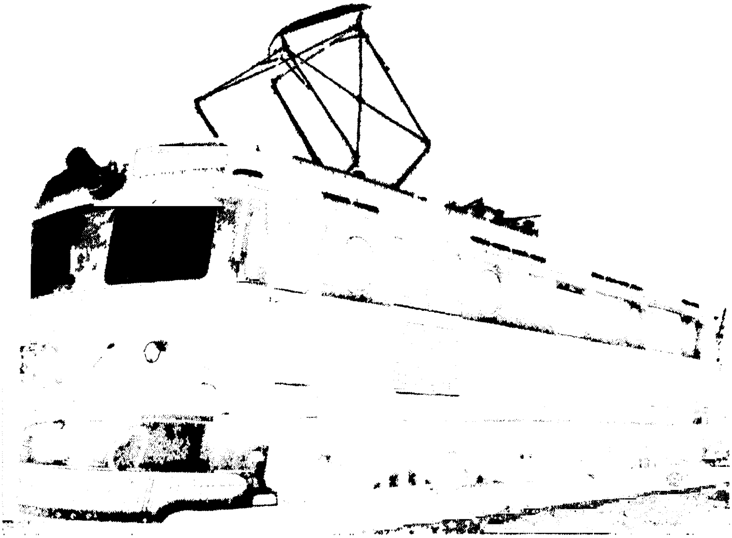


FIG 235A 2,660 H P, 22 kV LOCOMOTIVE WITH 50 c/s SERIES MOTOR EQUIPMENT, BELGIAN CONGO RAILWAYS (A C E C CHARLEROI)

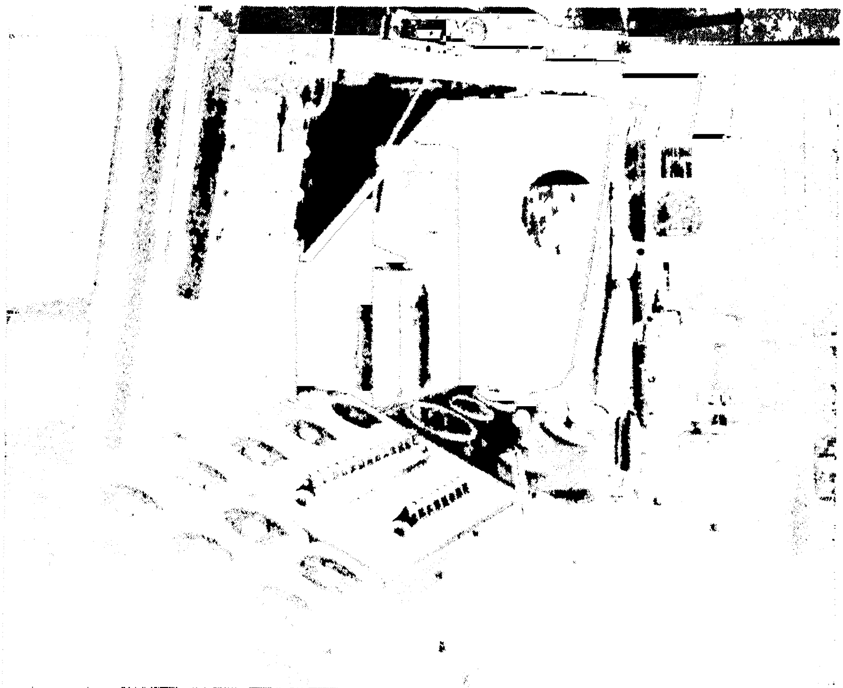


FIG. 235B DRIVING CAB OF LOCOMOTIVE OF FIG 235A (A C E C)

The nine instruments are arranged to measure the following—(i) current in each motor, (ii) exciter current (during braking), (iii) line voltage and current, (iv) battery voltage and current

50-c/s series motors are in service on the Katanga lines (3 ft 6 in. gauge) of the *Belgian Congo Railways*.* A view of a locomotive is shown in Fig. 235A, the driving cab is shown in Fig. 235B, and one of the bogies, with a twin motor being placed in position, is shown in Fig. 235C. The twin motors are mounted on the frames of the bogies and each is geared to a quill which is coupled to each driving wheel by six spiral springs.

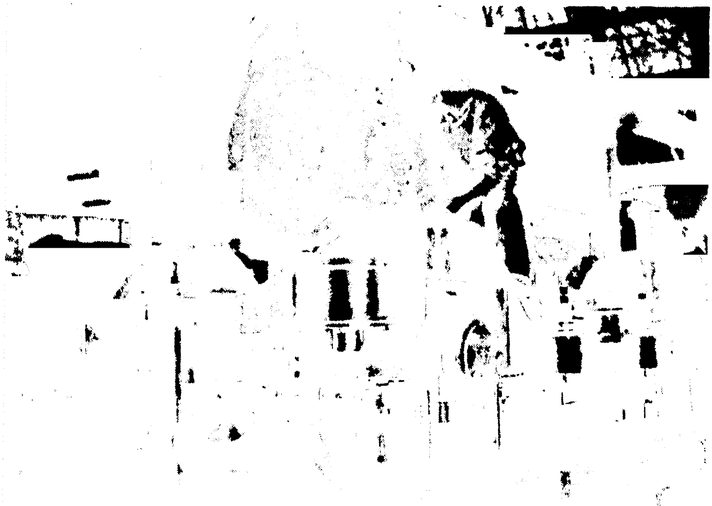


FIG. 235C. TWIN MOTOR BEING PLACED IN POSITION ON BOGIE. (A.C.E.C.)

Each of the individual motors of a twin motor is rated at 208 h.p., 240 V, the armature speed being 1,580 r.p.m. The maximum armature speed is 2,330 r.p.m. and the corresponding locomotive speed, with 1,150 mm (45·2-in.) wheels, is 43·5 m.p.h. The two motors are connected permanently in series, so that the full-load terminal voltage of a twin motor is 480 V, the corresponding current being 850 A, and the efficiency and power-factor being 82·5 per cent and 0·93 respectively. The armatures have high-resistance connexions between the coils and commutator segments. The stators have ten poles with excitation, compensating and shunted commutating-pole windings.

Starting and speed control is effected by a group of 16 contactors (Fig. 141A) according to the scheme of Fig. 131, the four twin motors being connected in parallel.

As the railway has a long gradient of 1·25 per cent provision is made for rheostatic braking. The excitation windings of all the motors are connected in series to a d.c. exciter, and the armature circuits of each twin motor are connected to separate loading rheostats.

All auxiliary motors are of the single-phase induction type.

III. PHASE-CONVERTER, MOTOR-GENERATOR AND RECTIFIER LOCOMOTIVES

On these locomotives the single-phase energy from the contact wire is converted

* Details of this electrification are given in *Bulletin of the International Railway Congress Association*, Vol. IV, p. 49. The locomotives and sub-stations were equipped by A.C.E.C. (Charleroi).

into either three-phase alternating current or direct current for use in the traction motors. The object of such conversion is to overcome the limitations of 25-c/s and 50-c/s single-phase series-motor equipments when heavy starting duty or large-scale regenerative braking are required.

Phase-converter Locomotives. Locomotives equipped with a *fixed-frequency* phase converter and change-pole motors were developed some years ago by the Westinghouse Corporation—for the Norfolk and Western, and the Virginian railways—and by Ganz & Co. for the Hungarian State Railways.

The operating conditions on the two American railways involved the haulage of heavy coal and freight trains over mountain ranges, regenerative braking being necessary on the down grades. The locomotives—class 2 (1B + B1), 135 tons, 6,000 h.p., 25 c/s—were equipped with eight 8/4 pole, three-phase, slip-ring induction motors, arranged in pairs, each pair being geared to a jack-shaft from which the wheels were driven by side-rods. The operating speeds were about 14 and 28 m.p.h.

Electrical operation on the Norfolk and Western lines has been abandoned, and locomotives recently acquired by the Virginian Railway have been of the motor-generator type to give greater flexibility in speed control.

On the Hungarian railways the single-phase trolley wire is supplied, through transformers, from a three-phase 50-c/s transmission network. The operating conditions involve the hauling of passenger and freight trains over about 120 miles of route with a long, but not severe, gradient.

The original (26) four-speed locomotives—class 1D1, 93 tons, 2,500 h.p., 50 c/s—were equipped with a synchronous phase converter and a single change-pole traction motor (Fig. 65), the coupled wheels being driven by inclined connecting rods.

The phase converter has an oil-cooled stator, and a water-cooled rotor. The single-phase primary winding is designed for the line voltage (16 kV), the secondary winding (which supplies the traction motor) is a three-six phase winding, and a light three-phase tertiary winding supplies the auxiliaries.

In recent *Ganz multi-frequency locomotives*—class B_0C_0, 83 tons, 3,200 h.p.—the phase converter is coupled to a frequency converter, and frequencies of 25, 75, 100, 125 c/s are obtained by changing the number of poles in the latter machine and reversing the direction of the rotating field. The scheme is shown diagrammatically in Fig. 236. The traction motors are of the slip-ring type with non-pole-changing windings. Five synchronous speeds are obtained corresponding to the frequencies 25, 50, 75, 100, 125 c/s. The higher frequencies and the individual-axle drives have enabled a considerable reduction to be made in the weight of the equipment compared with former locomotives in spite of the extra converting machine.

This multi-frequency scheme may be extended by employing a continuously variable frequency, and if the voltage applied to the motors is proportional to the frequency, the motors will operate with constant flux. In such a case squirrel-cage motors may be employed with a consequent reduction in weight. Developments along these lines are in progress in France.

Motor-generator Locomotives (a.c./d.c.). This type of locomotive is particularly suitable for heavy freight traffic on mountain grades on account of the flexibility and smoothness of speed control during both motor operation and regenerative braking. It is also particularly suitable for dual operation at full power on both a.c. and d.c. systems.

In each case the d.c. traction motors—connected either all in parallel or in series-parallel groups—are supplied at variable voltage from a separately-excited generator, or a group of generators, driven by an a.c. motor, the excitation being controlled by a rheostat in the field circuit of the exciter. For regenerative braking the field windings of the traction motors are separately excited (from another exciter) and the recuperated power is supplied to the generator (which now acts as a motor) and returned to the supply system via the a.c. side of the motor-generator. Control of the recuperated power is effected chiefly by adjustment of the excitation of the exciter.

Powerful locomotives are in operation on the Cascade section of the Great Northern Railway (U.S.A.) and the Virginian Railway.* A dual-operation

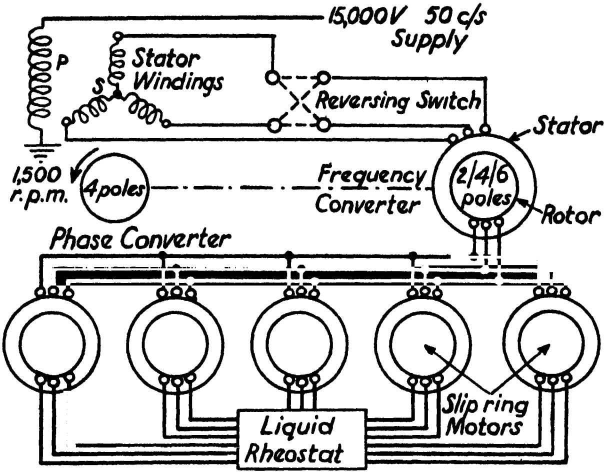


FIG. 236. GANZ MULTI-FREQUENCY SCHEME FOR SPLIT-PHASE LOCOMOTIVE

trial locomotive (20 kV, single-phase, 50 c/s and 1,500 V d.c.) has been built for the French National Railways.

The *Virginian* locomotives are of the double-unit articulated type and have a continuous rating at the rail of 6,800 h.p. at a speed of 15·75 m.p.h. The total weight (all adhesive) is 460 tons which is distributed between 16 axles, each of which is equipped with a nose-suspended motor. The eight motors of one unit are supplied by two generators driven by a 4,000-h.p. single-phase synchronous motor, each generator supplying four motors.

Rectifier Locomotives. Recent developments in the design and construction of mercury-arc rectifiers have enabled them to be adapted to locomotive (and also motor-coach) service. Two types are in use—one having all the anodes in a single tank, and the other having each anode in a separate tank (ignitron type).

The unsmoothed output current of a single-phase rectifier, however, if applied to normal d.c. traction motors would cause overheating and bad commutation, due to the pulsations of the flux. Hence either the 100-c/s ripple must be reduced (by means of a "low-pass" filter) to a value which is not

* For details of the design and equipment, see *Trans. A.S.M.E.* (1949); *General Electric Review*, Vol. 51 (June/July, 1948).

harmful (i.e. about 20 per cent, or less, of the fundamental), or the construction of the motor must be modified to allow the pulsations in the commutating flux to follow instantaneously those of the current, and the series field winding must be shunted non-inductively to bypass the a.c. component of the current and thereby obtain a non-pulsating flux.*

Locomotives incorporating these principles and different schemes of rectifier operation have been built by European and American manufacturers.†

Mixed Traffic Bogie Locomotive with Pumpless, Air-cooled, Single-tank Rectifiers. This locomotive (class B_0B_0, 78 tons, 2,840 h.p., 50 c/s, reference

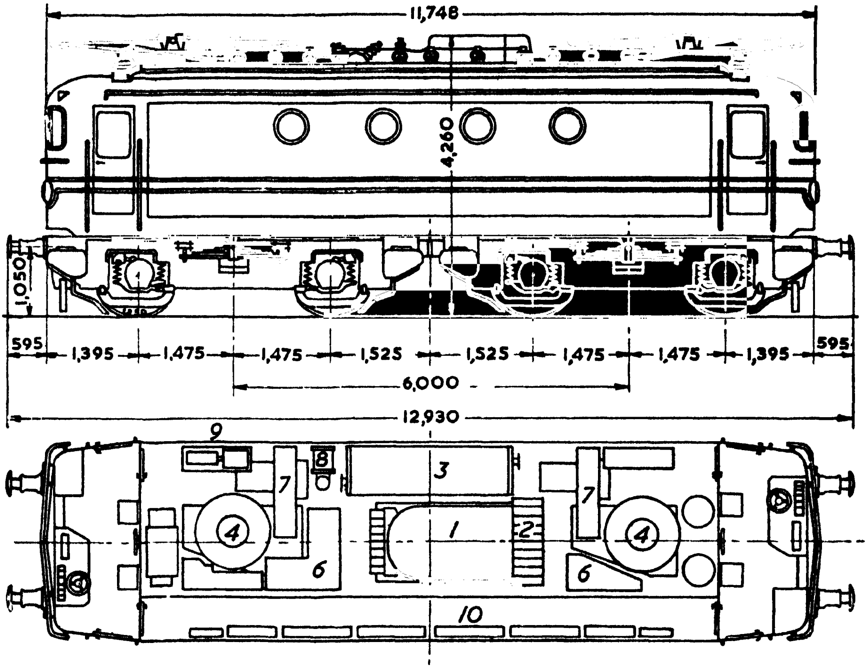


FIG. 237. GENERAL ARRANGEMENT AND LAYOUT OF RECTIFIER LOCOMOTIVE (ALSTHOM)

Index to equipment—1, transformer; 2, contactors; 3, oil cooler for transformer; 4, rectifiers; 5, air compressor; 6, control apparatus; 7, blowers for traction motors; 8, oil pump (for transformer), 9, auxiliary equipment for rectifiers; 10, battery (for control circuits).

No. 10) built by Alsthom is in service on the 20-kV, 50-c/s line of the French National Railways. The leading dimensions and the layout of the equipment in the cab are shown in Fig. 237, from which it will be observed that there are two rectifiers and a common transformer.

* Details of researches into the operation and performance of series motors fed with pulsating current are given in the following papers by Monsieur M. Blondet—"Quelques problémes particuliers aux locomotives à redresseurs," *Bulletin, Soc. de la Française des Électriciens*, 7th series, Vol. I, (1951), p. 563; "Le moteur à courant ondulé," *Revue Électrique et Mecanique*, No. 86 (1951), p. 1.

† Details of American locomotives are given in a paper, by Messrs. Whittaker and Hutchison on "Pennsylvania Railroad Ignitron Rectifier Locomotive." *Trans. A.I.E.E.*, Vol. 71, p. 37.

Fig. 238 shows one of the rectifiers which is rated at 830 A, 1,350 V continuously and 1,200 A for 10 min. The six main anodes are connected in two groups, each with three anodes in parallel, to form a single-phase full-wave rectifier. The anode groups are supplied with variable voltage according to the scheme shown in Fig. 239. The voltage applied to each anode is the algebraic sum of a constant portion (due to the secondary windings S_1, S_2 of the main

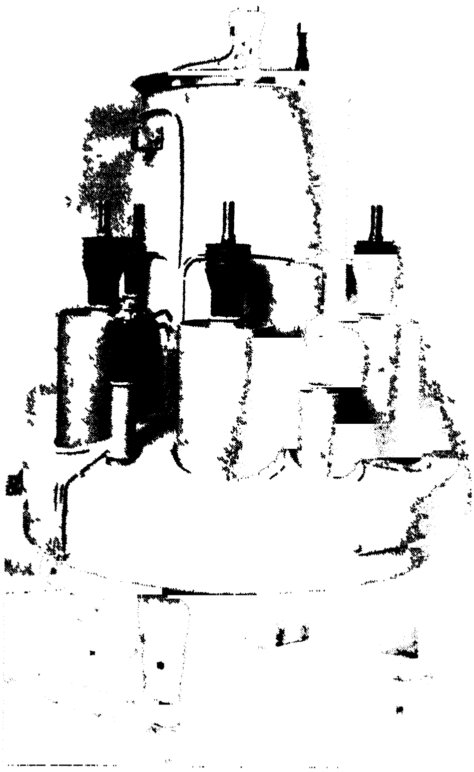


FIG. 238 MULTI-ANODE RECTIFIER FOR LOCOMOTIVE (ALSTHOM)

transformer) and a variable portion derived from the tapped winding S_3, the polarity of which may be reversed relatively to the e.m.f's. of S_1 and S_2. An auto-transformer, S_4, provides the centre tap for the rectifier circuit.

The excitation anodes are supplied with direct current. Another pair of auxiliary anodes, fed from an auxiliary transformer, supply rectified current at 175 V for the motor driving the air compressor.

The two motors on each bogie are supplied by separate rectifiers. Each motor has a 1-hour rating of 710 h.p. at 675 V, the corresponding locomotive speed being 36·4 m.p.h. Nose suspension and resilient gear-wheels are employed. The electrical design incorporates the features previously mentioned for pulsating-current operation, and the series-field windings are permanently shunted non-inductively. An iron-cored reactor is connected in series with

each pair of motors, as shown in Fig. 239, to reduce the current ripple to a value which can be tolerated in so far as the performance of the motor is concerned.

IV. Three-phase Locomotives

Three-phase locomotives for operating on low-frequency, medium-voltage systems are characterized by their extreme simplicity, owing to the absence of transformers and tap-changing gear. Typical freight and passenger locomotives in service on the *Italian State Railways* are illustrated in Fig. 240.

The *freight locomotive* (Class E, 3,300 V) is equipped with two 1,200 h.p. motors, which are controlled on the cascade-parallel system, the synchronous speeds—corresponding to normal frequency ($16\frac{2}{3}$ c/s)—being 15·5 m.p.h. and

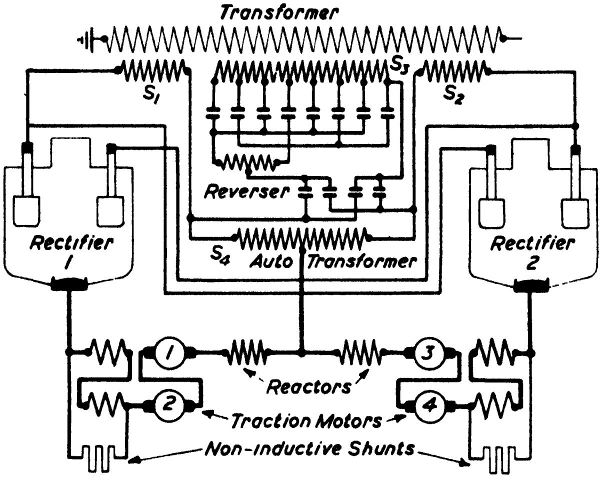


Fig. 239. Scheme of Connexions for Rectifier Locomotive (Alsthom)

31 m.p.h. A large number of these locomotives are in service on the Giovi-Genoa lines, on which the gradients are long and heavy, the maximum gradient being 3·5 per cent. The total weight of the locomotive is 76 tons, all of which is on the driving wheels. Two locomotives (one hauling and the other pushing) are capable of handling freight trains weighing 330 tons (excluding the locomotives) over the Giovi lines at a speed of 30·3 m.p.h. up the gradients. On the return journey (down the gradients) two locomotives are coupled together at the front of the train, and the gradients are descended at a speed of about 33 m.p.h. with the motors acting as induction (asynchronous) generators, from 600 to 800 kW being returned to the supply system by each locomotive. The performance of these freight locomotives has been highly satisfactory, and their adoption on the Giovi lines has enabled the capacity of these lines under steam conditions to be nearly trebled, this increase in capacity being due to the higher speeds and the heavier trains.*

The *passenger locomotive* (class 1D1, 3,300 V) is equipped with two 1,300 h.p. three-speed induction motors, which are controlled on the changeable-pole

* For interesting data relating to the Giovi lines and the performance of the electric locomotives, see *The Engineer*, Vol. 117, pp. 89, 115, 143, 194. See also a paper, by G. Bianchi, on "Some Data concerning Railway Electrification in Italy," *Journ. I.E.E.*, Vol. 69, p. 325.

cascade-parallel system. The four synchronous speeds—corresponding to a frequency of $16\frac{2}{3}$ c/s are—23, 32, 46, and 64 m.p.h. The corresponding tractive

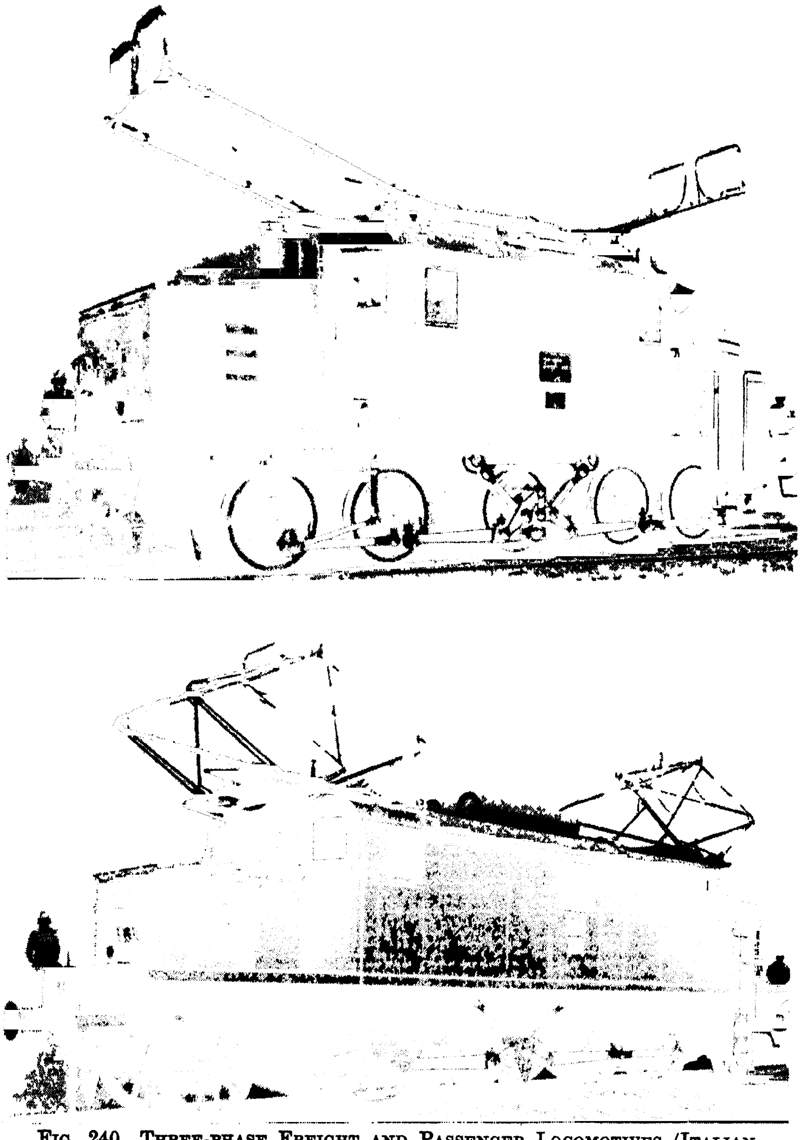


FIG. 240. THREE-PHASE FREIGHT AND PASSENGER LOCOMOTIVES (ITALIAN STATE RAILWAYS) WITH BIANCHI DRIVE (BROWN BOVERI AND BREDA)

efforts (continuous ratings) are 18,740, 26,455, 26,455 and 15,430 lb. The weight of the locomotive is 92 tons of which 64 tons is adhesive.

TABLE VIIA

DATA OF D.C. ELECTRIC LOCOMOTIVES—4′ 8½″ GAUGE

Reference No.	1	2	3	4	5
Railway	British	British	Italian	French	Nederlands
Class*	$C_o C_o$	$B_o + B_o$	$B_o B_o$	$C_o C_o$	$1 D_o 1$‡
Length over body	55′ 6″	47′ 0″	47′ 2″	57′ 10″	49′ 2″
Width overall	8′ 5″	9′ 0″	8′ 7″	9′ 9″	9′ 8″
Height over collector (min)	12′ 8″	13′ 9″	14′ 6″	13′ 9″	15′ 3″
Total weight (tons)	105	89	72	102	100
Adhesive weight (tons)	105	89	72	102	72
Total wheel base	44′ 6″	35′ 0″	34′ 1″	47′ 3″	39′ 0″
Bogie wheel base	16′ 0″	11′ 6″	10′ 4″	15′ 11″	8′ 0½″
Diameter of driving wheels	42″	50″	49¼″	49¼″	61″
Diameter of pony wheels	—	—	—	—	43½″
Distribution voltage	660	1,500	3,000	1,500	1,500
Normal motor voltage	400	750	1,500	750	750
Number of motors	6	4	4	6	8
H.p. of each motor (1 hr)	245	465	530	765	560
Type of geared drive	single direct	single direct	single quill	twin quill	S.L.M.
Gear ratio	3·83	4·12	4·06	2·606	3·56
Tractive effort at 1-hr rating (lb)	19,500	15,400†	23,700	35,000	25,140
Speed at 1-hr rating (m.p.h.)	28·5	45†	33·5	43·5	63·4
Maximum speed (m.p.h.)	75	65	90	125	100
Number of running speeds	26	6		18	15

* Initial letters A, B, C, D, E denote 1, 2, 3, 4, 5 *coupled* driving axles, respectively, in a truck. The subscript $_o$ denotes that the driving axles of a truck are not coupled. Numerals denote the number of pony axles. The plus (+) sign denotes that trucks are connected together by a link or hinge.

† Weak field. ‡ Manufacturer's classification. Alternative classification $1AB_oA1$.

TABLE VIIB

DATA OF SINGLE-PHASE ELECTRIC LOCOMOTIVES—4′ 8½″ GAUGE

Reference No.	6	7	8	9	10
Railway	Swiss Federal	Lotschberg	Pennsylvania	French	French
Class	$B_o B_o$	$B_o B_o$	$B_o B_o$	$C_o C_o$	$B_o B_o$
Length over body	44′	45′ 10″	52′	53′	38′ 9″
Width over all	8′ 10″	8′ 10″	10′	9′ 10″	10′
Height over collector (min)	14′ 9″	14′ 9″	15′	14′ 3″	14′ 6″
Total weight (tons)	56	80	107	104	78
Total wheel base	35′ 6″	37′ 8″	39′ 10″	43′ 3″	29′ 4″
Bogie wheel base	9′	10′ 8″	11′	13′ 9″	9′ 8″
Diameter of driving wheels	41″	49¼″	48″	55″	49¼″
Distribution voltage	15,000	15,000	11,000	20,000	20,000
Frequency	16⅔	16⅔	25	50	50
Normal motor voltage	415	395	230	230	675*
Number of motors	4	4	4	6	4
H.p. of each motor (1 hr)	575	1,000	650	720	710
Type of geared drive	spring quill	cardan disc	direct	spring quill	direct
Gear ratio	2·85	2·22	3·95	2·68	4·375
Tractive effort at 1-hr rating (lb)	17,600	31,300	38,400	38,350	28,400
Speed at 1-hr rating (m.p.h.)	46·7	46·7	25·5	40·4	36·4
Maximum speed (m.p.h.)	78	78	65	62	65
Number of voltage steps	24	28	21	16	29

* D.C. motors fed from rectifiers.

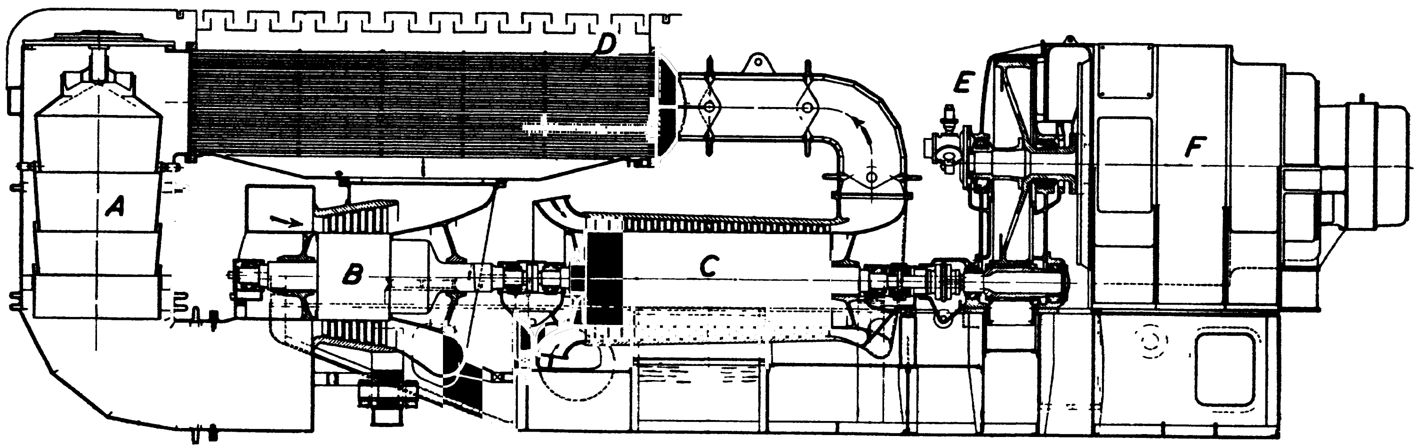


FIG. 241. SECTIONAL VIEW OF ARRANGEMENT OF GAS TURBINE AND GENERATOR (BROWN-BOVERI)

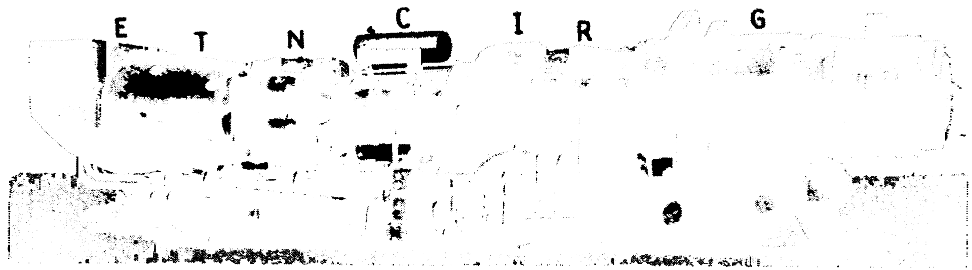


FIG. 242A. METROPOLITAN-VICKERS GAS TURBINE UNIT FOR LOCOMOTIVE

E, exhaust; *T*, turbine; *N*, fuel injection nozzles; *C*, compressor; *I*, air intake for compressor; *R*, gear box; *G*, generators.

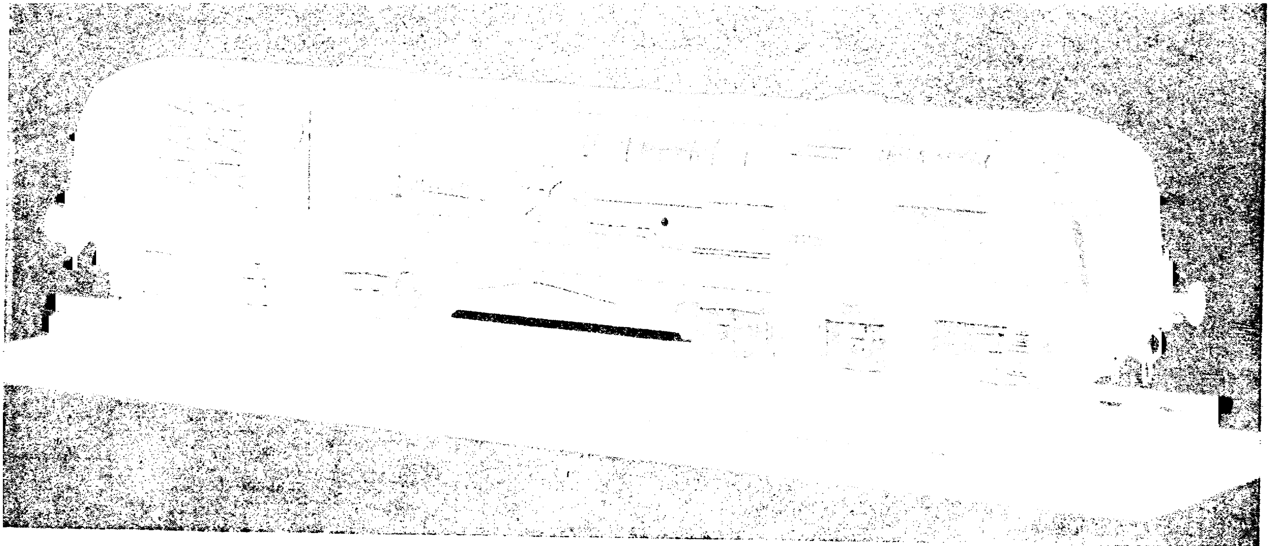


FIG. 242B. LAYOUT OF POWER PLANT ON BRITISH RAILWAYS' GAS TURBO-ELECTRIC TRIAL LOCOMOTIVE NO. 18100 (METROPOLITAN-VICKERS EQUIPMENT)

The view shows a cut-away scale model of the actual locomotive. The compartments on each side of the central power-unit (Fig. 242A) contain the auxiliary equipment which comprises storage battery (48 lead-acid cells, 384 Ah), oil-fired boiler for train heating, two blowers for ventilating the traction motors, oil cooler, air reservoir, air compressor and exhausters, control apparatus. The storage tanks for the oil fuel and water are fixed to the underframe between the bogies.

Chief Data of Locomotive—class C_6 C_6; 130 tons; 2,500 h.p. (continuous, at rail, for speed range 40–60 m.p.h.); max. speed 90 m.p.h.; max. tractive effort 60,000 lb; gas turbine 3,000 h.p. (net) at 7,000 r.p.m.; main generators (3), 1,100 A, 660 V, 1,600 r.p.m.; auxiliary generator 65 kW, 110 V; exciter 10·5 kW, 55 V; traction motors (6, with resilient nose suspension), 550 A, 666 V (max. 1,100 A, 825 V); gear ratio 58 : 21 (resilient gear wheels); wheel diameter 44 in.

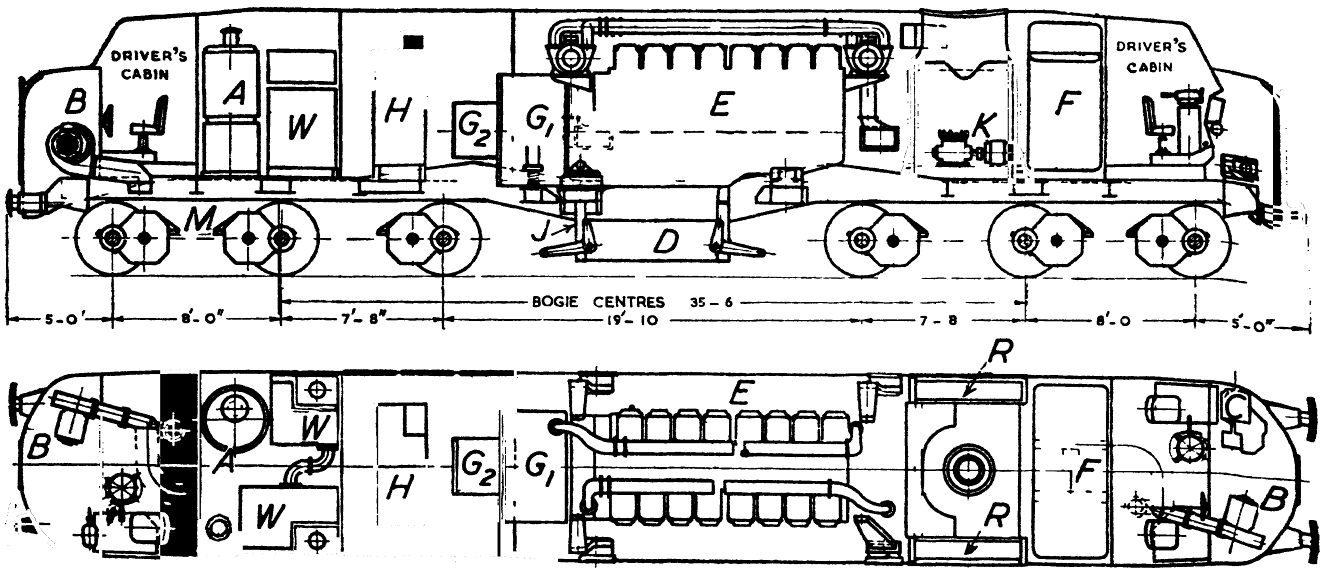


FIG 243 LAYOUT OF PLANT AND EQUIPMENT ON MAIN LINE DIESEL ELECTRIC LOCOMOTIVE (BRITISH RAILWAYS)

A, oil fired boiler for train heating *B* blowers for traction motors *C* air compressor *D* battery *E* engine *F* fuel tank G_1 G_2 main and auxiliary generators *H* control gear cubicle *J* vacuum brake cylinder *K* exhauster *M* traction motors *R* cooler for engine circulating water *W* water storage tank

V. Self-propelled (Thermo-electric) Locomotives

In this category are included locomotives with electric driving motors supplied from a power plant on the locomotive itself, the prime mover being either a steam or gas turbine or an internal combustion engine.

The steam turbo-electric locomotive is only suitable for large powers, and although locomotives have been built there is little prospect of further developments because of the future possibilities of the gas turbo-electric locomotive and the extended application of the Diesel-electric locomotive, in the multiple-unit form, to large powers.

The gas turbo-electric locomotive is in the development stage, trial locomotives having been built in Switzerland, America and Great Britain. These single-unit locomotives are of high power (2,000 to 4,500 h.p.) and are designed for long-distance passenger and freight traffic.* The highest ratings have a lower weight and a shorter overall length than a Diesel-electric locomotive of equivalent output.

Principle of Gas Turbo-electric Locomotive. The power plant, Figs. 241, 242, consists of a gas turbine, *B*, Fig. 241, running at a speed of about 7,000 r.p.m., coupled to an axial-flow compressor, *C*, and a speed-reducing gear, *E*, from which one or more variable-voltage d.c. generators, *F*, are driven, the output of which is utilized by the traction motors, four or more being employed, each with an individual-axle drive.

After being started and brought up to about one-third of normal speed by an auxiliary motor, the turbine operates on the open-cycle system in the following manner. Low-grade fuel oil and hot air from the compressor (at a pressure of 4 to 5 atmospheres) in the correct proportions are ignited in an ignition chamber (forming part of the combustion chamber). The gas so formed is mixed with a considerably larger volume of compressed air and supplied to the turbine, the inlet temperature being about 600 to 700°C. Although at full load the compressor absorbs about 65 per cent of the power developed by the turbine the thermal efficiency of the turbine-unit and gearing is of the order of 20 per cent.

As the turbine, in common with other internal combustion engines, is best suited for constant output conditions—the torque having a definite limiting value—the control system must provide for the regulation of the generator output to a constant value under normal running conditions. The governor of the turbine therefore tends to maintain the speed constant by regulating the fuel supply, and the generator is designed with a drooping volt-ampere characteristic in conjunction with a field rheostat, servo-operated from the governor gear.

Diesel-electric Main-line Locomotives. Loading-gauge and track restrictions usually limit the size of power-unit (i.e. engine and generators) which can be installed on a locomotive to about 1,600 h.p. (continuous rating), and when more power is required two or more locomotive-units are employed. In North America locomotive-units of 1,600 h.p. have been standardized and are employed extensively for passenger and freight traffic, usually with two, three or four

* Technical information is given in articles and papers in—*Engineering*, Vol. 173, pp. 161, 193; *Engineer*, Vol. 189, p. 608; Vol. 193, pp. 176, 237, 283; *Trans. A.I.E.E.*, Vol. 69, pp. 504–18; *Brown Boveri Review*, Vol. 32, p. 353; Vol. 33, pp. 263, 270; *Metropolitan-Vickers Gazette*, Vol. 24 (1952), p. 187.

units close-coupled and controlled from one driving position. The overall length of one 1,600-h.p. unit is about 53 ft and the weight is about 107 tons.

The power unit of such a locomotive consists of a twelve-cylinder (V formation) pressure-charged, four-stroke-cycle engine, with a normal speed of about 1,000 r.p.m., direct coupled to two d.c. generators—main (variable voltage) and auxiliary (constant voltage). The main generator supplies the traction motors and the auxiliary generator supplies the control and auxiliary circuits.

British-built 1,600-h.p. locomotives have a sixteen-cylinder engine with a normal speed of 750 r.p.m. The layout of the plant on a locomotive is shown in Fig. 243.

Control System. With main-line locomotives the full power of the engine may be required for a large portion of the running period. As the engine has a

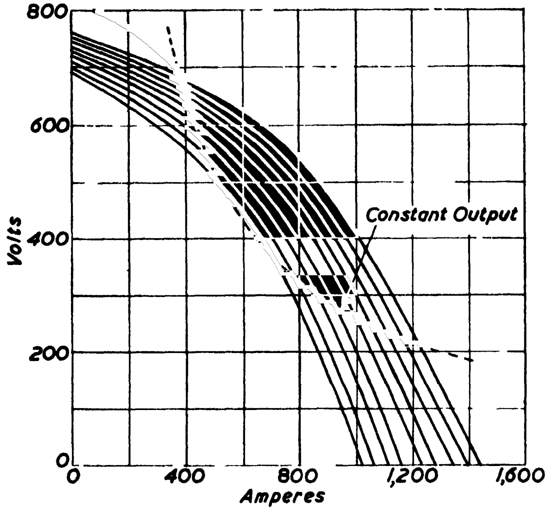


FIG. 244. VOLT-AMPERE CHARACTERISTICS OF GENERATOR WITH SELF-EXCITED AND SEPARATELY-EXCITED DIFFERENTIAL FIELD WINDINGS

Each load characteristic corresponds to a definite (constant) exciting current in the separately-excited field winding. The stepped curve shows how a constant output may be obtained by a change of excitation at a particular point on each characteristic.

definite limiting torque and a relatively small overload capacity the generator voltage must be controlled so that the output at the particular engine speed is within the capabilities of the engine.

Although a drooping volt-ampere characteristic can be obtained at constant speed by self-excited shunt and differential-series windings, such a characteristic has not the correct shape to utilize the full output of the engine over a wide range of train speeds. A characteristic of the desired shape may be obtained either by the addition of a separately-excited winding to the self-excited shunt generator (the exciting current being controlled by a load-sensitive servo-operated field rheostat) or by supplying the whole of the excitation from an amplidyne exciter with transductor (saturable-core reactor) control. The former method is employed in this country and by European manufacturers of Diesel-electric locomotives: the latter method is employed in Alco-G.E.

(U.S.A.) locomotives.* Fig. 244 shows how the drooping volt-ampere characteristics of a generator with self-excited and separately-excited differential field windings may be modified, by suitably controlling the current in the latter, to give constant output, at constant engine speed, over a wide range of train speeds. A similar constant-output characteristic may be obtained with cumulative windings by suitably controlling the current in the separately-excited winding. In this case the individual load characteristics corresponding to definite (constant) currents in the separately-excited windings converge at the horizontal axis and diverge at the vertical axis.

When reduced power is required for propulsion the engine speed is reduced to economize fuel consumption, but the speed reduction must be related to the power requirements and the limiting current of the generator. For express passenger traffic under British conditions, three engine speeds (55, 80, 100 per cent) are sufficient.

Automatic Regulator. Several forms of servo-mechanism are in use for operating and controlling the power-driven field rheostat which regulates the

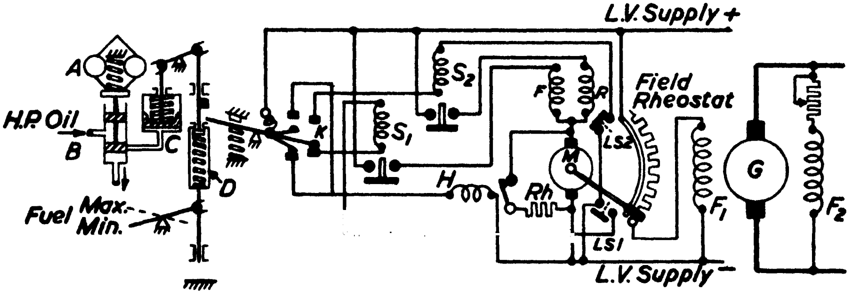


FIG. 245. CONNEXIONS OF AUTOMATIC REGULATOR (ENGLISH ELECTRIC)

current in the separately-excited field winding of the generator. The moving contact may be operated either electrically or hydraulically (using oil from the engine lubricating system), but the control is always by an oil-operated servo mechanism.

Fig. 245 shows a scheme of electric operation. The moving contact of the field rheostat is driven, through gearing and a slipping clutch, by a small series motor, M, which has a separate field winding for each direction of rotation. The armature is normally shunted by a resistor, Rh, but when rapid motion is required this circuit is opened by a small contactor, H. The motor is controlled by two relays S_1, S_2 and the limit switches, LS_1, LS_2, on the field rheostat, the operating coils of the relays being controlled by contacts K actuated by the engine governor, these contacts being spring-biased to the lower position.

The servo mechanism consists of a pilot valve, B, operated by the governor, A, which controls the flow of oil to a cylinder, C, the piston of which actuates the fuel valve through linkwork which includes a telescopic link, D. When the fuel valve is fully open further movement of the governor, due to increased load, causes the moving contact K to move upwards and close the control circuit of relay S_2, thereby operating the rheostat and reducing the effective excitation of the generator. When the engine speed returns to normal the

* For details see a series of papers in *Trans. A.I.E.E.*, Vol. 66, pp. 233–246.

moving contact K occupies a neutral position between the fixed contacts. A sudden drop or rise in engine speed causes the inner moving contacts of K to energize contactor H thereby producing rapid movement of the field rheostat. The field windings, F_1, F_2, act cumulatively.

One form of hydraulically-operated field rheostat, largely used on Continental Diesel-electric plants is shown in Fig. 246. The lever A is connected by linkwork to the engine governor and actuates the valves controlling the servomotor

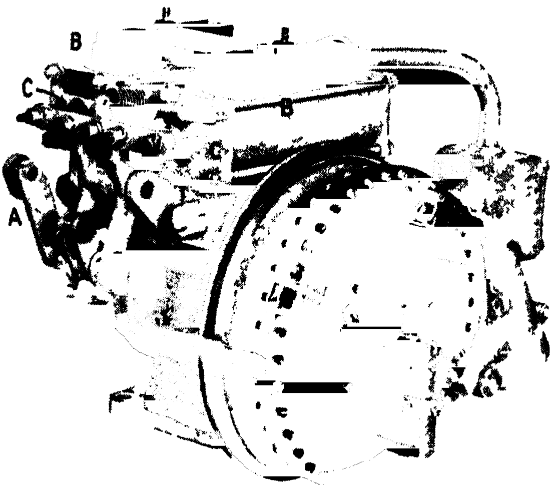


FIG. 246. BROWN BOVERI SERVO OPERATED REGULATOR

operating the field rheostat. Different torque settings are obtained by means of solenoids, B, energized from the master controller, actuating the plungers, C, connected to the lever system.

Engine Starting. The cranking of the engine at starting is performed by operating the main generator as a motor, a special series winding being provided. Power is supplied from a storage battery.

Diesel-electric Shunting Locomotives. These locomotives have superseded steam locomotives for shunting operations in many marshalling yards. A typical locomotive weighs 47 tons and is equipped with a 350-h.p., 6-cylinder engine and a pair of traction motors which drive, through double-reduction gearing, three coupled axles.*

The operating conditions of shunting locomotives do not warrant such elaborate control systems as are necessary for locomotives handling traffic, and a simple control system, involving field control of the main generator and speed control of the engine, has given satisfactory results in practice.

* Some details, including a drawing, are given in a paper, by F. A. Harper, on "Diesel Electric Locomotives of the British Railways," *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 188. See also a paper, by Messrs. Mardis and Jowett, on "The Electrical Equipment of Diesel-electric Locomotives and Motor-coach trains," *Ibid*, p. 262.

CHAPTER XVIII

TRAIN RESISTANCE

TRAIN resistance is the term applied to the forces resisting the motion of a train when it is running at uniform speed on a straight and level track. Under these conditions the whole of the energy output from the driving axles is expended against train resistance. Thus a portion is expended against friction internal to the rolling stock (which consists of friction at the journals, guides, bogies, buffers, etc.); another portion is expended against the external resistances between the rolling stock and the track (e.g. rolling friction between the wheels and rails, flange friction between the wheels and rails, resistances resulting from the temporary deflection of the track due to the passage of the train over it); and the remaining portion is expended against air resistance.

Components of Train Resistance. The internal and external resistances together constitute the *mechanical resistance* component of train resistance. These resistances do not admit of detailed analysis on account of their varied and uncertain nature. For example, flange friction depends largely upon accidental conditions such as oscillation of the coaches, lateral wind pressure, etc., while the track resistance is influenced by the condition of the track, the strength of the rails, and the nature of the ballast.

It is probable that some of these resistances increase with the speed, while others may be unaffected or may even decrease with the speed. At low and moderate speeds (between 5 m.p.h. and 40 m.p.h.) we are probably correct in assuming that the mechanical resistance increases directly with the speed,* but at higher speeds there is evidence to show that this relation does not hold good. In fact the train-resistance tests† carried out on the Marienfelde-Zossen experimental track indicate that, for the particular coaches experimented with, the mechanical resistance is practically constant between speeds of 90 and 125 m.p.h. At these high speeds, however, the train resistance consists principally of air resistance, and the mechanical resistance is only a small fraction of the total.

The mechanical resistance is generally assumed to be proportional to the weight of the train, which assumption, for a given class of rolling stock, is probably correct.

The *air resistance* is generally assumed to vary as the square of the velocity of the train, and may be divided into two components, one associated with the ends of the train and the other with the length of the train. The former includes the head resistance and the suction effect at the rear, while the latter includes the air friction on the sides, top, and underside of train, and is termed "skin friction."

The *head resistance* depends upon the exposed surface at right angles to motion: it is largely influenced by the shape of the leading portion of the train and the direction and velocity of the wind. With trains hauled by locomotives

* This law does not hold good for the very low speeds incidental to starting, as the resistance under these conditions is very much greater than that at speeds above 4 to 5 m.p.h.—due to increased track resistance and journal friction.

† See *Journ. I.E.E.*, Vol. 33, p. 894.

the largest portion of the head resistance is encountered by the locomotive, but with electric trains operated with motor coaches and trailers the whole of the head resistance is encountered by the leading coach. By suitably shaping the end of this coach, it is possible to obtain a considerable reduction in head resistance.

The *suction resistance* is also affected by the shape of the end coach, but as the magnitude of this resistance is only about one-tenth of the head resistance, the shape of this portion of the train is not so important as that of the opposite (or leading) end.

The manner in which air resistance is influenced by the contour of the front and rear portions of the train is shown by the curves of Fig. 247. These

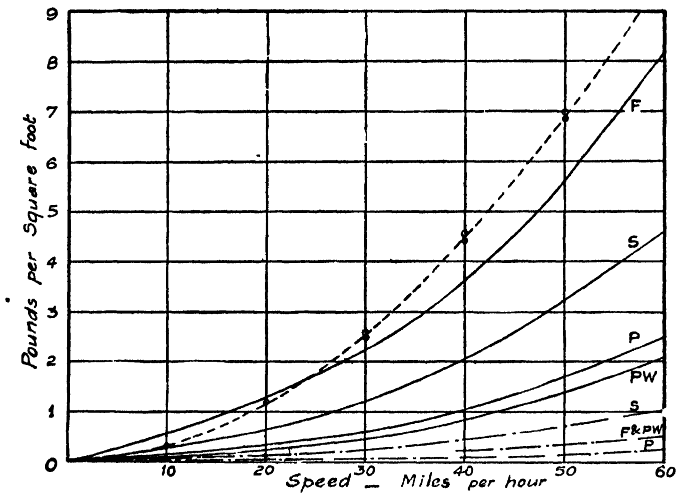


FIG. 247. AIR-RESISTANCE CURVES FOR MOTOR-COACH WITH VARIOUS TYPES OF VESTIBULE

F—flat end. *P*—parabolic.
S—partially rounded (standard U.S.A.). *PW*—parabolic wedge.

Note.—Full lines show the head resistance; chain-dotted lines show the suction resistance. The dotted curve is drawn through points calculated from equations—

$p = 0{\cdot}0028\ V^2$ and $p = 0{\cdot}003\ V^2$

curves indicate the results obtained by the St. Louis Railway Test Commision* on an experimental motor coach fitted with vestibules of different forms, (i) flat; (ii) partially rounded (the standard type on U.S. inter-urban cars); (iii) parabolic; (iv) parabolic wedge, the relative shapes being shown in Fig. 248.

The dotted curve represents the results of measurements of the air pressure on the exposed surface of a coach, at right angles to the direction of motion. Tests by Aspinall gave the law—$p = 0{\cdot}003V^2$, and the high-speed Marienfelde-Zossen tests gave the law—$p = 0{\cdot}0028V^2$, where p denotes the pressure in lb per square foot and V the speed in m.p.h. The divergence between this

* *Report of Electric Railway Test Commission* (St. Louis), p. 534. The car on which the air resistance tests were conducted had the following dimensions: Length over corner posts, 32 ft; width, 8 ft 4 in.; height from side sills to top of roof, 9 ft 5 in.; projected area of each vestibule (at right angles to motion), 96 sq. ft. The car body was specially mounted on dynamometers, so that the total resistance and the head resistance could be measured directly.

curve and that for the flat vestibule is accounted for by the different methods (indirect and direct) employed and the probability of reduced pressure near the edges of the vestibule.

The *skin-resistance* component of the air resistance depends on the length of the train, the type of coaches, the nature of the external fittings, projections, etc. It is affected so some extent by side winds, and in the case of long trains becomes an important item in the air resistance.*

Methods of Determining Train Resistance. The methods of conducting train resistance tests with steam trains are: (i) by determining the draw-bar pull of the locomotive and the speed of the train under conditions of uniform

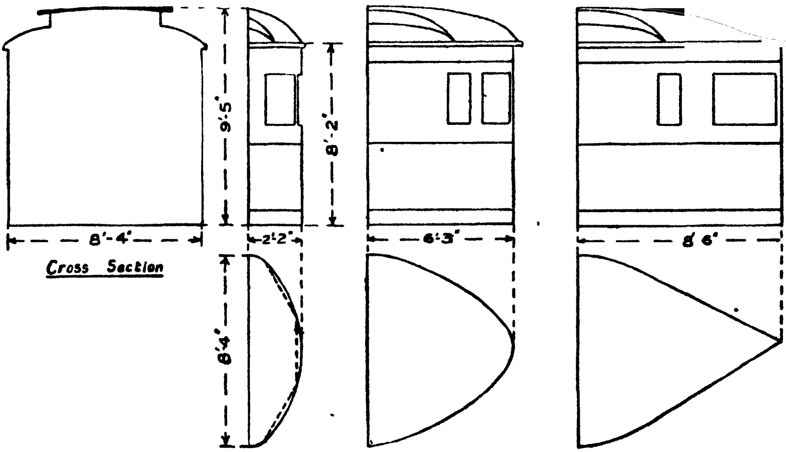


FIG. 248. TYPES OF VESTIBULE USED FOR AIR-RESISTANCE TESTS

speed; (ii) by allowing the train to coast (without the locomotive), and obtaining an accurate record of the retardation.

The first method requires a dynamometer car, and has the disadvantage that only a portion of the head resistance is included in the dynamometer reading. This disadvantage is not apparent when considering trains hauled by locomotives, as the largest portion of the head resistance is encountered by the locomotive, and would be included in the resistance of the locomotive. The train resistance is equal to the draw-bar pull, which, together with the speed, is recorded graphically.

The second method has the drawback (which is associated with all coasting tests on trains) that the whole of head resistance is encountered by the leading coach, and therefore the retarding force on this coach is greater than that on the following coaches. Consequently there is a tendency for the coaches to crowd together, which produces greater oscillation and flange friction than when the couplings are tight. The train resistance is obtained from the retardation in the following manner—

The force necessary to produce a retardation of 1 m.p.h.p.s. on an effective

* Wind-tunnel tests on model trains have been carried out at the National Physical Laboratory. Some of the results are shown graphically in a paper, by F. Lydall, on "Tractive Resistance of Electric Trains," *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 96.

mass of 1 ton is 102 lb (p. 20). Hence, if β_c is the retardation in m.p.h.p.s., and W_e is the effective mass of the train, the total retarding force will be $102\beta_c W_e$ lb, which will be equal to the train resistance, provided that the train is on a level track. It is essential that the contour of the track be accurately known, as an "up" gradient of 1 in 1,000 corresponds to a retarding force of $2\frac{1}{4}$ lb per ton of train weight.

With electric trains the total train resistance can be determined by observing the voltage and current input to the motors when the train is running at uniform speed. The tractive effort and speed can then be deduced from the characteristic curves of the motors: the tractive effort will correspond to the train resistance, provided that the train is on the level and is not being accelerated.

With the usual motor equipment the free running speed of the train occurs on the steep portion of the speed curve of the motor, so that a small error in reading the current may result in a relatively large error in the speed. Moreover, as free running is approached very slowly with the full motor equipment, a long stretch of level track would be required in order to eliminate the above sources of error. This objection can be overcome by using a train with a small motor equipment. Thus, with several motors controlled on the multiple-unit system, the train can be accelerated, by the whole equipment, to approximately the speed required, and then a number of motors may be cut out, so that the train may be kept running at uniform speed.

The coasting method of determining train resistance may also be used with electric trains. In this case, however, the total resistance to motion includes not only the train resistance, but also the friction in the motors and gears, which, in trains equipped with several motors, may amount to a considerable percentage of the total resistance. The effect of the revolving parts (armatures, gears and wheels) must be taken into account in deducing the train resistance from the observed retardation as the stored energy in these parts may, in some cases, amount to over 10 per cent of that for the whole train.

Train Resistance Formulae. In view of the large number of variables involved in train resistance, it is not surprising to find a large number of formulae of varied forms to express the law of variation of train resistance with speed. These formulae, when applied to a given train, will be found to give widely divergent results. Hence train resistance formulae must be used with discrimination, as, although each formula may be correct for the conditions under which it was derived, the probability of similar conditions for the tests of different investigators is very remote. Such items as the type of coach, the nature of the track, and the method of testing would be quite sufficient to cause large variations in the results.

Aspinall, many years ago, carried out tests on 22-ton British bogie coaches hauled by a steam locomotive with a dynamometer car.* The formula he deduced from the test results was—

$$r = 2{\cdot}5 + V^{5/3}/(51 + 0{\cdot}0278L) \qquad . \quad . \quad . \quad . \ (33)$$

where r is the specific train resistance† in lb per ton of train weight, V is the speed in miles per hour, and L is the length of the train in feet.

* Paper by Sir John Aspinall on "Train Resistance" (*Min. of Proc. I.C.E.*, Vol. 147, p. 155). A collection of formulae is given on pp. 189–192.

† The customary method of expressing train resistance is in *lb per ton of train weight*, which may be termed the "specific train resistance."

The results of tests on modern and heavier main-line bogie coaches, however, show considerable divergence from Aspinall's results when expressed in terms of specific train resistance. For example, Stanier*, from tests on British 36-ton bogie coaches, derived the formula—$r = 4(V + 12)/(V + 4{\cdot}5) + V^{1{\cdot}87}/322$ lb per ton, which, for speeds between 20 and 90 m.p.h., can be expressed by the more appropriate formula—$r = 5{\cdot}35 + 0{\cdot}0016V^2$. Again, Clarke,† from tests in India on passenger stock, has derived the formulae—$r = 3{\cdot}2 + 0{\cdot}047V + 0{\cdot}00235V^2$ for 25-ton coaches; $r = 3{\cdot}2 + 0{\cdot}0375V + 0{\cdot}00205V^2$ for 30-ton coaches and $r = 3{\cdot}2 + 0{\cdot}0239V + 0{\cdot}00161V^2$ for 40-ton coaches.

From analyses of recent tests on passenger rolling stock in this country and India, Lydall has suggested the following formula for the total tractive resistance (in lb) of main-line trains hauled by electric locomotives.‡

$$R = 5{\cdot}5W + (0{\cdot}35 + 0{\cdot}05n)V^2 \quad . \quad . \quad . \quad (33a)$$

where W denotes the total weight of the train (coaches plus locomotive) and n the number of coaches.

However, in view of the wide differences in the bodies and running gear of electric locomotives, it is desirable to estimate the tractive resistances of locomotive and coaches separately, using the formulae given later.

General Equations for Train Resistance. From a consideration of the separate components of train resistance, we should expect the law of variation of resistance with speed to be of the form—

$$R = a + bV + cV^2$$

where R is the total train resistance in lb, V is the speed of the train in m.p.h., and a, b, c, are constants related to the particular train and track. In this equation the first two terms represent the mechanical resistances and the last term represents the air resistance.

(*a*) *Locomotive-hauled Trains.* The author has examined the results of Aspinall's tests to ascertain if they conform to the law

$$R = a + bV + cV^2$$

It was found that the curves closely approximated this law, and the following equations were obtained for the 5, 10 and 20 coach trains—

$$\begin{aligned} \text{5 coach train: } R &= 230 + 10{\cdot}3V + 0{\cdot}322V^2 \\ \text{10 coach train: } R &= 402 + 20{\cdot}6V + 0{\cdot}547V^2 \\ \text{20 coach train: } R &= 800 + 35{\cdot}2V + 0{\cdot}86V^2. \end{aligned}$$

As the last term in these equations represents the air resistance, we can, by estimating the head and suction resistances, arrive at an approximate value for the skin resistance. Moreover, this value may be checked by determining the difference in the air resistances for trains of different lengths. Treating the tests on the 5 coach and 10 coach trains in this manner, we obtain, for the skin resistance, an average value of $0{\cdot}000035V^2$ lb per square foot of longitudinal exposed surface.

* Address to British Association, 1936. *See Engineer*, Vol. 162, p. 319.

† Technical paper No. 288M, Great Indian Peninsula Railway. See *Engineer*, Vol. 159, p. 640 for abstracts and comments by C. F. Dendy Marshall.

‡ *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 98.

For coaches with elliptical roofs, of the usual proportions on the main-line railways of this country, the longitudinal exposed surface (S) per coach is given approximately by

$$S \text{ (square feet)} = 0{\cdot}35LA$$

where L is the length of the coach in feet and A is the transverse cross-section of the coach body in square feet.

Hence, if the head resistance is taken at $0{\cdot}0028V^2$ lb per square foot of transverse exposed surface, the coefficient c in the general equation for train resistance becomes

$$\begin{aligned} c &= 0{\cdot}0028kA\lambda + 0{\cdot}0000122nLA \\ &= A(0{\cdot}0028k\lambda + 0{\cdot}0000122nL) \qquad (34) \end{aligned}$$

where k is a coefficient to include the effect of the shape of the end of the coach (*see* below), λ* is the ratio exposed transverse surface/cross-section of coach body, n is the number of coaches in the train, L is the length of each coach in feet, and A is the transverse cross-section of the coach body in square feet. The following values, based on the curves of Fig. 247, may be taken for k.

Type of End of Coach (*see Fig.* 248)	k
Flat	1·0
Partially rounded (Standard U.S. inter-urban cars)	0·65
Parabolic	0·3
Parabolic wedge	0·28

A general expression for the total mechanical resistances can be deduced from the above equations for the resistances of 5, 10 and 20 coach trains.

The values obtained for the constants a and b, per ton of train weight, however, are inconsistent with those obtained from recent tests on modern British main-line coaches, the latter giving a between $1{\cdot}5W$ and $1{\cdot}7W$, and b between $0{\cdot}052W$ and $0{\cdot}064W$,† R being the total resistance in lb, and r the specific train resistance in lb per ton of train weight.

Hence for locomotive-hauled modern main-line bogie coaches—

$$R = W(1{\cdot}6 + 0{\cdot}055V) + AV^2(0{\cdot}0028k\lambda + 0{\cdot}0000122nL) \qquad (35)$$

$$r = 1{\cdot}6 + 0{\cdot}055V + AV^2(0{\cdot}0028k\lambda + 0{\cdot}0000122nL)/W \qquad (36)$$

(*b*) *Motor-coach Trains*. The increased resistance of motor-coach trains is manifested by the greater wear which these trains produce on the track rails,‡ and may be accounted for by (i) the heavy weight of bogie trucks with motors, (ii) the unspring-borne weight on the axles of the motor trucks, (iii) the low centre of gravity of the motor coaches, (iv) the small diameter of the driving wheels. These conditions are not conducive to good riding qualities, and, in consequence, a large amount of flange friction and "nosing" (lateral oscillation) takes place, while the track is subjected to direct blows of considerably greater magnitude than those which occur with locomotive hauled trains of trailer coaches.

Now it is extremely important to remember that, for electric trains

* This term is introduced to include the shielding effect of the locomotive. For motor-coach trains $\lambda = 1{\cdot}0$.

† *Engineer*, Vol. 159, p. 641.

‡ See *Min. of Proc. I.C.E.*, Vol. 179, pp. 99, 143; Vol. 197, p. 79. *Proc. I.M.E.* (1909), p. 438. *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 50.

consisting of motor coaches and trailers, there are *two train resistances* to be considered, (i) the *true train resistance* when the power is "on"; (ii) the *apparent train resistance* when the power is "off" and the train is coasting. In the latter case the motors are being driven by the train, and, in addition to the true train resistance, there are friction losses in the motor-axle bearings, gears, armature bearings, brushes, and the windage loss in the motors. These losses are all attributed to the motors when the power is "on" (the characteristic curves of the motors being calculated for the output at the tread of driving wheels), and it would be impracticable to do otherwise, as the loss in the gearing will necessarily depend upon the power being transmitted.

The additional retarding force due to motor and gear friction depends upon the size and type of motor, the number of motors per train, the gear ratio, and

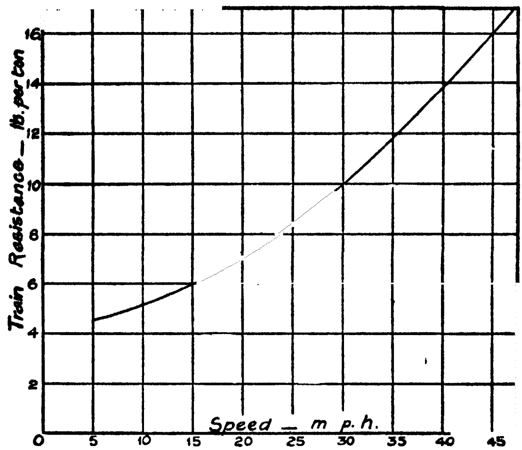


FIG. 249. CORRECTED CURVE FOR THE RESISTANCE OF A TWO-COACH ELECTRIC TRAIN

the diameter of the driving wheels. In the case of trains operating on urban railways, where the motors are geared for a low free-running speed, the motor and gear friction may be of the order of from 4 to 5 lb per ton of train weight. For suburban trains, operating at higher speeds, the motor and gear friction, at free-running speed, may be of the order of from 2 to 3 lb per ton.

In order to derive a formula for the resistance of motor-coach trains, the author has analysed the curves which have been published* for the electric trains on the Liverpool–Southport section of the Midland Region, British Railways. Corrections have been applied for motor and gear friction and the effect of head winds. The corrected curve for the resistance of a two-coach train (consisting of one motor coach and one trailer coach) is plotted in Fig. 249, and follows the law

$$r = 4{\cdot}1 + 0{\cdot}055V + 0{\cdot}0045V^2,$$

where r is the specific train resistance in lb per ton of train weight, and V is the speed in m.p.h.

A general equation, derived from Fig. 249, however, will enable the train

* See *Proc. I.M.E.* (1909), p. 473; *Journ. I.E.E.*, Vol. 52, p. 446.

resistance of motor-coach trains to be estimated with sufficient accuracy. This equation* follows the law

$$R = W(4{\cdot}1 + 0{\cdot}055V) + AV^2(0{\cdot}0028k + 0{\cdot}0000122nL) \quad . \quad . \ (37)$$

or

$$r = 4{\cdot}1 + 0{\cdot}055V + AV^2(0{\cdot}0028k + 0{\cdot}0000122nL)/W \quad . \quad . \ (38)$$

where the symbols have the same significance as in previous equations.

Train resistance tests have recently been made by Lydall on motor-coach trains on the Metropolitan line, London.† An analysis of the results shows that the total resistance closely approximates to that obtained by the application of equation (37).

Electric Locomotive and Train (*American Coaches*). An extended investigation on this subject has been carried out by Dr. F. W. Carter,‡ using the results obtained during the 50,000 miles endurance tests on the original New York Central gearless locomotives built by the General Electric Co., Schenectady. Some hundreds of runs were made with trains composed of different numbers of coaches, of the standard American saloon type—some of the trains consisting of four-wheel bogie coaches (55 ft long overall, 47 ft 6 in. over body, 9 ft 11½ in. width, 13 ft 6½ in. overall height, 36-in. wheels, loaded to 26·2 tons), and others consisting of six-wheel bogie coaches (66 ft long overall, 60 ft 2 in. over body, 9 ft 11½ in. in width, 13 ft 8½ in. height, 36-in. wheels, loaded to 45·3 tons).

The investigation showed that the resistance of the locomotive and train at a given speed was a function of the number of coaches, the relationship between total tractive resistance at a given speed and number of coaches following a straight line law for trains consisting of two or more coaches. Moreover, the relationship between the resistance for each coach added to the train and the speed was also found to be a straight line. Thus if n is the number of coaches in the train, the tractive resistance in lb due to these coaches is given by $n(100 + 5{\cdot}22V)$ for the 66 ft, 6-wheel, bogie coaches, and $n(84 + 4{\cdot}28V)$ for the 55 ft, 4-wheel, bogie coaches.

Tractive Resistance of Freight Trains. The mechanical resistance usually forms the chief component of the tractive resistance, although with very long trains the air resistance (principally skin resistance and eddies with open wagons) may not be negligible at the higher operating speeds (40 to 45 m.p.h.). Tests have shown that the mechanical resistance (in lb) at a given speed of a given type of wagon varies with the load carried by the wagon, and for a fully-laden wagon may be 50 per cent higher than that for the empty wagon. If, however, the *specific* resistances (lb per ton) are compared in the two cases, the specific resistance of the empty wagon may appear to be 100 per cent greater than that of the fully-laden wagon.

Data obtained by Clarke from dynamometer-car tests on open and closed wagons, empty and loaded, have enabled the following formulae to be deduced—

$$r = (4{\cdot}74 - 0{\cdot}052W_1) + 0{\cdot}0007W_1V + (0{\cdot}0381/W_1)V^2$$

and

$$r = (5{\cdot}3 - 0{\cdot}1W_1) + 0{\cdot}0007W_1V + (0{\cdot}0369/W_1)V^2$$

for trains with closed and open wagons respectively, where W_1 denotes the weight per wagon (tare plus load).

* In these equations the value of k should be chosen from 15 per cent to 20 per cent higher than the values given on p. 320 to allow for the increased air resistance caused by the motors.

† *Loc. cit.*, p. 100.

‡ *Min. of Proc. I.C.E.*, Vol. 201, p. 243.

Train Resistance in Tunnels. The resistance of trains in tunnels is naturally higher than that of trains in the open on account of the increased air resistance. It is of particular importance in *tube railways*, where the clearance between the train and the tube is very small.

For example, in the London deep-level tube railways the clearance between the present-day rolling stock and the older (11 ft 8¼ in. dia.) tunnels is 5½ in. and is 7⅜ in. for the newer (12 ft dia.) tunnels. Tests with this rolling stock in both tunnels and in the open have given the following results for the tractive resistance of a 210-ton, 7-car train (5 motor coaches and two trailers) at speeds of 20, 30 and 40 m.p.h.—

In 11 ft 8¼ in. tunnel—1,800, 3,200, 5,100 lb;
in 12 ft tunnel—1,400, 2,170, 3,270 lb;
in the open—1,000, 1,420, 2,250 lb.

Train Resistance at Curved Track. This is greater than the resistance on straight track, due to greater flange friction, etc. The additional resistance will depend upon the radius of the curve, the wheel-base of the trucks, and the end play between the wheel flanges and the rail. Very little data is available on the resistance at curved track,* which is probably due to the fact that to obtain accurate results a long stretch of track of uniform curvature is required.

A method adopted by American engineers for estimating the additional resistance at curves is to consider that each degree† of curvature increases the train resistance 0·6 lb per ton (2,000 lb) of train weight. In this country curves are usually expressed in terms of the radius (R), and if this is given in feet the additional train resistance will be

$$0{\cdot}6 \times (2{,}240/2{,}000) \times 2 \sin^{-1}(50/R) = 1{\cdot}35 \sin^{-1}(50/R) \text{ lb per ton.}$$

Tractive Resistance of Tramcars. The resistance of tramcars operating through streets will be much higher than that of railway trains, on account of differences in the construction of the track, cars and trucks.

The nature of the service on tramways and the low operating speeds, however, do not warrant an accurate estimation of the resistances to motion, and an average value of 25 lb per ton may be assumed for general conditions.‡

Tractive Resistance of Trolleybuses. British manufacturers of electrical equipment have collaborated in a series of carefully conducted tests to determine the tractive resistance of double-deck buses of British design. The tests were made by the coasting method and the values of tractive resistance include the losses in the transmission gearing. Standard curves have been deduced from these tests, from which the following values of the specific tractive resistance (in lb per ton) at speeds of 10, 20, 30, 40 m.p.h. have been taken—

2-axle bus, 27, 36·5, 47·5, 64;
3-axle bus, 39, 47·5, 59, 75.

The values of the resistance during coasting at these speeds are—

2-axle bus, 36, 45, 60, 80

* Some data are given by McMahon in *Min. of Proc. I.C.E.*, Vol. 147, p. 216. Recent data obtained by Clarke using a dynamometer car shows that on a 1 degree curve with 2 $\frac{1}{16}$ in. super-elevation the increased resistance is about 2 lb per ton at 15 m.p.h. but may become zero at a particular speed.

† In this system curvature is given by the angle (in degrees) which a chord 100 ft long subtends at the centre of curvature. Hence if θ is the curvature in degrees and R is the radius of the curve in feet, then $\sin \frac{1}{2}\theta = 50/R$, or $\theta = 2 \sin^{-1} 50/R$.

‡ *Min. of Proc. I.C.E.*, Vol. 198, p. 24.

Tractive Resistance of Battery Vehicles. Careful tests* on a ½-ton delivery van with solid rubber tyres and a back-axle drive have shown that on asphalt, macadam, and wood paving in good and dry condition the tractive resistance is between 20 and 30 lb per ton at a speed of 12 m.p.h. With the surface in poor condition, the resistance may be of the order of 50 lb per ton.

Tractive Resistance of Electric Locomotives. Data of the tractive resistance of locomotives with individual-axle drive have been obtained on the newly developed Brown-Boveri gas turbo-electric locomotive.† The tests were made with the locomotive hauling a dynamometer car and an electric locomotive which acted as a "load" when operating regeneratively. The resistance of the locomotive was determined from measurements of the current input to the motors and the pull at the drawgear. The average values from a number of tests are—

Speed (m.p.h.)	5	10	20	30	40	50	60	70
Resistance (lb per ton)	7·6	7·65	8·3	9·8	11·3	13·25	15·75	18

* *Trans. A.I.E.E.*, Vol. 35, p. 925 (Tests on ½-ton Electric Delivery Van).

† *Brown Boveri Review*, Vol. 31, p. 200. Data of locomotive—Weight 93 tons (63·6 adhesive); class $1AB_0A1$; frame-mounted motors with spring drive; length of body, 49 ft; cross section of body, 118 ft²; shaped ends similar to *S*, Fig. 248.

CHAPTER XIX

THE CALCULATION OF SPEED-TIME CURVES AND ENERGY CONSUMPTION FOR ELECTRIC TRAINS

PART I—SPEED-TIME CURVES

THE importance of the speed-time curve in electric railway engineering has been considered in Chapter II. Although the simplified speed-time curve discussed in that chapter is convenient for preliminary calculations, it does not correspond to the actual operating conditions. Moreover, an accurate speed-time curve is required for energy calculations.

The calculations for the speed-time curve and the energy consumption are usually carried through together, since certain quantities—e.g. the current and the time—are common to both calculations. However, to simplify matters, we shall at present consider only the calculation of speed-time curves.

For the calculation of the speed-time curve we require—

1. Complete information of the train service.
2. A survey of the route, showing the gradients, curves, stations, etc.
3. Sufficient particulars of the rolling stock and electrical equipment to enable the train resistance to be estimated and the accelerating weight or effective mass to be computed.
4. The characteristic curves of the motors.

The method of calculating the speed-time curve involves only the application of elementary mechanics, the chief feature of the method being the adoption of the speed as the independent variable. The time intervals corresponding to certain increments of the speed are therefore obtained indirectly from the acceleration. The process is essentially a *point-to-point* one, and the accuracy of any point is governed by the accuracy with which the preceding points have been obtained.

The *method of procedure* is best illustrated by working through an example.

Thus, consider that a service of motor-coach trains has to be run at a schedule speed of 16 m.p.h. over a straight and level track for which the average distance between the stations is 2,560 ft. There is a stop of 20 seconds at each station.

The trains are composed of two motor coaches, weighing 42·5 tons each (without passengers), and four trailer coaches, weighing 22·5 tons each (without passengers), the total seating capacity of a six-coach train being 324. Each coach body has a length of 52 ft, a maximum width of 8 ft 9 in., and a transverse cross-section of 87·5 ft^2. The height of the bottom of the side sills above the rails is 2 ft 11 in. Each coach is mounted on two four-wheel bogie trucks with 36-in. wheels, and each truck of the motor coaches is equipped with two 600-volt d.c. geared motors, the armatures of which are 18 in. in diameter and weigh 1,500 lb. The gear ratio is 3·5 : 1, and the characteristics of the motors, calculated for this gear ratio, 36-in. wheels, and normal voltage are—

Amperes	50	75	100	150	200	225
Speed (m.p.h.)	36	26·8	22·7	19·5	17·5	16·8
Tractive effort (lb)	300	700	1,120	2,050	3,000	3,500
Efficiency (per cent)	70	83	86·8	88·2	87·9	87·5

The mean current input to each motor during rheostatic acceleration is 225 amperes, and the average rate of braking is 2 m.p.h.p.s.

The *preliminary calculations* which have to be made before the actual calculation of the speed-time curve can be commenced are (i) the effective mass of the train, (ii) the train resistances when running with power and when coasting, (iii) the accelerating tractive effort at various speeds.

The *effective mass* of the train is calculated by the aid of equation (8); the loaded weight of the train (including the 324 passengers) being estimated at 195 tons; and the weight of each wheel being assumed at 900 lb. Thus

$$W_e = 195 + 1{\cdot}2 \times 24 \times \frac{900}{2{,}240} + 0{\cdot}49 \times 8 \times \frac{1{,}500}{2{,}240} \times 3{\cdot}5^2 \times \left(\frac{9}{18}\right)^2$$
$$= 214{\cdot}6 \text{ tons, i.e. } (214{\cdot}6/195 =)1{\cdot}1\ W$$

The *train resistance* is calculated from equation (38). In the present case the transverse cross-section of the coaches is 78·5 ft², and the motors increase this by about 11·5 ft². Allowing for the chamfered ends of the coaches, the coefficient k in equation (34) may be taken at $(0{\cdot}9(78{\cdot}5 + 11{\cdot}5)/78{\cdot}5 =)\ 1{\cdot}05$.

Hence the specific train resistance is given by the equation

$$r = 4{\cdot}1 + 0{\cdot}055V + 78{\cdot}5V^2(0{\cdot}0028 \times 1{\cdot}05 + 0{\cdot}0000122 \times 6 \times 52)/195$$
$$= 4{\cdot}1 + 0{\cdot}055V + 0{\cdot}00272V^2$$

the evaluation of which gives the following values for train resistance—

Speed, m.p.h. (V) . .	10	15	20	25	30	35
Specific train resistance, lb per ton (r)	4·92	5·53	6·3	7·2	8·2	9·34

The *apparent train resistance* during coasting will be greater than the above values, on account of the motor friction and gear losses. For the class of equipment under consideration the following values are representative of the friction and gear losses per motor—

Armature speed, r.p.m.	250	500	750	1,000	1,150
Friction and gear loss, kW	1·3	3·0	5·1	7·4	8·9

Rearranging these values to correspond to the retarding force per motor, we obtain*—

Speed of train (m.p.h.) .	10	15	20	25	30	35
Retarding force per motor (lb)	87·5	98	107	114	121	126·5

Hence the apparent train resistance during coasting is obtained by adding this additional retarding force to the true train resistance. The steps in the process are shown below—

Speed of train (m.p.h.)	10	15	20	25	30	35
Specific train resistance (r-lb per ton) .	4·92	5·53	6·3	7·2	8·2	9·34
Total train resistance (195r-lb) . .	960	1,070	1,230	1,405	1,600	1,820
Retarding force due to 8 motors (lb) .	700	784	865	911	967	1,010
Total retarding force (lb)	1,660	1,854	2,095	2,316	2,567	2,830
Apparent train resistance (lb per ton) or retarding force per ton (lb) . . .	8·5	9·5	10·75	11·88	13·15	14·5

* The method of converting the loss into retarding force is as follows: With a gear ratio of 3·5 : 1 and 36-in. wheels, the relation between the armature speed (r.p.m.) and the train speed (m.p.h.) is—

$$\text{m.p.h.} = (\text{r.p.m.}/3{\cdot}5) \times (60/5{,}280) \times 3\pi = \text{r.p.m.}/32{\cdot}65$$

Hence the retarding force (in lb) corresponding to the gear and friction losses (in kW) at this speed

$$= (\text{kW/m.p.h.}) \times [33{,}000 \times 60/(0{\cdot}746 \times 5{,}280)] = 503 \text{ kW/m.p.h.}$$
$$= 16{,}440 \text{ kW/r.p.m.}$$

The train resistance and the apparent train resistance during coasting are plotted in Fig. 250.

The *tractive effort available for acceleration* at various speeds is readily obtained from the characteristic curves of the motor by deducting the train resistance from the speed/tractive-effort curve. Instead of deducting the total train resistance from the total tractive effort of the eight motors, we deduct one-eighth of the total train resistance from the tractive effort of one motor and thus obtain the accelerating force per motor, as shown in Fig. 251. By

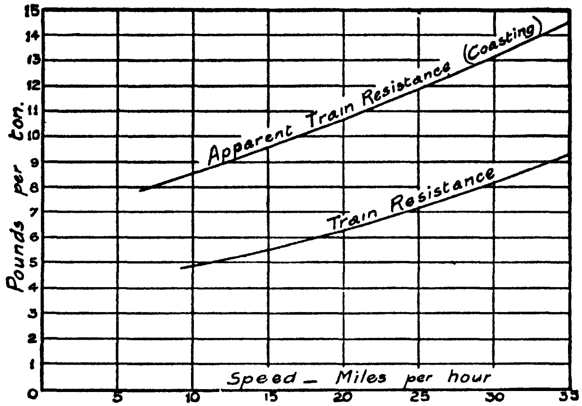


FIG. 250. TRAIN RESISTANCE AND APPARENT TRAIN RESISTANCE OF SIX-COACH, 195-TON ELECTRIC TRAIN

considering this force to act upon one-eighth of the mass of the train, we obtain the same conditions as if the total accelerating force of the eight motors acted upon the whole mass of the train.

We have now all the data necessary for the calculation of the speed-time curve.

I. **Period of Initial Acceleration.** During this period the starting resistance is cut out to maintain the average accelerating current at 225 amperes per motor. When all resistance has been cut out in the parallel combination of the motors, the speed of the train (assuming normal voltage) will be 16·8 m.p.h. The mean train resistance during this period should not be taken from the above values on account of the rapidly varying speed. An average value of 8 lb per ton will, therefore, be assumed.

Hence the average tractive effort available for acceleration—

$$= 3{,}500 - \tfrac{1}{8} \times 195 \times 8 = 3{,}500 - 195 = 3{,}305 \text{ lb}$$

(NOTE. 3,500 is the tractive effort corresponding to a current of 225 A.)

Therefore, mean acceleration $= 3{,}305/(\tfrac{1}{8} \times 214{\cdot}6 \times 102)$

$= 1{\cdot}21$ m.p.h.p.s.

Duration of accelerating period $= 16{\cdot}8/1{\cdot}21 = 13{\cdot}9$ seconds

Distance run during this period $= \tfrac{1}{2} \times 16{\cdot}8 \times 13{\cdot}9 \times 5{,}280/3{,}600$

$= \tfrac{1}{2} \times 16{\cdot}8 \times 13{\cdot}9 \times 1{\cdot}4\dot{6}7 = 170$ ft

II. **Period of Speed-curve Running.** A series of increments of speed are selected and the mean acceleration for each interval is calculated, after which the time and distance are readily obtained.

Thus, consider the increment from 16·8 m.p.h. to 19 m.p.h. The mean accelerating tractive effort (from Fig. 251)

	$= \frac{1}{2}(3{,}305 + 2{,}090) = 2{,}697$ lb
and the mean acceleration	$= 2{,}697/(26{\cdot}8 \times 102) = 2{,}697/2{,}735$
	$= 0{\cdot}987$ m.p.h.p.s.
Time increment	$= (19 - 16{\cdot}8)/0{\cdot}987 = 2{\cdot}23$ seconds
Total time from start	$= 13{\cdot}9 + 2{\cdot}23 = 16{\cdot}13$ seconds
Distance run during interval	$= \frac{1}{2}(16{\cdot}8 + 19) \times 2{\cdot}23 \times 5{,}280/3{,}600$
	$= 17{\cdot}9 \times 2{\cdot}23 \times 1{\cdot}467 = 58{\cdot}6$ ft
Total distance from start	$= 171 + 58{\cdot}6 = 229{\cdot}6$ ft

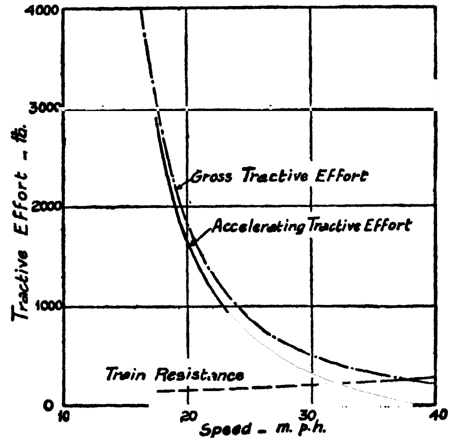


FIG. 251. CURVE SHOWING RELATIONSHIP BETWEEN SPEED AND ACCELERATING TRACTIVE EFFORT PER MOTOR FOR SIX-COACH TRAIN

This process is repeated until the free-running speed (38·5 m.p.h.) is reached, which speed is obtained directly from Fig. 251.

The results of these calculations are given in Table VIII.

In the case of short runs, however, free-running speed is seldom reached. It is advisable, therefore, to plot the speed-time and distance-time curves after a few points have been calculated.

Now, for a schedule speed of 16 m.p.h., the running time for a distance of 2,560 ft is $(2{,}560 \times 3{,}600/(5{,}280 \times 16) - 20 =)$ 89 seconds.

The accelerating and braking portions of the speed-time curve can now be drawn, and are represented in Fig. 252 by *OA* and *DE* respectively, *OD* representing the running time—89 seconds. *DE*, of course, makes an angle of $(-\tan^{-1}2)$ with the time axis.

The *points at which the power must be cut off and the brakes applied* must now be determined. We know that the area of the speed-time diagram must represent the distance travelled during the running time—in the present case 2,560 ft. Hence the coasting line *BC* (Fig. 252) must be drawn so that the area *OBCD* represents 2,560 ft, and the inclination of *BC* to the time axis must correspond to the mean retardation during the coasting period. This process

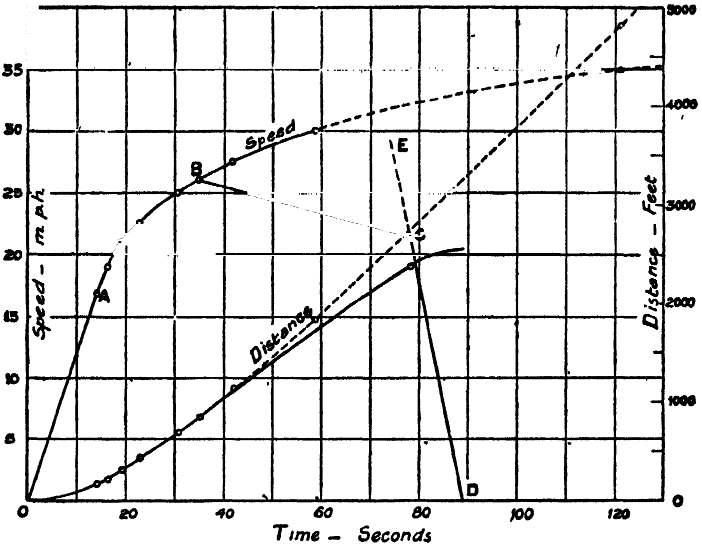


FIG. 252. SPEED-TIME CURVE FOR MOTOR-COACH TRAIN
(2,560-ft run on level track at 16 m.p.h. schedule speed)

TABLE VIII

CALCULATION OF SPEED-TIME CURVE FROM START TO FREE RUNNING FOR RUN ON LEVEL TRACK

Speed	Net Tractive-Effort	Speed Increment	Mean Accelerating Tractive-Effort	Mean Acceleration	Time Increment	Time from Start	Mean Speed	Distance Increment	Total Distance
m.p.h.	lb	m.p.h.	lb	m.p.h.p.s.	sec.	sec.	m.p.h.	ft	ft
0	3,305					0			0
—	—	16·8	3,305	1·21	13·9	—	8·4	171	—
16·8	3,305					13·9			171
—	—	2·2	2,697	0·987	2·23	—	17·9	58·6	—
19	2,090					16·13			230
—	—	2·0	1,730	0·633	3·16	—	20	92·7	—
21	1,370					19·29			322
—	—	1·5	1,210	0·433	3·39	—	21·75	108·1	—
22·5	1,050					22·68			430
—	—	2·5	880	0·322	7·76	—	23·75	270	—
25	710					30·44			700
—	—	2·5	600	0·219	11·38	—	26·25	438	—
27·5	490					41·82			1,138
—	—	2·5	410	0·15	16·66	—	28·75	702	—
30	330					58·5			1,840
—	—	5	218	0·08	62·8	—	32·5	2,990	—
35	105					121·3			4,830
—	—	3·5	52·5	0·0192	182·2	—	36·75	9,810	—
38·5	0					303·5			14,640

involves trial and error, but with some experience the correct position can usually be obtained on either the first or the second trial.

For instance, suppose we cut off power at 35 seconds when the speed is 26·1 m.p.h. The distance travelled is then 875 ft, so that the train must coast for a certain period before the brakes are applied. To obtain the duration of the coasting period, a value must be assumed for the average speed during the coasting period, in order that an appropriate value for the train resistance may be obtained. In the present case the average speed will be assumed as 24 m.p.h., which corresponds to an apparent train resistance of 11·6 lb per ton.*

Hence the retardation $= 11{\cdot}6 \times 24{\cdot}35/(102 \times 26{\cdot}8) = 0{\cdot}103$ m.p.h.p.s. The coasting line BC (Fig. 252) is now drawn on the curve-sheet, and the intersection of this line with the braking line gives the point at which the brakes must be applied.† This point is found to be at 78·2 seconds, when the speed is 21·6 m.p.h. Before proceeding further it is necessary to check our assumption of the average speed for the coasting period. This is $\frac{1}{2}(26{\cdot}1 + 21{\cdot}6) = 23{\cdot}85$ m.p.h., so that the value obtained above for the retardation will be substantially correct.

If the area of the diagram $OBCD$ be determined by a planimeter, it will be found to represent 2,555 ft, which is sufficiently near the actual distance (2,560 ft) of the run to justify the method of our trial.

The alternative method of ascertaining whether or not the point of cut-off has been chosen correctly is to determine the distances by calculation, as follows—

The time of the coasting period	$= 78{\cdot}2 - 35 = 43{\cdot}2$ seconds
and the time of the braking period	$= 89 - 78{\cdot}2 = 10{\cdot}8$ seconds
Hence the distance run during coasting	$= 23{\cdot}85 \times 43{\cdot}2 \times 1{\cdot}467 = 1{,}510$ ft
and the distance run during braking	$= \frac{1}{2} \times 21{\cdot}6 \times 10{\cdot}8 \times 1{\cdot}467$
	$= 170$ ft

The total distance run is, therefore, $875 + 1{,}510 + 170 = 2{,}555$ ft.

The complete speed-time curve, together with the distance-time curve, is shown in Fig. 252.

* The mean train resistance during coasting is slightly greater than the train resistance at the mean coasting speed, but the conditions of operation and other variable features do not warrant a closer estimation than that given above.

† The point of application of the brakes may be ascertained analytically by obtaining the co-ordinates of the point of intersection of the coasting and braking lines. Thus the equations of the coasting and braking lines are, respectively, given by

$$V' = U_1 - \beta_c t' \qquad (39)$$

and

$$V'' = U_2 - \beta t'' \qquad (40)$$

where V', V'', denote the speeds at times t', t'' respectively; β_c, β denote the respective retardations during coasting and braking; U_1 and U_2 denote the hypothetical speeds at zero time (i.e. the intercepts on the vertical axis). If V''', t''' denote the co-ordinates of the point of intersection of the coasting and braking lines, then $V''' = U_1 - \beta_c t''' = U_2 - \beta t'''$;

whence

$$t''' = (U_2 - U_1)/(\beta - \beta_c) \qquad (41)$$

V''' is, of course, obtained by substitution.

Applying this method to the above example we obtain the values of U_1 and U_2 by substituting known values for V', V'', t', t''. Thus, at the point of cut-off, $V' = 26{\cdot}1$, $t' = 35$; while at the end of the braking period $V'' = 0$, $t'' = 89$. Hence, adopting the above value for the retardation during coasting, we obtain $U_1 = 26{\cdot}1 + 0{\cdot}103 \times 35 = 29{\cdot}7$; $U_2 = 0 + 2 \times 89 = 178$. Therefore $t' = (178—29{\cdot}7)/(2 - 0{\cdot}103) = 78{\cdot}2$ sec., while $V''' = 178 - 2 \times 78{\cdot}2 = 21{\cdot}6$ m.p.h.

Effect of Gradients and Curves. In practice we are not generally favoured with ideal track conditions—as assumed in the above example—but we have curves and gradients of varying amounts. When considering the electrification of a particular railway, the energy consumption must be calculated for each section of the route before an accurate value of the average energy consumption over the whole route can be obtained. Hence it will be necessary to determine the speed-time curves for the actual track conditions, and if there are numerous gradients and curves the calculation by the above method will usually consume much time and patience. The effect of the gradients and curves on the train resistance can readily be allowed for, but the time of running on the various gradients and curves must be determined by trial. An example—representing typical conditions on a suburban railway—will best illustrate the method of procedure.

Suppose the above 195-ton motor-coach has to operate over a section 4,800 ft long at a scheduled speed of 20 m.p.h., with a stop of 20 seconds at the

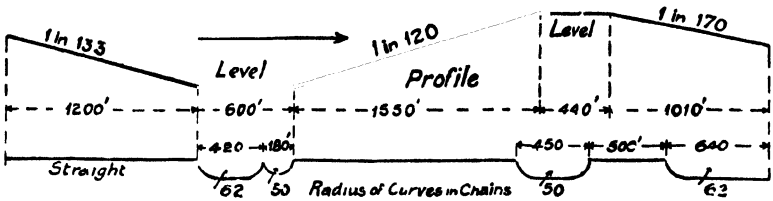


FIG. 253. PROFILE OF TRACK FOR SPEED-TIME CURVE IN FIG. 254

station. The profile of the section is shown in Fig. 253, and the speed-time curve will be calculated for the direction of running indicated by the arrow. The mean accelerating current is 225 amperes (as above), and the average rate of braking *on level track* is 2·0 m.p.h.p.s.

From an inspection of Fig. 253 it will be apparent that the brakes must be applied when the train is on the falling gradient of 1 in 170. This gradient is equivalent to an accelerating force of $(22{\cdot}4 \times 100/170 =)13{\cdot}2$ lb per ton of train weight. Hence the actual retardation during braking will equal $(2 - 13{\cdot}2/(1{\cdot}1 \times 102) =)\ 1{\cdot}88$ m.p.h.p.s.

The falling gradient of 1 in 133, on which the train is started, is equivalent to an accelerating force of $(22{\cdot}4 \times 100/133 =)\ 16{\cdot}8$ lb per ton of train weight, or $(16{\cdot}8 \times \frac{1}{8} \times 195 = 16{\cdot}8 \times 24{\cdot}35 =)\ 410$ lb per motor.

Hence during the initial acceleration (up to a speed of 16·8 m.p.h.) the mean accelerating tractive effort per motor—on the assumption of 8 lb per ton for the average train resistance—is $(3{,}500 - 8 \times 24{\cdot}35 + 410 =)\ 3{,}715$ lb.

$$\text{Therefore the mean acceleration} = 3{,}715/(102 \times 26{\cdot}8) = 3{,}715/2{,}735$$
$$= 1{\cdot}36 \text{ m.p.h.p.s.}$$

Duration of the period of initial acceleration

$$= 16{\cdot}8/1{\cdot}36 = 12{\cdot}35 \text{ seconds}$$
$$\text{Distance run during this period} = \tfrac{1}{2} \times 16{\cdot}8 \times 12{\cdot}35 \times 1{\cdot}467$$
$$= 152{\cdot}2 \text{ ft}$$

We now continue the calculation in the same manner (but allow for the effect of the gradient) as in the above example, until a distance of 1,200 ft has been run. The results of the calculations are given in Table IX, and it is only necessary to remark that the last increment of speed (for this period) must be obtained by trial.

We have next 600 ft of level, but curved, track. For the first 420 ft there is a curve of 62 chains radius ($= 2 \sin^{-1} 50/(62 \times 66) = 1{\cdot}4$ degrees), and for the remainder of the distance there is a curve of 50 chains radius ($= 2 \sin^{-1} 50/(50 \times 66) = 1{\cdot}73$ degrees). These curves increase the train resistance by 1·0 and 1·2 lb per ton respectively, or an increase in resistance of 24 and 29 lb respectively per motor.

The accelerating tractive effort is therefore obtained by deducting these values from the appropriate values given in Fig. 251.

A few trials will probably be necessary before the correct distances are obtained.

Next we have to negotiate the rising radient of 1 in 120. This gradient is

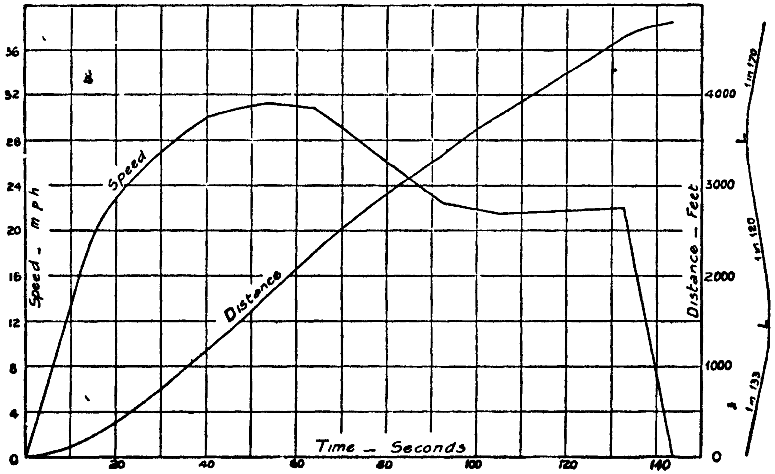


FIG. 254. SPEED-TIME CURVE FOR MOTOR-COACH TRAIN OPERATING ON A TRACK WITH CURVES AND GRADIENTS

(4,800-ft run at 20 m.p.h. schedule speed)

equivalent to a retarding force of ($22{\cdot}4 \times 100/120 =$) 18·7 lb per ton, or ($18{\cdot}7 \times 24{\cdot}35 =$) 455 lb per motor. Hence the mean accelerating (or retarding) tractive effort is obtained by deducting this force—due to the gradient—from the appropriate values given in Fig. 251. Thus, consider the speed decrement 31·43 to 29·5 m.p.h. The mean accelerating force on level track, obtained from Fig. 251, is $\frac{1}{2}(250 + 350) = 305$ lb. Hence the net retarding force $= 455 - 305 = 150$ lb. The retardation is therefore ($150/2{,}735 =$) 0·055 m.p.h.p.s.; whence the time and distance follow in the usual manner.

Before making further calculations the speed-time and distance-time curves should be plotted for the purpose of making trials to determine the point of cut-off.

The results of the calculations for the speed-time and distance-time curves up to 89 seconds (i.e. until the second stretch of level track is reached) are given in Table IX.

We now make a trial of cutting off power at 64 seconds, when the speed is 30·8 m.p.h. The distance run during this time is 2,257 ft. Consequently the position of the train is 457 ft up the 1 in 120 gradient. There is, therefore, a further distance of ($1{,}550 - 457 =$) 1,093 ft to be run up this gradient. The

retarding force due to the gradient is 18·7 lb per ton, and, assuming the mean train resistance to be 12·4 lb per ton, we obtain the retardation as [(18·7 + 12·4)/(1·1 × 102) =] 0·277 m.p.h.p.s.

The time required to run the distance of 1,093 ft with this retardation and an initial speed of 30·8 m.p.h. can be obtained by the application of the general dynamical equation

$$D_1 = V_1 t - \tfrac{1}{2}\beta t^2 \quad . \quad . \quad . \quad . \quad . \quad (42)$$

where V_1 is the initial velocity, β is the retardation, and D_1 is the distance run during the time interval t, all in foot-second units.

TABLE IX

CALCULATION OF SPEED-TIME CURVE FOR RUN ON TRACK WITH GRADIENTS AND CURVES

Speed	Net Tractive-effort on level and straight Track	Tractive-effort due to Gradient or Curve	Mean Accelerating Tractive-effort	Speed Increment	Mean Acceleration	Time Increment	Time from Start	Mean Speed	Distance Increment	Total Distance
m.p.h.	lb	lb	lb	m.p.h.	m.p.h.p.s	sec	sec	m.p.h	ft	ft
0	3,305	+ 410					0			0
—	—	—	3,715	16·8	1·36	12·35	—	8·4	152	—
16·8	3,305	+ 410					12·35			152
—	—	—	3,107	2·2	1·135	1·94	—	17·9	51	—
19	2,090	+ 410					14·3			203
—	—	—	2,140	2·0	0·782	2·66	—	20	75	—
21	1,370	+ 410					16·85			278
—	—	—	1,620	1·5	0·592	2·53	—	21·75	81	—
22·5	1,050	+ 410					19·4			359
—	—	—	1,290	2·5	0·472	5·3	—	23·75	185	—
25	710	+ 410					24·7			544
—	—	—	1,010	2·5	0·37	6·8	—	26·25	261	—
27·5	490	+ 410					31·5			805
—		+ 410	810	2·7	0·296	9·12	-	28·85	386	-
30·2	310						40·6			1,191
—	—	− 24	262	0·9	0·096	9·4	--	30·65	423	--
31·1	262						50·0			1,614
—	—	− 29	227	0·33	0·083	4·0	—	31·27	184	—
31·43	250						54·0			1,798
—		− 455	− 150	1·93	−0·055	35·0	--	30·46	1,562	—
29·5	360						89·0			3,360

In this equation D_1 is given in feet, if V_1 is expressed in feet per second, t in seconds, and β in feet per second per second. For our purpose we require D_1 in feet when V_1 is expressed in m.p.h. and β is expressed in m.p.h.p.s. Hence, transforming the equation to these units and solving for t, we obtain

$$t = \{1{\cdot}467 V_1 - \sqrt{[(1{\cdot}467 V_1)^2 - 2 \times 1{\cdot}467 \times \beta \times D_1]}\}/1{\cdot}467\beta \quad . \quad (43)$$

Inserting the above values for V_1, D_1, and β, we obtain $t = 28{\cdot}4$ sec.

The speed decrement for this interval is (28·4 × 0·277 =) 7·85 m.p.h. Hence the speed at the end of the interval is (30·8 − 7·85 =) 22·94 m.p.h., and the mean speed is 26·87 m.p.h. The train resistance corresponding to the mean speed is 12·4 lb per ton (from Fig. 250), so that the assumption of 12·4 lb per

ton is correct, although, in the present instance, an error of 1 lb per ton would have affected the retardation by only 3 per cent.

The run along the level stretch of 440 ft is next calculated, and we finally reach the falling gradient of 1 in 170 on which the train is to be brought to rest. The train commences the descent of this gradient at a speed of 21·47 m.p.h., the time being 105·8 seconds. The gradient produces an accelerating force of 13·2 lb per ton, and if the mean train resistance is assumed to be 11·1 lb per ton we have a net accelerating force, due to the gradient, of (13·2 − 11·1 =) 2·1 lb per ton, which produces an acceleration of 0·0187 m.p.h.p.s.

Now the running time = 4,800/(20 × 1·467) − 20 = 143·5 sec.

Hence the time on this (1 in 170) gradient = 143·5 − 105·8 = 37·7 sec, which includes the time for braking.

The point at which the brakes must be applied may be determined either

TABLE X

Calculation of Coasting and Braking Portion of Speed-Time Curve for Run on Track with Gradients and Curves

Speed	Mean Train Resistance	Retarding Force due to Gradient or Curve	Mean Retarding Force	Mean Retardation	Time Increment	Speed Decrement	Distance Increment	Mean Speed	Time from Start	Total Distance from Start
m.p.h.	lb per ton	lb per ton	lb per ton	m.p.h.p.s.	sec	m.p.h.	ft	m.p.h.	sec	ft
30·8	—	—	—	—	—	—	—	—	64	2,259
	12·4	18·7	31·1	0·277	28·4	7·85	1,093	26·87		
22·94	—	—	—	—	—	—	—	—	92·4	3,350
	11·1	1·2	12·4	0·11	13·4	1·47	440	22·2		
21·47	—	—	—	—	—	—	—	—	105·8	3,790
	11·1	− 13·2	− 2·1	− 0·0187	26·6	− 0·05	848	21·72		
21·97	—	—	—	—	—	—	—	—	132·4	4,638
				1·88	11·1	21·97	179	10·98		
0	—	—	—	—	—	—	—	—	143·5	4,817

from the curve sheet or by a few trial calculations: it will be found to be at 132·4 sec, when the speed is 21·97 m.p.h.

Hence the duration of the braking period is 11·1 sec, and the time of coasting down the gradient is 26·6 sec.

Therefore the distance run during braking is

$$\tfrac{1}{2} \times 21{\cdot}97 \times 11{\cdot}1 \times 1{\cdot}467 = 179 \text{ ft},$$

and the coasting distance is

$$\tfrac{1}{2}(21{\cdot}47 + 21{\cdot}97) \times 26{\cdot}6 \times 1{\cdot}467 = 848 \text{ ft}$$

Summing up the various distances, we obtain a total of 4,817 ft.

The complete speed-time and distance-time curves are given in Fig. 254, while the results of the calculations for the coasting and braking periods are given in Table X.

In cases where speed-time curves are required for runs extending over many gradients, the calculations can be carried out more expeditiously by

using either the universal speed-time and speed-distance curves developed by Dr. F. W. Carter* or a graphical method (involving the use of an adjustable graduated set-square) developed by the Traction Department of the General Electric Co., London, in which the speed-distance curve is derived by direct construction.†

Part II—Energy Consumption (Direct-current Equipments)

One of the characteristic features of electrical engineering is that the energy input to a motor, or a group of motors, performing a definite cycle of operations can be predetermined with a high degree of accuracy when the characteristic curves of the machines are available and the mechanical resistances are known. This feature also applies to electric railway engineering. For instance, the performance of a given electric train, operating to a given schedule in suburban service, can be predetermined with precision, since the uncertain factors (such as train resistance) connected with the problem influence the final results to only a small degree. Hence, in making guarantees for the energy consumption of suburban electric trains it is only necessary to add a small allowance (to cover unforeseen contingencies) to the calculated figures; in fact, this allowance is, in many cases, only of the order of 5 per cent. It is now our purpose to show the manner in which these calculations are made.

In order to calculate the energy required by an electric train when operating to a given schedule, it is necessary to have available the speed-time curve corresponding to the conditions of service and the characteristic curves of the driving motors, while a knowledge of the method of control will also be necessary.

In principle, the *method of calculation* is similar to that adopted for the calculation of the speed-time curve, i.e. the speed is considered as the independent variable, and the increments in the time and current, corresponding to increments in the speed, are obtained. The increments in the energy are then calculated, and the total energy supplied follows by a process of summation.‡

Example. The method of procedure is best illustrated by calculating the energy consumption for the 2,560 ft run for which the speed-time curve was calculated in the earlier part of this chapter.

In the calculations which follow, the energy supplied to one motor is calculated, and the total energy supplied to the train is obtained by multiplying by the number of motors. This method possesses an advantage over the direct method of calculating the total energy, since the standard characteristic curves of the motor can be used without modification.

I. **Period of Rheostatic Acceleration** (i.e. from the start until a speed of 16·8 m.p.h. is reached). During this period the current per motor will be assumed to be maintained constant at 225 amperes. With series-parallel control, the input (from the conductor rails) to a pair of motors during the first half of this period (when the motors are in series) will be $225 \times 600 = 135$ kW, and during the second half of the period (when the motors are in parallel) the input will be $2 \times 225 \times 600 = 270$ kW. Hence the energy input per motor

* See *Transactions of the American Institute of Electrical Engineers*, Vol. 22, p. 133.

† This method is described in *Railway Gazette*, Vol. 94, p. 207.

‡ The total energy supplied may also be obtained by plotting a power-time curve and integrating this by means of a planimeter.

for the whole of the initial accelerating period will be* $\frac{1}{2}(135 \times \frac{1}{2} \times 13\cdot9 + 270 \times \frac{1}{2} \times 13\cdot9) = 1{,}409$ kW-seconds, or $(1{,}409 \times 1{,}000/3{,}600 =)$ 391 watt-hours.

II. **Period of Speed-curve Running.** The energy input during this period is obtained by selecting a series of increments of the speed, calculating the average energy input for each increment, and summing the results.

Thus the interval from 16·8 m.p.h. to 19 m.p.h. occupies 2·23 sec, and the average current is $\frac{1}{2}(225 + 160) = 192$ amperes, so that the average energy input $= (192 \times 600 \times 2\cdot23)/3{,}600 = 71\cdot4$ watt-hours.

Similarly, for the intervals until cut-off, we obtain the following results—

Speed Increment (m.p.h.)	Time Interval (seconds)	Average Current (amperes)	Average Energy Input (watt-hours)
19 to 21	3·16	141	74·2
21 to 22·5	3·29	114	64·4
22·5 to 25	7·76	93	120
25 to 26·1	4·6	83·5	64

The average energy input per motor for the whole run is, therefore, $(391 + 71\cdot4 + 74\cdot2 + 64\cdot4 + 120 + 64 =)$ 785 watt-hours. Hence the total energy consumption for the train is $(785 \times 8/1{,}000 =)$ 6·28 kW-hours, or $(6\cdot28 \times 5{,}280/2{,}560 =)$ 12·95 kW-hours per train mile.

The specific energy consumption $= 12\cdot95 \times 1{,}000/195 = 66\cdot4$ watt-hours per ton mile.

Analysis of Energy Consumption. It is instructive to analyse this energy consumption into its several components. Thus—

Energy expended during braking† $= \dfrac{0\cdot0283 \times (21\cdot6)^2 \times 214\cdot6 \times 5{,}280}{2{,}560 \times 195}$

$= 29\cdot9$ watt-hours per ton mile

Energy expended against train resistance (while power is on)‡ $=$ 4·7 watt-hours per ton mile

Energy expended against apparent train resistance during coasting (difference between kinetic energy at 26·1 m.p.h. and 21·6 m.p.h.) $=$ 13·8 watt-hours per ton mile

Losses in starting rheostats|| $=$ 10·5 watt-hours per ton mile

Losses in motors and gearing (by difference) $=$ 7·5 watt-hours per ton mile

* The series and parallel portions of the initial accelerating period are here considered to be of equal duration. For a 5 per cent voltage drop in each motor the respective times on the series and parallel notches are 6·58 and 7·32 seconds, so that the energy input per motor during the initial accelerating period is 1,432 kW-seconds instead of 1,409. The specific energy consumption, however, only differs 0·5 per cent in the two cases.

† Obtained from equation (10). See p. 25.

‡ Obtained as follows: Average train resistance between 16·8 m.p.h. and 26·1 m.p.h. = 6·6 lb per ton. Total distance up to the point of cut-off is 875 ft, whence from equation (11), p. 25, energy expended against train resistance (while power is on) $= (2 \times 8 \times 170/2{,}560) + 2 \times 6\cdot6 \times (875 - 170)/2{,}560 = 4\cdot7$ watt-hours per ton mile.

|| Obtained as follows: Assuming a 5 per cent voltage drop in each motor and a constant supply voltage of 600 volts, the times on the series and parallel notches are, respectively, 6·58 and 7·32 seconds. The mean voltage drop in the rheostats during series notching

The energy utilized during the run = 4·7 + 13·8 = 18·5 watt-hours per ton mile, or 28 per cent of the energy supplied from the conductor rails. The remaining 72 per cent is accounted for as follows: 45 per cent is dissipated in the brake shoes, 12 per cent is dissipated in the starting rheostats, and 12 per cent is dissipated in the motors and gearing.

Of the kinetic energy possessed by the train at the point of cut-off, 31·6 per cent is utilized during coasting, and the remaining 68·4 per cent is dissipated in the brake shoes.

Effect of Acceleration and Rate of Braking on Energy Consumption. We will investigate the effect of, say, a 17·5 per cent reduction in the initial acceleration for the above service, the schedule speed, rate of braking, and other conditions remaining unchanged.

The mean tractive effort per motor to give an acceleration of 1·0 m.p.h.p.s. (allowing 8 lb per ton for train resistance) is $(1{\cdot}0 \times 102 \times 26{\cdot}8 + 8 \times \frac{1}{8} \times 195 =)$ 2,928 lb, which corresponds to a current of 195 amperes. The speed of the train, corresponding to this current and normal voltage, is 17·6 m.p.h. Hence the duration of the initial accelerating period $= 17{\cdot}6/1{\cdot}0 = 17{\cdot}6$ seconds.

The average energy input (per motor) during the initial accelerating period $= \frac{1}{2}(195 \times 600 \times \frac{1}{2} \times 17{\cdot}6 + 2 \times 195 \times 600 \times \frac{1}{2} \times 17{\cdot}6)/3{,}600 = 429$ watt-hours. By calculating the speed-time curve in the manner indicated above we find that power must be cut off at 41 seconds when the speed is 26·8 m.p.h., and the brakes must be applied at 77·5 sec when the speed is 23 m.p.h. The energy input (per motor) for the whole run is 840 watt-hours, or 7 per cent greater than that for the higher acceleration.

The *energy account* for this run is as follows—

Energy dissipated in the brake shoes	= 33·9 watt-hours per ton mile
Energy expended against train resistance (while power is on)	= 6·8 watt-hours per ton mile
Energy expended against (apparent) train resistance during coasting	= 12·1 watt-hours per ton mile
Losses in starting rheostats	= 11·5 watt-hours per ton mile
Losses in motors and gearing	= 6·8 watt-hours per ton mile

The lower acceleration, however, will result in a *lower maximum output per motor* and a lower maximum load on the sub-station. Thus, in the first case, the maximum output per motor, calculated from equation (9), $= 0{\cdot}002 \times 3{,}500 \times 16{\cdot}8 = 156$ h.p.; and in the second case, this becomes $0{\cdot}002 \times 2{,}928 \times 17{\cdot}6 = 137{\cdot}5$. Similarly, the *maximum input* to the train is, in the first case $(225 \times 600 \times 8/1{,}000 =)$ 1,080 kW; and in the second case $(195 \times 600 \times 8/1{,}000 =)$ 936 kW, which is 13·4 per cent lower than the former.

is $[\frac{1}{2} \times 600(1 - 2 \times 0{\cdot}05) =]$ 270 volts, while the value corresponding to the parallel notches is 150 volts. Hence the energy dissipated in the rheostats (per pair of motors) is $[(225 \times 270 \times 6{\cdot}58 + 2 \times 225 \times 150 \times 7{\cdot}32)/3{,}600 =]$ 248 watt-hours, which corresponds to $248 \times 4 \times 5{,}280/(195 \times 2{,}560) = 10{\cdot}5$ watt-hours per ton mile. If the internal resistance of the motors is neglected, the loss in the rheostats will be given by $\frac{1}{2}$ (kinetic energy of train at end of initial accelerating period + work done against train resistance). Applying this rule to the above example, we have—

Kinetic energy of train at 16·8 m.p.h. $= (0{\cdot}0283 \times (16{\cdot}8)^2 \times 214{\cdot}6 \times 5{,}280)/(2{,}560 \times 195) = 18{\cdot}1$ watt-hours per ton mile.

Work done against train resistance $= 2 \times 8 \times 170/2{,}560 = 3{\cdot}36$ watt-hours per ton mile.

Therefore the approximate loss in rheostats $= \frac{1}{2}(18{\cdot}1 + 3{\cdot}36) = 10{\cdot}73$ watt-hours per ton mile.

The current-time curves (for one motor) are given in Fig. 255.

The comparisons between the two runs may be summarized thus–

Initial acceleration (m.p.h.p.s.)	1·21	1·0
Rate of braking (m.p.h.p.s.)	2·0	2·0
Initial accelerating current per motor (amp)	225	195
R.m.s. current per motor (amp)	95·4	93·1
Specific energy consumption (Wh per ton mile)	66·4	71·1
Total energy consumption (kWh)	12·95	13·9
Maximum input from conductor rails (kW)	1,080	936
Average input from conductor rails (kW)	202	222
Maximum output from motors (h.p.)	1,250	1,100
Maximum speed (m.p.h.)	26·1	26·8
Speed at commencement of braking (m.p.h.)	21·6	23
Time from start to point of cut-off (sec)	35	41
Duration of coasting period (sec)	43·2	36·5
Duration of braking period (sec)	10·8	11·5
Energy utilized (Wh per ton mile)	18·5	18·9
Energy dissipated in brakes (Wh per ton mile)	29·9	33·9
Energy dissipated in starting rheostats (Wh per ton mile)	10·5	11·5
Energy dissipated in motors and gears (Wh per ton mile)	7·5	6·8

It is apparent that, from the energy point of view, the adoption of the higher acceleration has considerable advantages. The disadvantages are: (i) a higher peak load on the sub-stations, (ii) a slightly increased maintenance on the rolling stock and equipment.

The *influence of the rate of braking on the energy consumption* is shown by the following figures, which have been calculated for the above train. The initial acceleration has been chosen at 1·21 m.p.h.p.s., so that the results can readily be compared with those obtained previously, and which are given below for comparison.

Average rate of braking (m.p.h.p.s.)	2·0	2·75	1·5
Initial acceleration (m.p.h.p.s.)	1·21	1·21	1·21
Time at which power is cut off (sec)	35	33	43
Time at which brakes are applied (sec)	78·2	81·6	69·5
Maximum speed (m.p.h.)	26·1	25·6	27·6
Speed at commencement of braking (m.p.h.)	21·6	20·3	24·5
Duration of coasting period (sec)	43·2	48·6	26·5
Duration of braking period (sec)	10·8	7·4	19·5
Specific energy consumption (Wh per ton mile)	66·4	64	75·3
Energy dissipated in brakes (Wh per ton mile)	29·9	26·4	38·5

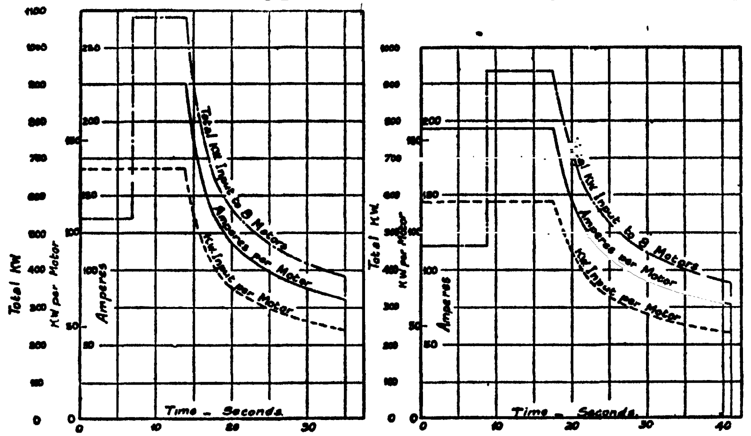
Thus, considering the rate of braking to be normally 2·0 m.p.h.p.s. a reduction of 25 per cent increases the energy consumption by 13 per cent; and an increase of 37·5 per cent reduces the energy consumption by 4 per cent.

The relationship between the rate of braking and the energy consumption is shown better in Fig. 256, which is plotted from the above results. It is apparent, therefore, that for the above service very little advantage is gained by the adoption of a braking rate above 2·0 m.p.h.p.s.

Influence of Length of Run on Energy Consumption. Fig. 257* is plotted from test results of the energy consumption for a number of runs with 175-ton motor-coach trains when operating at constant schedule speed. As the runs were all made with the same driver, the effect of the personal element on the performance is practically the same for each run.

* From a paper by Mr. Roger T. Smith on "Some Railway Conditions Governing Electrification" (*Journal of the Institution of Electrical Engineers*, Vol. 52, p. 293).

Effect of Gear Ratio and Method of Control on Energy Consumption. Since the energy consumption of a train operating to a given schedule is influenced by the duration of the coasting period, it is clear that any conditions of operation



(a) (b)

FIG. 255. CURRENT AND POWER CURVES FOR 195-TON MOTOR-COACH TRAIN

(a) Accelerating current = 225 A per motor. (b) Accelerating current = 195 A per motor.

which will increase the coasting period will result in reduced energy consumption.* When the length of the run, the schedule speed, and the rate of braking are fixed, the coasting period will be affected only by (i) the initial acceleration, and (ii) the free-running speed. With short runs the initial acceleration will

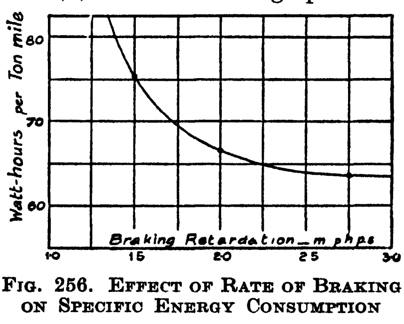


FIG. 256. EFFECT OF RATE OF BRAKING ON SPECIFIC ENERGY CONSUMPTION

(2,560-ft run at 16 m.p.h. schedule speed; acceleration = 1 21 m.p.h p s.)

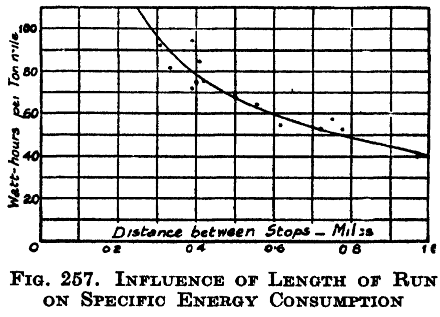


FIG. 257. INFLUENCE OF LENGTH OF RUN ON SPECIFIC ENERGY CONSUMPTION

(Weight of train = 175 tons. Schedule speed = 17 m.p.h.)

have the greater influence on the energy consumption, but with longer runs the energy consumption will be largely dependent on the free-running speed.

Now, if with a given equipment the gear-ratio be changed, the initial acceleration and the free-running speed will be changed; consequently the saving resulting from, say, a higher initial acceleration will be offset by the longer duration of the speed-curve running period, due to the lower free-running speed (assuming the schedule speed to be unaffected by the change). Whether

* This statement assumes that the increased coasting is obtained by altering either the initial acceleration or the braking, or both; the acceleration on the speed-curve being unaltered. In other words, the gear ratio is assumed to remain constant.

or not the energy consumption will be affected by the change of gearing will depend upon the relative values of the energy consumption for the accelerating and speed-curve running periods in each case, and a definite decision can only be arrived at by working through the speed-time curves and calculating the energy consumption in the usual manner.

In order to illustrate this point, we show, in Fig. 259, speed-time curves—for the above 195-ton motor-coach train—corresponding to gear ratios of 2·5 : 1,

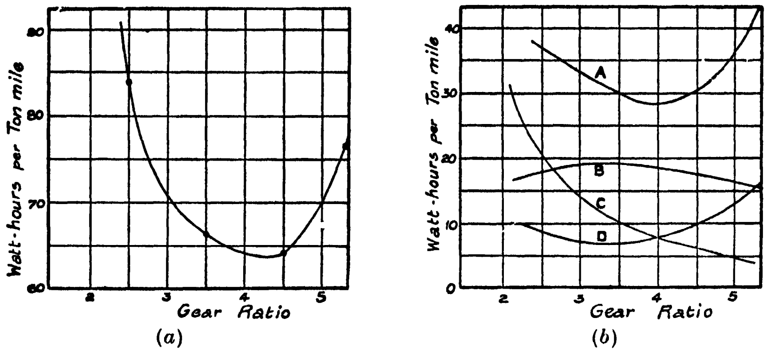


FIG. 258. EFFECT OF GEAR RATIO ON ENERGY CONSUMPTION OF 195-TON MOTOR-COACH TRAIN

(2,560-ft run at 16 m.p.h. schedule speed)

(*a*) Energy consumption. (*b*) Components of energy consumption. *A*—energy dissipated in brake shoes. *B*—energy expended against train resistance. *C*—energy dissipated in rheostats. *D*—energy dissipated in motors and gearing.

3·5 : 1, and 4·5 : 1, the schedule speed, distance of run, rate of braking, and accelerating current being the same in each case. The energy consumption has also been calculated, and the values are given in Table XI.

Before giving these results, it will be desirable to indicate the modifications required to the above data to allow for the change of gear ratio.

The characteristics of the motors (p. 325) are modified in the following manner. If V denotes the speed corresponding to a given current and gear ratio γ, then for a gear ratio γ_1, the speed (V_1) at the same current is given by $V_1 = V\gamma/\gamma_1$. Again, if F, F_1 denote the tractive efforts corresponding to a given current, then $F_1 = F\gamma_1/\gamma$.*

The modified values of the speed and tractive effort are as follow—

Current (amperes)	225	175	125	100	75	50
Speed, 2·5 : 1 gear; 36-in. wheels (m.p.h.)	23·5	25·6	29·3	32·5	38·1	51·4
Speed, 4·5 : 1 gear; 36-in. wheels (m.p.h.)	13·07	14·25	16·25	18	21·2	28·6
Tractive effort, 2·5 : 1 gear; 36-in. wheels (lb)	2,500	1,800	1,130	805	500	214
Tractive effort, 4·5 : 1 gear; 36-in. wheels (lb)	4,500	3,240	2,030	1,405	900	385

* This assumes that the gear and axle-friction losses are the same in each case, which assumption may be considered to be approximately correct, since, although the speeds of the axle and gearing (corresponding to a given speed of the armature) are increased with a reduction in the gear ratio, the tooth and bearing pressures are reduced.

The change in the gear ratio will affect the apparent train resistance during coasting. The values modified are as follows—

Speed of train (m.p.h.)	10	15	20	25	30	35	40
Apparent train resistance for gear ratio 2·5 : 1 (lb per ton)	7·6	8·16	9·13	10·25	11·45	12·7	14·1
Apparent train resistance for gear ratio 4·5 : 1 (lb per ton)	10	11·1	12·4	13·7	15·1	—	—

The effective mass of the train will also be affected by the change in the gear ratio due to the change in the ratio of the speed of the train to the speed of the armatures. By the application of equation (8), p. 22, the effective mass is found to be 210·6 tons when the gear ratio is 2·5 : 1, and 217 tons when the gear ratio is 4·5 : 1.

Hence, for an accelerating current of 225 amperes per motor and a gear ratio of 2·5 : 1, the initial acceleration is

$$(2{,}500 - 195)/(\tfrac{1}{8} \times 210{\cdot}6 \times 102) = 0{\cdot}86 \text{ m.p.h.p.s.}$$

With a gear ratio of 4·5 : 1 the initial acceleration is

$$(4{,}500 - 195)/(\tfrac{1}{8} \times 217 \times 102) = 1{\cdot}55 \text{ m.p.h.p.s.}$$

TABLE XI

SUMMARY OF THE RESULTS OF CALCULATIONS FOR A 195-TON MOTOR-COACH TRAIN OPERATING AT A SCHEDULE SPEED OF 16 M.P.H. ON A RUN OF 2,560 FT ON LEVEL TRACK, WITH STOPS OF 20 SECONDS DURATION

Gear ratio	2·5	3·5	4·5
Initial accelerating current per motor (amp)	225	225	225
Rate of braking (m.p.h.p.s.)	2·0	2·0	2·0
Initial acceleration (m.p.h.p.s.)	0·86	1·21	1·55
Specific energy consumption (Wh per ton mile)	84	66·4	64
Maximum input from conductor rails (kW)	1,080	1,080	1,080
Average input from conductor rails (kW)	262	202	200
Maximum output from motors (h.p.)	156	156	156
Free-running speed of train (m.p.h.)	43·5	38·5	32·5
Maximum speed of train during run (m.p.h.)	28	26·1	25·5
Speed of train when rheostats are cut out (m.p.h.)	23·5	16·8	13·1
Speed of train at commencement of braking (m.p.h.)	24	21·6	21·8
Mean retardation during coasting (m.p.h.p.s.)	0·096	0·103	0·117
Time from start to point of cut off (sec)	35	35	46·5
Duration of coasting period (sec)	42	43·2	31·6
Duration of braking period (sec)	12	10·8	10·9
Energy utilized (Wh per ton mile)	18	18·5	17·5
Energy dissipated in brakes (Wh per ton mile)	36·9	29·9	30·5
Energy dissipated in starting rheostats (Wh per ton mile)	20·6	10·5	6·4
Energy dissipated in motors and gears (Wh per ton mile)	8·5	7·5	9·6
R.m.s. current per motor (amp)	121·5	95·4	80·8
Peripheral speed of gearing at free-running speed (ft per min)	2,260	2,260	2,000
Diameter of pitch circle of gear wheel (in.)	22·5	24·5	25·8

The *relationship between the energy consumption and the gear ratio* for the above conditions of operation is shown in Fig. 258 (*a*). We observe that the energy consumption is not materially affected by changes in the gear ratio between 3·75 : 1 and 4·5 : 1, but is increased considerably for gear ratios below 3·0 : 1 and above 4·5 : 1. For example, for a gear ratio of 5·3 : 1 (which is the

highest gear ratio with which the service can be run, power being kept on for 76 seconds, and the brakes applied immediately power is cut off), the energy consumption reaches 78·6 watt-hours per ton mile.

The comparison between the above runs is greatly facilitated by plotting the values of the energy expended in the various parts of the equipment against the gear ratio, as shown in Fig. 258 (*b*). We have now an explanation

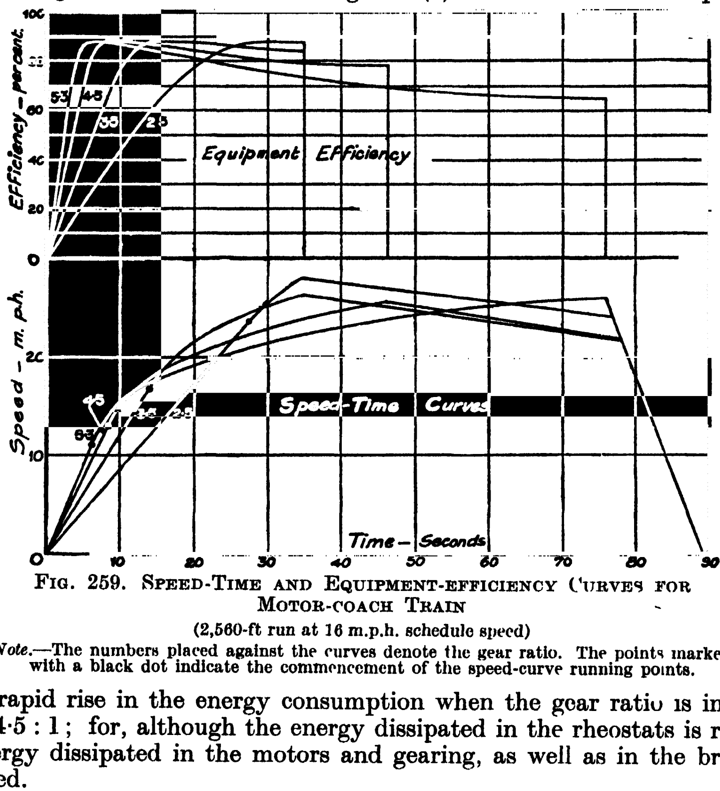


FIG. 259. SPEED-TIME AND EQUIPMENT-EFFICIENCY CURVES FOR MOTOR-COACH TRAIN

(2,560-ft run at 16 m.p.h. schedule speed)

Note.—The numbers placed against the curves denote the gear ratio. The points marked with a black dot indicate the commencement of the speed-curve running points.

of the rapid rise in the energy consumption when the gear ratio is increased above 4·5 : 1; for, although the energy dissipated in the rheostats is reduced, the energy dissipated in the motors and gearing, as well as in the brakes, is increased.

The increased losses in the motors are principally friction losses, consequent upon the high armature speed. For instance, in the particular motor for which the characteristics are given on p. 325, the friction and gear losses at an armature speed of 1,200 r.p.m. (corresponding to a train speed of 26 m.p.h. with a gear ratio of 5·3 : 1) are approximately 20 per cent of the input to the motor. With a high gear ratio, a large portion of the run must be made at speeds in the neighbourhood of free running, and as this condition usually corresponds to a high armature speed, the average equipment efficiency during this period is low. This point is clearly shown by the curves of Fig. 259, which refer to the above runs, with an additional curve for the maximum gear ratio (5·3 : 1) added.* Comparing the runs with gear ratios of 3·5 : 1 and 5·3 : 1, the mean

* The points for the portion of the efficiency curve corresponding to speed-curve running are obtained directly from the motor efficiency curve (p. 325), while the points for the portion corresponding to the initial acceleration are obtained from the ratio of the output to the input, the output being (kinetic energy of train + work done against train resistance).

efficiencies during the periods of speed-curve running are 86·8 per cent and 73·4 per cent respectively.

The curves of Fig. 259 also indicate the *ideal conditions for obtaining a low energy consumption* with the standard method of series-parallel control. Thus, suppose it were possible to adopt the 5·3 : 1 gear for the initial accelerating period, and to change this gear to 3·5 : 1 at a speed of 17·25 m.p.h. (corresponding to the intersection of the speed-time curves for these gear ratios). We should then be able to cut off power earlier (e.g. at 30 seconds), to coast longer, and to apply the brakes at a lower speed than if we made the run with the 3·5 : 1 gear throughout. The specific energy consumption for this method of operation is 57·6 watt-hours per ton mile, of which 27 watt-hours per ton mile are expended in the brakes.

Of course, such a method of operation is quite impracticable, but with motors designed for "tap-field" control, we can obtain conditions somewhat similar to the above. Thus, at starting, the full field winding would be used, thereby giving a low speed at which the rheostats are cut out, while for the speed-curve running period the field tappings would be used, thereby enabling this portion of the run to be made at a moderately high speed, so as to obtain a long coasting period.

Energy Consumption of Equipments Operating on Various Services. In the above discussion we have considered conditions of service which are typical for city lines. But, with an extensive system of electrification—involving the electrification of city, suburban, and interurban lines—the traffic department would require the suburban and interurban trains to be scheduled for a faster service than the city trains. Hence, if the same trains are operated over both city and interurban routes, the gear ratio cannot be selected to give the most economical operation on each system. For instance, suppose the average run of 5,000 ft is to be made at a scheduled speed of 24·5 m.p.h., with stops of 20 seconds' duration. The running time is 119 seconds. A reference to p. 329 will show that the above equipment, with a gear ratio of 3·5 : 1 and 36-in. wheels, is quite incapable of operating to this schedule. A lower gear ratio (between 2·0 : 1 and 2·5 : 1) must, therefore, be employed; and to avoid an excessive gear velocity at free-running speed (which in this case would be of the order of 45 to 50 m.p.h.), the diameter of the driving wheels require to be increased to 43·5 in. A reference to Fig. 258 (*a*) will show that such an equipment would have a high energy consumption when operating on the city service.

Moreover, if a compromise in the gear ratio be adopted, the energy consumption for the suburban and interurban services will be higher than that of a train equipped with the correct gear ratio for these services.

These disadvantages are to some extent minimized by employing *equipments with tapped or shunted field control.* The full field winding would be used for the initial acceleration, and the weakened (tapped or shunted) field would be used for obtaining the higher speeds required for the suburban and interurban routes. The gear ratio would be selected to give a moderately high acceleration, so that the train could be operated economically in city service.

Field control, therefore, *considerably extends the flexibility of the equipment,* and, with a suitable selection of the motor, *one class of equipment will be capable of operating on widely different services without the energy consumption, on any*

of the services, being excessive. This feature of field-control equipments can be better illustrated by an example.

Thus, consider a train of two 36-ton motor coaches, with equipments arranged for tapped-field control, to operate on level track to various schedules, the braking retardation being 2 m.p.h.p.s. in all cases. The coach bodies are of the same dimensions as those in the above examples. The trucks have 43½-in. wheels, and each truck is equipped with two 75 h.p., 500-volt, geared motors, the gear ratio being 3·9 : 1. The weight of the train loaded with passengers is 79 tons, and the effective mass is 86·5 tons.

The characteristics of the motors at normal voltage and for 43½-in. wheels, 3·9 : 1 gear ratio, are as follows—

Amperes	40	60	80	100	130
Speed, full field* (m.p.h.)	27·5	22·5	20	18·4	17
Speed, normal field* (m.p.h.)	38	28·7	24	21·6	19·5
Speed, weak field* (m.p.h.)	50	33·7	27·8	24·6	22
Tractive effort, full field (lb)	285	570	850	1,130	1,560
Tractive effort, normal field (lb)	200	460	725	1,000	1,410
Tractive effort, weak field (lb)	127	375	625	900	1,300
Efficiency, full field (per cent)	78·3	83	83	82·2	80·5
Efficiency, normal field (per cent)	75	84·2	86·2	85·8	84·5
Efficiency, weak field (per cent)	67	83	86·2	87	86·2

The mean accelerating current with full field is 130 amperes per motor.

The train resistance, calculated from equation (38), is given by $r = 4{\cdot}1 + 0{\cdot}055V + 0{\cdot}0042V^2$, and the apparent train resistance during coasting is obtained by making allowance for the motor and gear friction losses during this period. The values of train and coasting resistances at various speeds are—

Speed of train (m.p.h.)	10	20	30	40	50
Train resistance (lb per ton)	5·1	6·9	9·6	13	17·4
Apparent train resistance (lb per ton)	8·5	10·8	14	17·8	22·4

First, consider this train to operate on a service with 2·06 stops per mile, at a schedule speed of 16 m.p.h., the duration of each stop being 20 seconds. The distance between the stops is 2,560 ft, and the running time is 89 seconds.

For this run the motors will be operated with the full field winding throughout. The initial acceleration (corresponding to a mean current of 130 A per motor) is 1·35 m.p.h.p.s. and occupies 12·6 seconds. Power is cut off at 30·1 seconds from the start, when the speed is 26 m.p.h. The train coasts for 48·4 seconds, and the brakes are applied when the speed is 21 m.p.h. The specific energy consumption is 68·7 watt-hours per ton mile, and the total energy consumption is 5·4 kW-hours per train mile. The r.m.s. current per motor for the run is 51·7 amperes.

Second, consider the train to operate on a service in which the average run of 3,900 ft has to be made at a schedule speed of 20·6 m.p.h., with stops of 20 seconds' duration. The running time is 109 seconds.

For this run the initial acceleration is made with the full-field winding, and the acceleration on the speed-curve is made with the normal field winding, the transition from full field to normal field being made at a speed of 17 m.p.h. Power is cut off at 44·5 seconds (when the speed is 32·5 m.p.h.), and the brakes

* The effective turns in the field winding corresponding to full, normal, and weak field are in the ratios 1·0 : 0·667 : 0·5. The free-running speeds on level track with the above train are 33·5, 42·5, 47·5 m.p.h. for the three field strengths.

are applied at 95·8 seconds from the start (when the speed is 26·4 m.p.h.). The specific energy consumption is 68·7 watt-hours per ton mile, the total energy consumption is 5·41 kW-hours per train mile, and the r.m.s. current is 55·6 amperes.

Third, consider the train to operate on a service in which the average run of 5,100 ft has to be made at a schedule speed of 24·1 m.p.h., the duration of the stops being 20 seconds. The running time is 124 seconds.

In this case the speed-curve running period is made with the minimum

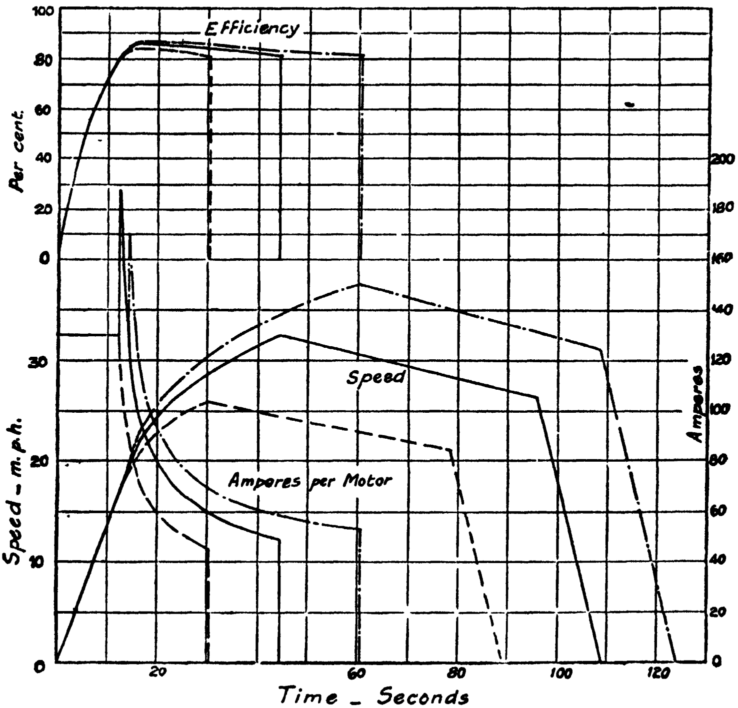


FIG. 260. SPEED, CURRENT AND EFFICIENCY CURVES FOR MOTOR-COACH TRAIN WITH FIELD-CONTROL EQUIPMENT

— — — — Full field ———— Normal field —·—· — Minimum field

number of field turns in circuit. The initial acceleration is made with the full field winding, the transition to the normal field winding is made at a speed of 17 m.p.h., and the transition from normal to weak field is made at a speed of 19·5 m.p.h.

Power is cut off at 60·7 seconds from the start (when the speed is 37·5 m.p.h.), and the brakes are applied at 108·5 seconds from the start (when the speed is 31 m.p.h.). The specific energy consumption is 72·6 watt-hours per ton mile, the total energy consumption is 5·72 kW-hours per train mile, and the r.m.s. current is 60 amperes.

Fig. 260 shows the speed-time curves for the above runs, together with the current and equipment-efficiency curves.

The equipment is also capable of operating this train to other fast suburban services, as shown below.

Distance between Stops		Duration of Stop	Schedule Speed	Running Time	Time of Cut off	Duration of Coasting Period	Specific Energy Consumption	R.M.S. Current per Motor
ft	miles	sec	m.p.h.	sec	sec	sec	Wh/t-mile	amp
5,370	1·02	20	25·5	124	79	27	80	62·2
6,150	1·17	,,	26·4	139	79	43·3	70	59·6
8,720	1·65	,,	30·3	176	127	29·4	69	58·6
10,560	2·0	,,	32·5	202	162	19	68	57·4

In all the above cases the equipment is not taxed to the limit of its capacity, and ample margin is allowed for making up time lost by slow-downs or signal checks. The hardest of the above schedules is the run of 5,370 ft at a schedule speed of 25·5 m.p.h. With easier schedules, the specific energy consumption would be lower, but the above may be considered as representative schedules for electric suburban services.

PART III.—ENERGY CONSUMPTION (ALTERNATING-CURRENT EQUIPMENTS)

The calculation of energy consumption with *single-phase equipments* is slightly more complicated than the above process, as the power factor has to be taken into account. Moreover, the current during the period of initial acceleration is not controlled by rheostats, but by the application of definite voltages to the motor. Hence every notch, if desired, can be used as a running notch.

The method of procedure is, in general, similar to that employed with d.c. equipments, but owing to the different method of speed control, the current during the initial period of acceleration is variable, and therefore the increment process must be applied to this period as well as to the succeeding periods. Curves of the accelerating tractive effort and speed (similar to Fig. 251) will, of course, be required for each operating voltage, and a knowledge of the currents at which the transitions are effected will also be necessary.

The calculations involve only the application of the principles already discussed in detail, and should, therefore, present no difficulty.

The curves of Figs. 261, 262, 263 have been calculated for a four-coach, 175-ton, suburban train (equipped with single-phase motors) when running on level track, between stations 4,600 ft apart at an average speed of 25·3 m.p.h. The current and power curves (Fig. 262) refer to the input to the high-voltage side of the transformer (i.e. the input from the 6,600-V trolley wire), the current curve, however, showing the equivalent current per motor and not the total current. The power factor and efficiency curves (Fig. 263) also refer to the high-voltage side of the transformer.

The energy consumption is 11·85 kW-hours per train mile, and the specific energy consumption is 67·4 watt-hours per ton mile.

The efficiency curve shows clearly that, although no rheostats are used during the starting period, the average efficiency is fairly low. Moreover, the average efficiency during the period of speed-curve running is below 80 per cent.

The low efficiency in the starting period is due to the low power factor and the relatively large losses in the motors.

The *sustained acceleration* (which is clearly shown in the speed-time curve

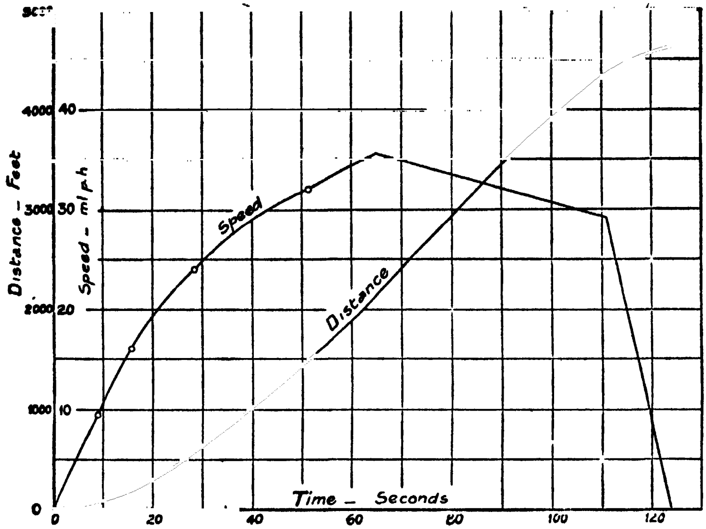


Fig. 261. Speed-Time Curve for Motor-coach Train with Single-phase Motors

Note.—The points marked by small circles indicate a change of controller notch.

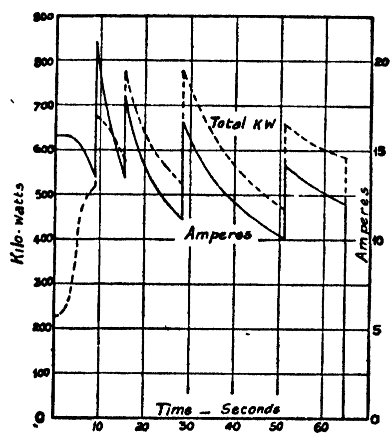


Fig. 262. Current and Power Curves for Motor-coach Train with Single-phase Motors

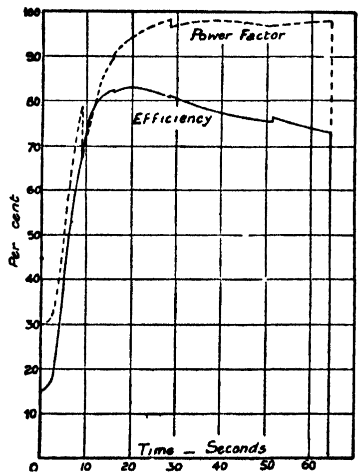


Fig. 263. Power-factor and Efficiency Curves for Motor-coach Train with Single-phase Motors

of Fig. 261) is a special feature of single-phase equipments, and is a result of the inherent characteristics of the motors and the method of voltage control.

With *three-phase equipments* having rheostatic control, the torque during

the initial accelerating period may be maintained approximately constant, and therefore this period can be calculated in a single step, similar to the direct-current case. The period of speed-curve running, however, is of extremely short duration, owing to the "flat" speed-torque characteristic of three-phase motors. This period is, therefore, calculated in a single step.

With multi-speed cascade equipments, starting is effected by rheostatic control—generally with automatically controlled rheostats—so that the calculations become very simple.

CHAPTER XX

TRAMWAY TRACK CONSTRUCTION

In this country street tramway track is laid with grooved girder rails on a solid foundation of 6 : 1 concrete, at least 6 in. thick. Standard railway practice, on the contrary, aims at a resilient track, the resiliency being obtained by supporting the rails, at frequent intervals, on chairs, and fixing the latter to wooden sleepers on a ballasted foundation. This difference in the track construction of tramways and railways is necessary in order that the tramway

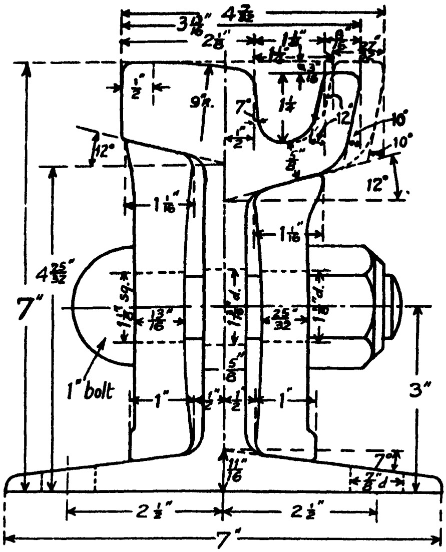


Fig. 264. Dimensions of B.S. No. 8 Tramway Rail and Fish-plates

Note.—Fish-plates are 2 ft long and are fastened with six bolts.

track may be suitable for ordinary vehicular traffic, and the grooved girder rail allows the paving to be brought up close to, and level with the top of, the rail.*

The *grooved girder rail* is standardized in four sections, varying from 96 to 113 lb per yard for straight track. Table XII gives the principal dimensions of these sections, and Fig. 264 shows details of one section. The *standard width of groove* for straight track is $1\frac{1}{8}$ in., but for curves below 150 ft radius the groove is widened to $1\frac{1}{4}$ in.

The lip, or check, is $1\frac{1}{4}$ in. below the tread on straight track, but is level with the tread on curved track. The position of the web of the rail and the width of the base are such that the resultant of the forces, due to the weight of the car and lateral pressure of the wheel flanges, falls well within the base.

* In tramways (or light railways) operating over a private right-of-way, the track is generally laid with sleepers and ballasted in the same manner as railway track.

Rails of standard section may be obtained in lengths of 35, 45, or 60 ft for straight track, and 35 ft for curved track. The 45- or 60-ft lengths are usually adopted on account of the reduced number of joints in the track.

The chemical composition is similar to that of steel for railway rails.

TABLE XII

DIMENSIONS OF BRITISH STANDARD TRAMWAY RAILS AND FISH-PLATES

Tramway Rails							Fish-plates	
B.S. Section	Weight of Rail (lb per yard)	Height of Rail	Width of Flange	Thickness of Web	Overall Width of Head and Lip	Width of Head at Tread	Weight of Inner Plate	Weight of Outer Plate
		in.	in.	in.	in.	in.	lb	lb
6	96·4	$6\frac{1}{2}$	$6\frac{3}{4}$	—	$3\frac{3}{4}$	$2\frac{1}{8}$	$22\frac{1}{2}$	27
6c	103·2	,,	,,	$\frac{13}{32}$, $\frac{15}{32}$	$4\frac{7}{32}$	,,	,,	,,
7	103·7	7	,,	—	$3\frac{13}{16}$	,,	26	$30\frac{1}{2}$
7c	109·7	,,	,,	$\frac{7}{16}$, $\frac{1}{2}$	$4\frac{7}{32}$	,,	,,	,,
8	112·9	,,	7	$\frac{5}{8}$	$3\frac{15}{16}$	,,	25	,,
8c	119·2	,,	,,	,,	$4\frac{7}{32}$	,,	,,	,,
10	95·4	5	6	$\frac{9}{16}$	$3\frac{13}{16}$	,,	$10\frac{1}{2}$	$13\frac{1}{2}$
10c	101·8	,,	,,	,,	$4\frac{7}{32}$	,,	,,	,,

NOTES. *c* denotes section for curved track. Sections Nos. 6–8*c* are in general use for street tramway track. The dwarf Sections, 10, 10*c* are intended for carrying heavy loads on sleeper track. The depth of groove is $1\frac{1}{16}$ in. for Nos. 6, 7; $1\frac{1}{4}$ in for Nos. 6*c*, 7*c*, 8, 10, and $1\frac{7}{16}$ in. for Nos. 8*c*, 10*c*. The lip is $\frac{1}{2}$ in. for No. 6; $\frac{9}{16}$ in. for Nos. 7, 8, 10; $\frac{23}{32}$ in. for Nos. 6*c*, 7*c*, 8*c*, 10*c*. Fish-plates are 2 ft long for Sections Nos. 6–8*c*, and 1 ft 6 in. long for Nos. 10, 10*c*.

Rail Joints. Individual rail lengths are connected together either mechanically by fish-plates or by welding to form one continuous length.

The *fish-plate joint* (Fig. 264) consists of two curved plates—placed one on each side of the web of the rail—with inclined or "fishing" surfaces fitting between the head and flange of the rail, and bolted together with six bolts. It is essential that the fishing surfaces should be a good fit, since the life and strength of the joint is dependent on this condition. As the life of a rail is largely influenced by wear at the joint, and the latter is inaccessible for inspection and adjustment, the importance of good joints on a tramway track is very great.

Welded joints are extensively used, as this method of jointing the rails eliminates track troubles due to faulty mechanical joints.

Special Trackwork. Trackwork at curves calls for special consideration where the radius is below 150 ft. In these cases the curve should be compounded with a spiral, in order to give an easier entrance, and to reduce the wear on wheel flanges and rails. With a spiral entrance the spreading of the track at the centre of the curve, to give the necessary statutory clearance of 15 in. between passing cars, is less than that required for a curve of uniform radius.

Points and crossings are required in connexion with special trackwork, such as loops or turnouts (passing places for cars on single lines), cross-overs, junctions, etc.

All points are made of manganese steel, with the tongues and grooves

shaped to standard radii (100, 150, 200, 300 and 350 ft). The overall length may be from 10 ft to 15 ft 6 in., with tongues from 7 ft 6 in. to 10 ft long.

Points can be divided into three classes, (i) open or fixed points, sometimes called "mates"; (ii) movable points; (iii) automatic or spring points.

Open points have no tongues, and are used in conjunction with movable points, but in some cases a pair of open points (or "mates") are used under trailing conditions.

A *movable point* has a tongue which is operated either by a crowbar or

(a) (b) (c)

FIG. 265. UNIVERSAL OPERATING MECHANISM OF HADFIELDS' POINT

(a) Movable point. (b) Automatic pulling point. (c) Automatic pushing point

through levers from the side of track. This class of point is used at turnouts, cross-overs, and junctions.

An *automatic point* has a tongue which is spring controlled, and is operated by the flanges of the car wheels. The tongue is kept in position for one direction of traffic by a spring, and is therefore self-setting when operated as a trailing point by a car coming from the other direction. An example is shown in Fig. 265. In this case the fulcrum of the rocking lever, A, is adjustable, and may occupy any of the three positions shown, according to the class of point desired.

The heel-end design of the tongue is shown in Fig. 266. The heel is held on its bearing surface (about 60 in.²) by means of a sliding block, and any wear is taken up automatically by means of a spring-operated adjusting wedge.

Crossings are usually single castings of manganese steel. *For turnouts and cross-overs they are standardized with* angles from 1 in $4\frac{1}{2}$ *to 1 in 8, and lengths varying from 7 ft to 11 ft.*

A *turnout* requires a crossing, an open point, and an automatic point at each end.

A *single junction* or *cross-over* requires at each end a crossing and a pair of

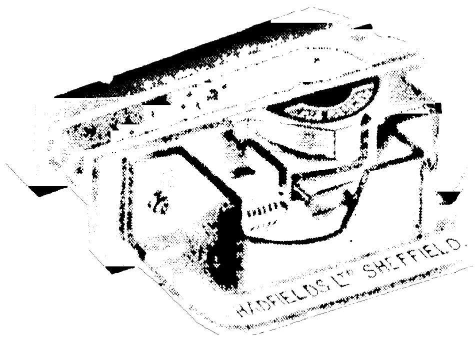


FIG. 266. ARRANGEMENT OF HEEL OF HADFIELDS' POINT

points, which may consist either of a pair of movable points or a movable point and an open mate.

Electrically-operated Points. At junctions in busy thoroughfares it is desirable to operate the points without manual labour, which can be arranged without difficulty on overhead systems. For example, the tongue of the point may be operated by an electro-magnet which is excited by the line current from a special overhead fitting in advance of the junction. If the point is required to be operated, the controller is kept in the first power position when passing this contact, while, if the car is to run through, the controller is kept "off."

CHAPTER XXI

THE TRAMWAY TRACK CONSIDERED AS AN ELECTRICAL CONDUCTOR

In tramways operating on the overhead system the track rails, in conjunction with feeder cables, form the return conductor. The electrical properties of the rails and track therefore require consideration.

The cross-sectional areas of rail sections Nos. 6, 7, 8 are 9·64 in.^{2}, 10·37 in.^{2}, and 11·29 in.^{2} respectively, and the resistances per foot are 9·33, 8·67, 7·96 microhms respectively.

Track as Electrical Distributing System. From these data we observe that the resistance of a mile of *continuous* rail is of the order of 0·05 ohm. But when a track is laid with commercial rail lengths jointed mechanically, the fish-plate joints have a high resistance relative to the rail. Therefore, to obtain good conductivity for the track as a whole, either this type of joint must be eliminated by welding the rails, or it must be supplemented with a good conductor or "bond."

A rail joint having a low resistance is necessary, not only for reasons of economy, but on account of statutory regulations, which limit the voltage drop in the rails to 7 volts and the voltage between any pipe and the rails to 1 volt positive and 3 volts negative.

The object of these regulations (which apply to tramways in Great Britain) is to minimize the corrosion of pipes and cable sheaths adjacent to the track which could be caused by leakage currents from the rails. Such currents, diverted to the pipes by high voltage drop in the rails, cause electrolytic corrosion to occur at places where the current leaves the pipes as, due to the moisture and salts in the surrounding soil, the combination of rail, soil and pipes forms an electrolytic cell, and therefore current passing through the cell causes metal to be removed from the anode, i.e. where the current leaves the pipe for the rails or other adjacent conductors.

In order to maintain the voltage drop in the rails within the limit of 7 volts, "negative" feeders will usually be necessary (i.e. cables connecting various points of the rails to the negative bus-bar). With a properly designed feeder system and the voltage drop in the rails within the above limit, there are practically no corrosion troubles, even on extensive tramway systems.

Bonds. Modern bonds are of the flexible type and are designed to be as short as the conditions of location will permit, in order to reduce to a minimum the resistance of the joint.

The bonds are usually fixed under the fish-plates, as shown in Fig. 267, as they are then protected from damage when repairs to the paving are being carried out. A bond having a cross-section of 0·166 in.^{2} consists of 24 strands of copper, 0·193 in. × 0·036 in., with terminals $\frac{7}{8}$ in. in diameter. The terminals are expanded into holes in the rails under a pressure of 20 to 30 tons.

In addition to bonds between the rail joints, the rails are *cross-bonded* at intervals of 40 yd, and on double track cross-bonds are fitted between the tracks at intervals of 80 to 100 yd. At junctions and special work, the rails should

be bonded independently of the points and crossings. For these purposes bonds consisting of stranded cable, with forged copper terminals, are employed, the heads of which are expanded into holes in the rail.

Resistance of Bonded Joint. The *contact resistance* of a bond terminal in a rail is largely influenced by the state of the hole and the terminal at the time of bonding. In order to obtain the minimum contact resistance it is necessary that the hole and the bond terminal be both clean, dry and bright. Tests have shown that the contact resistance of a $\frac{7}{8}$ in. terminal, in a rail with a web $\frac{1}{2}$ in.

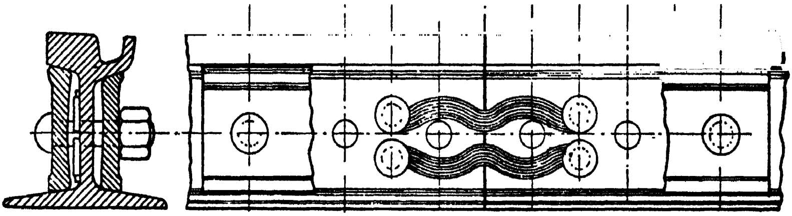


FIG. 267. LOCATION OF 8-IN. PROTECTED BONDS ON TRAMWAY RAIL

thick, may be as low as two microhms when the hole and bond are very clean. A thin film of oil or oxide on the bond or hole is sufficient to increase this resistance two to three times the original value. With fish-plates properly fitted, the resistance of a joint without bonds may be 20 microhms (or less), but this value will be considerably increased when any looseness occurs between the fish-plates and the rails. Taking the value of 20 microhms for the resistance of the fish-plate portion of the joint, the total resistance of a joint (for rails weighing 104 lb per yard), bonded with two 8 in., 0·166 in.^{2} bonds under the fish-plates, will be about 10 microhms,* which is equivalent to 15 in. of rail. If the contact resistance of the fish-plates increases, the resistance of the joint will increase, and would ultimately become (with infinite fish-plate resistance) 22 microhms, which is equivalent to 31 in. of rail.

* Obtained as follows: Conductor resistance of each bond = 38 microhms; contact resistances per bond (say), 7 microhms; resistance of two bonds fitted to rail, 22 microhms; combined resistance of bonds and fish-plates, 10·5 microhms (taking contact resistance of fish-plates as 20 microhms).

CONDUCTOR RAILS AND TRACK-WORK FOR ELECTRIC RAILWAYS

ON electric railways the track rails in many cases are used as the return conductor. Data of standard sections are given in Table XIII.

TABLE XIII

DATA OF STANDARD BULL-HEAD RAILWAY RAILS

B.S. section (indicating the weight in lb per yard	60	65	70	75	80	85	90	95	100
Height in rail (in.)	$4\frac{3}{4}$	$4\frac{7}{8}$	5	$5\frac{1}{8}$	$5\frac{3}{8}$	$5\frac{15}{32}$	$5\frac{35}{64}$	$5\frac{23}{32}$	$5\frac{29}{32}$
Width of head (in.)	$2\frac{5}{16}$	$2\frac{3}{8}$	$2\frac{7}{16}$	$2\frac{1}{2}$	$2\frac{9}{16}$	$2\frac{11}{16}$	$2\frac{3}{4}$	$2\frac{3}{4}$	$2\frac{3}{4}$
Thickness of web (in.)	$\frac{17}{32}$	$\frac{9}{16}$	$\frac{19}{32}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{21}{32}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
Cross-sectional area (in.^{2})	5·87	6·34	6·88	7·32	7·8	8·33	8·81	9·28	9·8
Perimeter (in.)	16·04	16·87	17·85	18·34	18·76	18·93	19·01	19·06	19·11

The *chemical composition* depends upon the process of steel manufacture and the carbon content. The limits for the impurities are—

Carbon	0·4 to 0·65 per cent
Manganese	0·7 to 1·0 per cent
Silicon	0·1 to 0·3 per cent
Phosphorus	0·05 to 0·06 per cent
Sulphur	0·05 to 0·06 per cent

The *resistivity* is about 13 times that of copper.

Conductor Rails. On low-voltage (600 V) d.c. railways the current is usually supplied to the trains through either one or two conductor rails, the latter being employed when the track rails are not used as the return conductor. The wear on conductor rails is only that due to the friction of the collector shoes (except at places where heavy currents have to be collected at starting and pitting occurs), and, as the strength of the rail is unimportant, the design, as far as the cost will permit, can be made from an electrical standpoint. The principal considerations, other than conductivity and cost, are: (i) the contact surface available for the collector shoes; (ii) the shape of the section (with reference to installation and insulators); (iii) the wearing qualities.

Steel is employed for reasons of economy, and the composition is arranged so that the highest conductivity is obtained consistent with the above conditions, the resistivity being about $6\frac{1}{2}$ times that of copper.

A typical composition is—iron 99·63 per cent, carbon 0·05 per cent, manganese 0·2 per cent, phosphorus 0·05 per cent, silicon 0·02 per cent, sulphur 0·05 per cent.

Classification of Conductor Rails. The conductor rails in general use may be divided into three classes according to the position of the contact surface, which may be at the top, bottom, or side of the rail.

The top-contact rail is adopted universally for low-voltage (600 V) electrifications in this country. The side-contact rail was developed for the 1,200-V Manchester–Bury section of the London Midland Region of British Railways. The under-contact rail is in use in America: it possesses the advantage over a top-contact rail that the contact surface is protected from snow, sleet and ice.

Type of Section and Mounting. Fig. 268 shows the type of rail and insulator which has been standardized for low-voltage electrifications in this country.

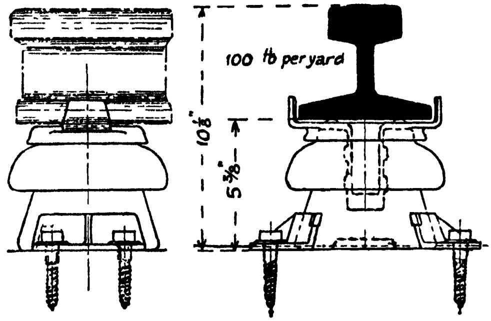
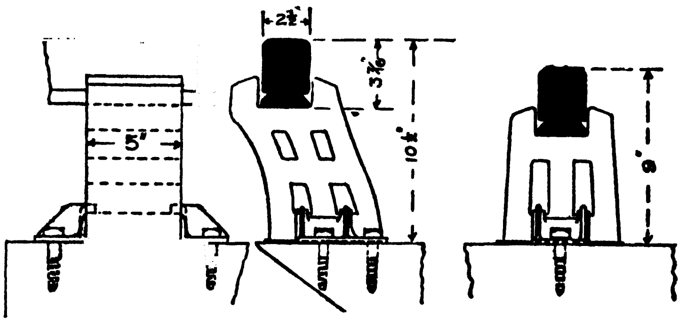


FIG. 268. METHOD OF MOUNTING FLAT-BOTTOM CONDUCTOR RAILS

The rail (weighing 100 to 150 lb per yard) rests on a malleable-iron cap which is fixed to the top of a petticoated "pedestal type" porcelain insulator, the base



(*a*) (*b*)

FIG. 269. METHOD OF MOUNTING RECTANGULAR CONDUCTOR RAILS
(*a*) Positive rail and insulator. (*b*) Negative rail and insulator.

of which rests directly upon one of the sleepers carrying the track rails, and is secured in position by two malleable-iron clamps.*

Fig. 269 shows the type of conductor rail and tubular earthenware insulator in use on the London tube railways. Pedestal-type insulators are also used.

The method of mounting low-voltage, under-contact conductor rails is shown in Fig. 270. The rail is supported from cast-iron brackets (fixed to the sleepers) by means of special porcelain insulators, which are in halves and are

* Proposed British Standard sections include flat-bottomed rails of 106 and 150 lb per yd, and a rectangular rail of 130 lb per yd. The width of the head is 3¼ in. for the flat-bottomed rails and 3½ in. for the rectangular rail.

held in position by a hook bolt. The portion of the rail between the insulators is protected by either wooden or fibre protection, so that only the lower (or contact) surface of the rail is exposed.

Fig. 271 shows the side-contact rail and method of mounting. The rail C

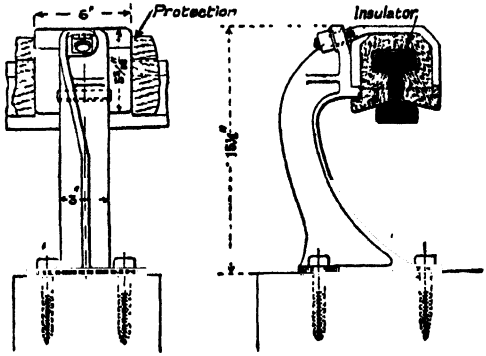


FIG. 270. METHOD OF MOUNTING UNDER-CONTACT CONDUCTOR RAILS

(weighing 85 lb per yard) rests upon a wooden block, B, recessed into the top of the porcelain insulator A. The guards, E, F, rest upon ledges on the insulator. Spring clips G, distance pieces, D, and keys, H, placed at intervals, maintain rail and guards in their correct positions.

In this country the following standard positions have been adopted for top-contact conductor rails—

(*a*) When the conductor rail is located between the track rails: centre-line of conductor rail to coincide with the centre-line of the track rails, and the top of the conductor rail to be 1½ in. above the top of track rails.

(*b*) When the conductor rail is located outside the track rails: centre-line of conductor rail to be 1 ft 4 in. from gauge line of nearest track rail, and the top of conductor rail to be 3 in. above the top of track rails.

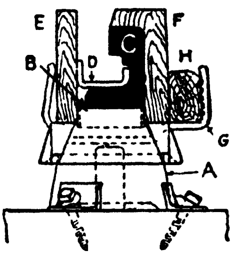


FIG. 271. SIDE-CONTACT CONDUCTOR RAIL

On American railways, where the under-contact rail, Fig. 270, is employed, the distance between the centre-line of the conductor rail and the gauge line of the nearest track rail is 2 ft 5 in., and the contact surface of the rail is 2¾ in. above the top of the track rail.

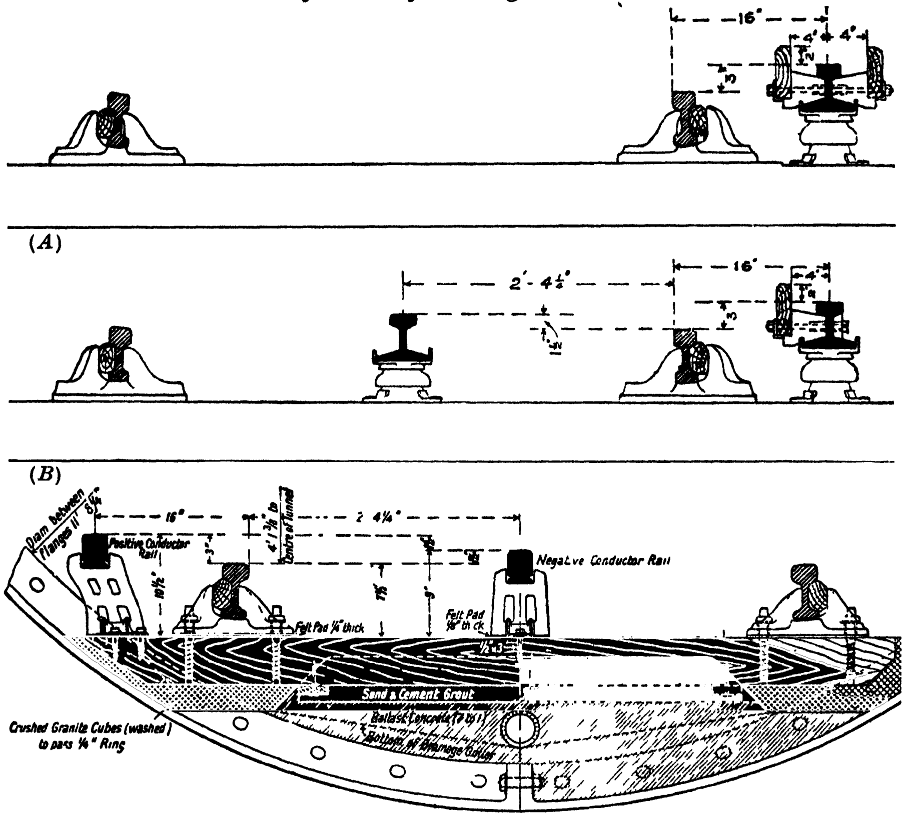
Fig. 272 shows cross-sections of the track on some British railways and the method of protecting the conductor rail (by wooden boards, 7 in. × 1½ in.) at stations, staff and other crossings.

Bonding of Conductor Rails. As the conductor rails are used solely as electrical conductors, and on a large system may have to carry currents of 2,000 amperes for short periods, it is necessary to bond the joints to the full current-carrying capacity of the rail, as shown in Fig. 273.

The Southern Region, British Railways, have recently developed special equipment, to facilitate quick renewals at night, for handling and installing 180-ft lengths of 150-lb rail (three 60-ft lengths being welded together at a depot), and welding at site these lengths to form a continuous rail ¼ mile long, thereby saving ½ ton of copper bonds. Each ¼-mile length is anchored and

adjacent lengths are connected by flexible bonds to allow for expansion and contraction due to changes of temperature.

Bonding of Track Rails. Each rail joint is bonded, and the rails of each single track, and also those of multiple tracks, are cross bonded at intervals. The bonds are now usually fixed by welding the terminal heads of the bond



(C)

FIG. 272. LOCATION OF CONDUCTOR RAILS ON SINGLE TRACK

A—Southern Region *B*—London surface and shallow-tunnel railways. *C*—London tube railways.

to the outer side of the heads of the rails. Impedance bonds are necessary at insulated joints in track-circuit signalling systems.

Feeder cables (and jumper cables) may be connected to the conductor rails by means of either several bonds with cable sockets, or a special copper plate (which is provided with cable sockets) bolted to the web of the rail.

At *cross-overs and special track-work* it is necessary to insert gaps in the conductor rails, the continuity of which is maintained by jumper cables. The ends of the rails are formed into ramps for the guidance of the collector shoes.*

* Details are given in "Conductor-rail Installation and Maintenance, and Collector Shoe Gear," Constant and Craig, *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 123.

In order that the supply of current to the train shall not be interrupted when it is passing over special track-work, the distance apart of the front and rear collector shoes must be greater than the longest gap in the conductor

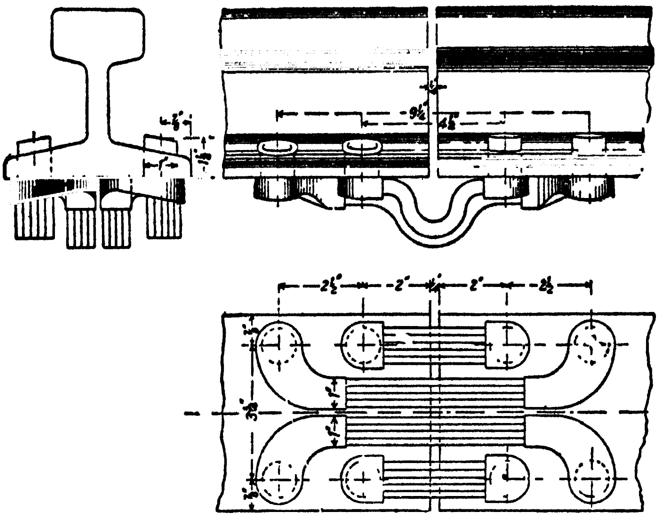


FIG. 273. BONDING FOR 100-LB CONDUCTOR RAILS

Note—Each bond has a cross-section of 0 35 in.[2]

rails. In some cases (for example, in locomotives, single motor coaches, and motor-coach trains for tube railways, where no "bus lines" are allowed on the train) it will not be possible to fulfil this requirement, and under these circumstances the train will have to coast over the gap.

The conductor rails are divided into *sections* of convenient length, and the

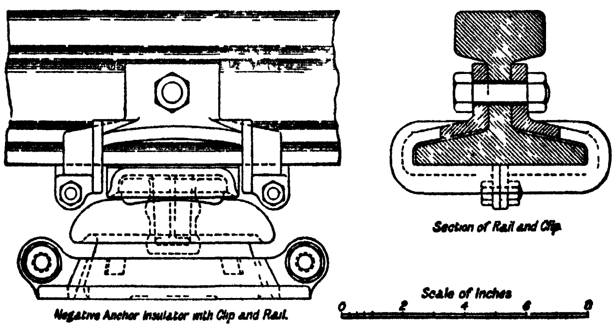


FIG. 274. DOULTON-SCOTT METHOD OF ANCHORING CONDUCTOR RAIL

section insulators are usually arranged near the sub-stations. (The feeding and switching arrangements for the conductor rails and section insulators are discussed in Chapter XXV.) Each section is anchored at one point, either by bolting to the rail a special anchor clip and anchoring one of the insulators, as shown in Fig. 274, or by means of insulated anchor ties.

CHAPTER XXIII

OVERHEAD CONSTRUCTION FOR TRAMWAYS AND TROLLEYBUS ROUTES

OVERHEAD construction has been standardized for a number of years, and modern installations show only improvements in a few details. Statutory regulations require the trolley wire to be erected at a minimum height of 17 ft above the street surface (except under bridges), and to be supported at intervals not greater than 120 ft. Further, each trolley wire must be divided into sections, not exceeding one-half of a mile in length, with an emergency switch between every two sections.

In addition to satisfying these requirements, the trolley wire must be designed to fulfil its function as an electrical conductor and to withstand the mechanical stresses due to temperature variation, etc.

Trolley Wire. The material in general use is hard-drawn copper, but alloyed wire has better wearing qualities, owing to its being uniformly hard throughout, whereas, with hard-drawn copper wire the hardness is confined to the outer skin.

The chief mechanical and electrical properties of copper and alloyed (cadmium-copper) wire are—

	Hard-drawn copper wire	Cadmium-copper wire
Ultimate tensile strength (tons per in.^{2})	23 to 26	30 to 36
Elastic limit (tons per in.^{2})	7·5 to 12·5	20 to 30
Young's modulus of elasticity (lb per in.^{2})	18×10^6	18×10^6
Specific resistance (60°F.) (ohm-in.)	$0{\cdot}69 \times 10^{-6}$	$0{\cdot}8 \times 10^{-6}$

The form of cross-section is grooved circular (Fig. 275), and the standard size has a cross-sectional area of 0·125 in.^{2}

Support of Trolley Wire. The trolley wire is supported and insulated from a transverse span wire by means of a steel bolt screwed into a gun-metal ear attached to the trolley wire, the bolt being insulated from, and held in, a hanger attached to the span wire. The span wire is supported from poles and is insulated therefrom.

A typical ear and hangers for the straight and curves are shown in Fig. 275.

On the straight the span wire supporting the hanger is above the level of the trolley wire, but at curves the span wire must be on a level with the trolley wire, otherwise the hanger will be pulled out of the vertical. Hangers for curves are called "*pull-offs*," the double pull-off being used when both sides of the span wire are in tension.

The *span wire* is of stranded galvanized or sherardized steel, having an ultimate tensile strength of 29 tons per in.^{2}, and elastic limit of 20 tons per in.^{2}. The sizes in common use are 7/0·125 in., 7/0·104 in., and 7/0·083 in., for which the average breaking loads are 4,550 lb, 3,950 lb, and 2,400 lb respectively.

The wire may extend the whole width of the road (in which case it is attached to poles on each side of the road), or a short length of span wire carrying the hangers may be attached to brackets carried from a horizontal arm which is

attached to a pole at one side of the road. The former arrangement is called *span-wire construction* and the latter *bracket-arm construction.*

Span-wire construction is suitable for any width of street. In narrow streets, with substantial buildings on either side, the span-wire may be attached to rosettes fixed to these buildings, thereby avoiding obstructions due to poles.

Bracket-arm construction has limited applications as the maximum length of the bracket-arm is 16 ft.

Typical insulators are shown in Fig. 276. The "globe" insulator consists of

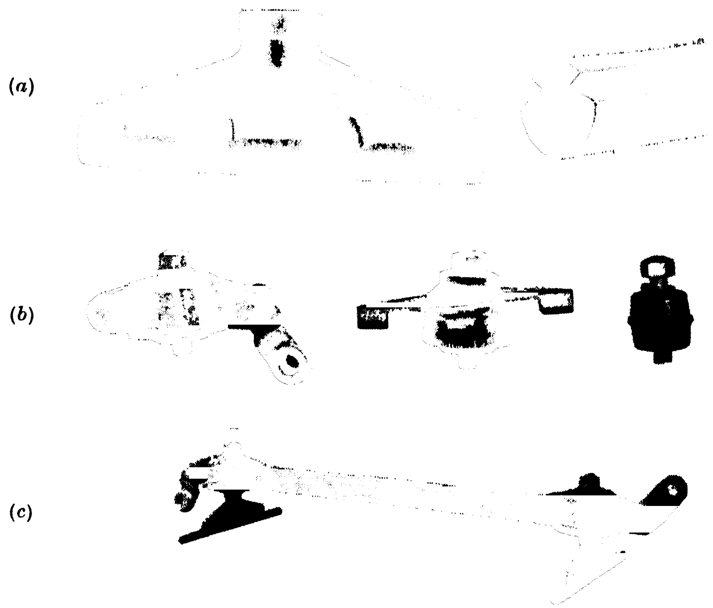


FIG. 275. TROLLEY WIRE AND SUSPENSION FITTINGS (BRITISH INSULATED CALLENDER'S CONSTRUCTION)

(*a*) Ear and trolley-wire, (*b*) pull-off, straight-line hanger and insulated suspension bolt, (*c*) twin hanger for trolleybus routes

two malleable iron castings, one of which is compressed over the other. The insulation between the castings is mica. Composition insulation is moulded around the central portion of the castings to protect the mica insulation from the weather and to increase the leakage surface. A typical insulator $2\frac{3}{4}$ in. in diameter has an average ultimate mechanical strength of 9,000 lb and a breakdown voltage, when dry, of about 14,000 volts. Corresponding values for the $4\frac{3}{8}$ in. $\times$ $2\frac{3}{8}$ in. size of porcelain insulator are 10,000 lb and 30,000 volts.

Construction at Curves. At curved track the trolley wire is maintained in position by means of pull-off wires. As the tension in some cases may be considerable, strain insulators must be employed for insulating the pull-off wires from the poles. The adjustment of the position of the trolley wire is effected by turnbuckles.

Fig. 277 is a diagram of the *overhead construction at a double-track right-angle*

curve. This diagram shows the arrangement of the pull-off wires and the method of anchoring the straight lengths of trolley wire so as to render the straight sections and curves mutually independent. Hence, if an accident occurs on one of the straight sections, its effects are not transmitted to the curve.

Special Fittings. At junctions, frogs and crossings are necessary for the

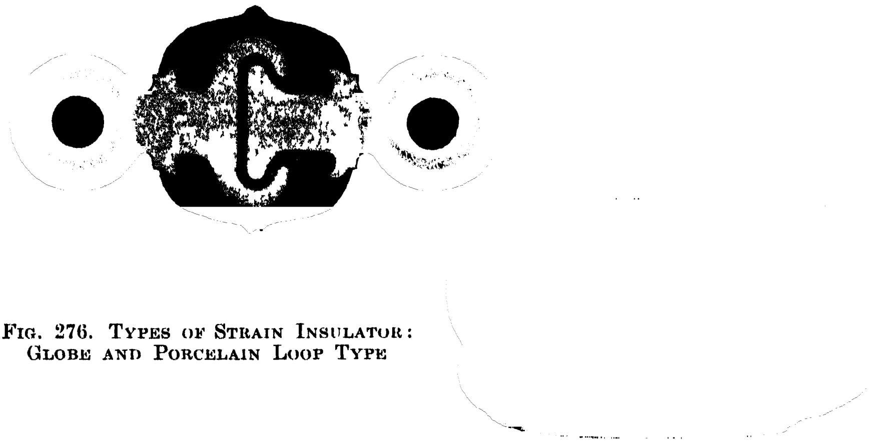


FIG. 276. TYPES OF STRAIN INSULATOR: GLOBE AND PORCELAIN LOOP TYPE

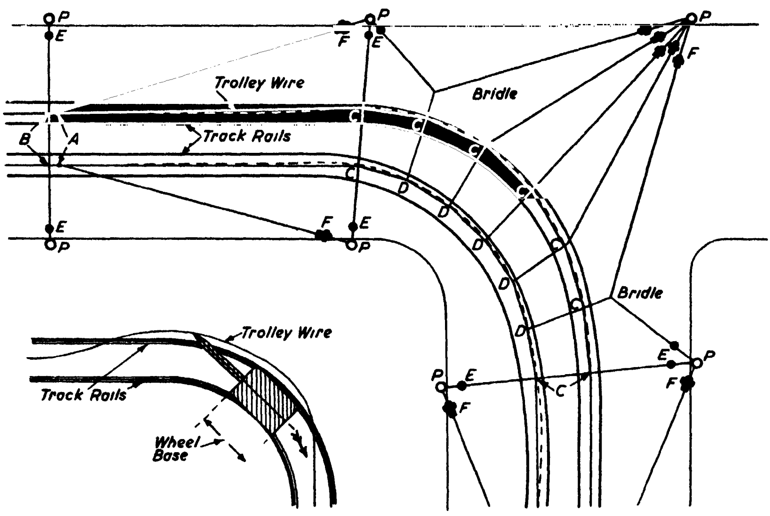


FIG. 277. LAYOUT OF TROLLEY-WIRES, SPAN WIRES AND FITTINGS AT A DOUBLE-TRACK RIGHT-ANGLE CURVE

A, anchor ear; *B*, straight-line hanger; *C*, double pull-off; *D*, single pull-off; *E*, strain insulator; *F*, adjustable strain insulator; *P*, pole. Inset shows method of locating position of trolley-wire on layout drawing.

guidance of the trolley wheel. *Frogs* are of two types, one being fitted with a movable tongue—for use at facing points—whilst the other is without a tongue and is intended for trailing use only. A *two-way switch frog* is shown in Fig. 278. The tongue is maintained in one position by a spring, and is moved to the other position by a wire operated by the point controller or pointsman.

A *section insulator* is shown in Fig. 279. The trolley wires are terminated at the ears, A, and an insulated central portion, B, provides a continuous surface for the collector. The strain due to the tension in the trolley wires is taken by rods, C, of insulating material.

Poles. These are of the tubular type, and usually consist of three steel tubes, of different diameters, shrunk together with telescopic joints. The

FIG. 278. TWO-WAY SWITCH FROG (BRITISH INSULATED CALLENDERS CONSTRUCTION)

overall length is 31 ft; the length of bottom section is 17 ft; the middle and top sections are each 8 ft 6 in. long; and each joint is 1 ft 6 in. long. Three standard sizes are in use, (i) "light," suitable for pulls up to 750 lb; (ii) "medium," suitable for pulls up to 1,250 lb, and (iii) "heavy," suitable for

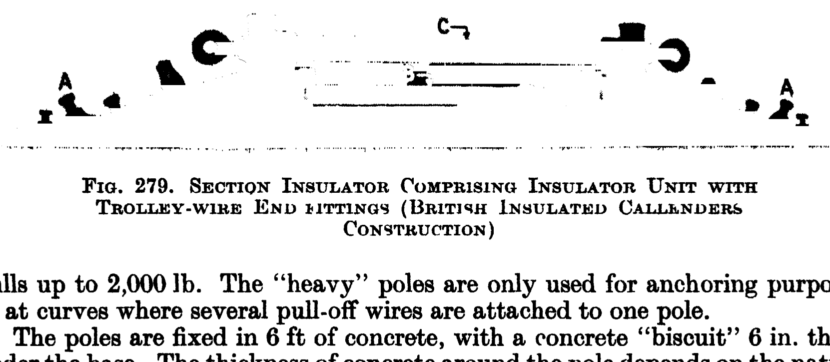


FIG. 279. SECTION INSULATOR COMPRISING INSULATOR UNIT WITH TROLLEY-WIRE END FITTINGS (BRITISH INSULATED CALLENDERS CONSTRUCTION)

pulls up to 2,000 lb. The "heavy" poles are only used for anchoring purposes or at curves where several pull-off wires are attached to one pole.

The poles are fixed in 6 ft of concrete, with a concrete "biscuit" 6 in. thick under the base. The thickness of concrete around the pole depends on the nature of the subsoil, and under normal conditions about 8 to 10 in. is sufficient. Each pole is set with sufficient rake so that the tension in the span wire will pull it vertical.

The *bracket arms* of side poles consist of steel tubing, $2\frac{1}{2}$ to 3 in. in diameter, fixed to the poles by means of collars and tie rods. On span-wire construction a short bracket arm is used.

CALCULATION OF STRESSES IN TROLLEY WIRE AND SPAN WIRE

Relationship between Sag and Tension for a Trolley Wire. A flexible wire, suspended horizontally and loaded only by its own weight, hangs in the form

of a catenary.* But if the sag is small in comparison with the span the catenary is practically identical with a parabola.† With tramway overhead construction, the sag in the trolley wire rarely exceeds 1 per cent of the span, and in these circumstances the wire may be considered to hang in the form of a parabola, in which case the sag will be proportional to the square of the span.

Thus, if w is the weight (in lb) of unit length of the wire and T is the tension (in lb), then, by taking moments about one of the supports (s, Fig. 280), we have

$$T\delta = wl \times \tfrac{1}{2}l,$$

whence $$T = wl^2/2\delta$$

and $$\delta = wl^2/2T \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The tension T is not uniform throughout the span, but increases slightly towards the supports. With small sags, however, the variation is practically negligible.

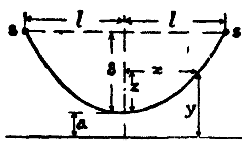


FIG. 280

In this country, trolley wire is usually erected with a sag of 9 in. in a span of 120 ft, at a temperature of about 65°F. Hence, applying equation (44) to these conditions, we have $l^2/2\delta = 60^2/(2 \times 0{\cdot}75) = 2{,}400$ so that $T = 2{,}400w$. The following values give the weight per foot of copper trolley wire—

Size of wire (S.W.G.)	00	000	0000
Weight per foot (lb)	0·367	0·419	0·484

Whence the tensions and stresses in these wires when erected (120 ft span, 9 in. sag) are—

Size of wire (S.W.G.)	00	000	0000
Diameter of wire (in.)	0·348	0·372	0·4
Area of cross-section (in.²)	0·095	0·1087	0·1257
Tension in lb (9-in. sag, 120 ft span) .	880	1,005	1,160
Stress (lb per in.²)	9,270	9,240	9,230

Effect of Temperature on Sag and Tension. A change in temperature produces changes in the sag and tension, but in calculating the sag at different temperatures the elasticity of the wire must be taken into account. For example, a decrease in temperature tends to reduce the sag, but the increased tension, due to the reduced sag, produces a stretching of the wire, so that the

* The general equation of the catenary is—

$$y = \tfrac{1}{2}a(\epsilon^{x/a} + \epsilon^{-x/a}) = a \cosh x/a,$$

where a is the distance from the origin to the vertex.

If z is the height of the point x, y, above the vertex (Fig. 280), then

$$z = y - a = a \cosh x/a - a = a\,(\cosh x/a - 1)$$

Expanding, we obtain

$$z = a(1 + (x/a)^2/2! + (x/a)^4/4! + \ldots - 1)$$
$$= (x^2/2a)[1 + (x/a)^2/12 + (x/a)^4/360 + \ldots]$$

which, if the second and succeeding terms are neglected, becomes

$$z = x^2/2a, \text{ or } x^2 = 2az,$$

which is the equation to a parabola.

† For example, if, in a catenary, the sag is 1 per cent of the span, the sag of a corresponding parabola will be $\frac{1}{75}$ of 1 per cent smaller than that of the catenary.

resulting change in the sag is smaller than that calculated by ignoring the elasticity of the material. Thus, consider a wire suspended between two horizontal supports, and let

$2l$ = distance between supports
L_0 = unstretched length of the wire at a temperature θ°F
L'_0 = unstretched length of the wire at a temperature θ_1°F
L = length of the wire at a temperature θ°F, when erected with a sag δ and a tension T
L_1 = length of the wire at a temperature θ_1°F, the corresponding values of the sag and tension being δ_1, T_1
α = coefficient of linear expansion of the material
E = Young's modulus of elasticity of the material
a = area of cross-section of the wire
w = weight of unit length of the wire

Then, at a temperature $\theta°$, the extension due to the elasticity of the wire is $L - L_0$, and the strain is $(L - L_0)/L_0$. The stress in the wire is T/a; and, as Young's modulus of elasticity = stress/strain, we have

$$E = (T/a)/[(L - L_0)/L_0]$$

from which the unstretched length is obtained as

$$L_0 = L(1 + T/aE)^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

or approximately $$L_0 = L(1 - T/aE) \quad . \quad . \quad . \quad . \quad . \quad (45a)$$

The length (L) corresponding to a sag of δ in a span $2l$ is given with sufficient accuracy by

$$L = 2l[1 + \tfrac{2}{3}(\delta/l)^2]^* \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Substituting in equation (45), we obtain

$$L_0 = 2l[1 + \tfrac{2}{3}(\delta/l)^2]\,(1 + T/aE)^{-1} \quad . \quad . \quad (45b)$$

Now, if the temperature increases to θ_1, the unstretched length becomes $L'_0 = L_0(1 + \alpha(\theta_1 - \theta))$, and when the wire is erected this length will be stretched to L_1, the corresponding values of the sag and tension being δ_1 and T'_1 respectively.

From equations (45, 46) we have

$$L_1 = L_0'(1 + T_1/aE)$$
$$= L_0[1 + \alpha(\theta_1 - \theta)]\,(1 + T_1/aE)$$

and $$L_1 = 2l[1 + \tfrac{2}{3}(\delta_1/l)^2]$$

* The length of any curve of which the equation is known is given by

$$L = \int\sqrt{[1 + (dy/dx)^2]}dx$$

Applying this to the catenary $y = a \cosh x/a$, we obtain $L = a \sinh x/a$.

Expanding, we have

$$L = a[x/a + (x/a)^3/3! + (x/a)^5/5! + \ldots]$$
$$= x[1 + \tfrac{1}{6}(x/a)^2]$$

if the third and succeeding terms are neglected.

In the case of a suspended wire, for which the span is $2l$, we have $x = 0$ at mid-span and $x = l$ at end of span. Hence, the length of wire in one-half of the span is given by

$$L = l[1 + \tfrac{1}{6}(wl/T)^2] = l[1 + \tfrac{2}{3}(\delta/l)^2]$$

Combining these equations with equation (45*b*) we obtain

$$2l\left\{1+\frac{2}{3}\left(\frac{\delta}{l}\right)^2\right\}\left(1+\frac{T}{aE}\right)^{-1}=2l\left\{1+\frac{2}{3}\left(\frac{\delta_1}{l}\right)^2\right\}\left(1+\frac{T_1}{aE}\right)^{-1}\left(\frac{1}{1+\alpha(\theta_1-\theta)}\right)$$

Substituting for δ and δ_1, we have

$$\left\{1+\frac{1}{6}\left(\frac{wl}{T}\right)^2\right\}\left(1+\frac{T_1}{aE}\right)(1+\alpha(\theta_1-\theta))=\left\{1+\frac{1}{6}\left(\frac{wl}{T_1}\right)^2\right\}\left(1+\frac{T}{aE}\right)$$

which, when simplified by neglecting the product of small quantities (such as $\alpha(\theta_1-\theta)(wl/T)^2$, $\frac{1}{6}T_1(wl/T)^2/aE$, etc.), reduces to

$$\alpha(\theta_1-\theta)+\tfrac{1}{6}(wl/T)^2-\tfrac{1}{6}(wl/T_1)^2+(T_1-T)/aE=0$$

or $$\alpha(\theta_1-\theta)=\tfrac{1}{6}(wl)^2(1/T_1^2-1/T^2)+(T-T_1)/aE \qquad (47)$$

From this equation we can easily calculate the change of temperature required to produce a given change in the tension. If, however, we require the tension T_1 corresponding to the temperature θ_1, the solution is not quite so easy, as it involves a cubic equation. Thus, expanding and re-arranging terms, we obtain

$$T_1^3+T_1^2aE[\alpha(\theta_1-\theta)+\tfrac{1}{6}(wl/T)^2-T/aE]=\tfrac{1}{6}aEw^2l^2$$

which may be written—

$$T_1^2\{T_1+aE[\alpha(\theta_1-\theta)+\tfrac{1}{6}(wl/T)^2-T/aE]\}=\tfrac{1}{6}aEw^2l^2$$

whence $$T_1=\sqrt{\frac{0{\cdot}166aE(wl)^2}{T_1+aE[\alpha(\theta_1-\theta)+\frac{1}{6}(wl/T)^2]-T}} \qquad (48)$$

or $$T_1=\sqrt{A/[T_1-(T-B)]}$$

where $$A=0{\cdot}166aE(wl)^2 \text{ and } B=aE[\alpha(\theta_1-\theta)+\tfrac{1}{6}(wl/T)^2]$$

This equation may be solved by assuming an appropriate value for T_1 and calculating the value of the radical, which should agree with the assumed value if the latter has been correctly chosen. A slide-rule greatly facilitates this method of solution.

Example. Determine the tension in a 120-ft span of copper trolley wire at temperatures of 40°F and 100°F having given that the trolley wire has been erected with a sag of 9 in. at a temperature of 65°F. Cross-sectional area, 0·1257 in.²

In this case

$$a=0{\cdot}1257 \text{ in.}^2,\ w=0{\cdot}484 \text{ lb},\ E=18\times10^6 \text{ lb per in.}^2$$

$$\alpha=0{\cdot}0000093, \text{ and from the example on p. 364, } T=1{,}160 \text{ lb}$$

Hence, substituting these values in equation (48), we obtain

$$T_1=1{,}000\sqrt{\{319/[T_1+21(\theta_1-65)+236-1{,}160]\}}$$

For a temperature of 40°F we have

$$T_1=1{,}000\sqrt{[319/(T_1-1{,}450)]}=1{,}578 \text{ lb}$$

At 100°F the equation for the tension becomes

$$T_1'=1{,}000\sqrt{[319/(T_1'-187)]}$$

which gives $T_1'=752$ lb.

The sags corresponding to these tensions are obtained from equation (44). Thus at 40°F, $\delta_1=0{\cdot}484\times60^2/(2\times1{,}578)=0{\cdot}552$ ft, or 6·63 in.; and at 100°F $\delta_1'=0{\cdot}484\times60^2/(2\times752)=1{\cdot}16$ ft, or 13·9 in.

Summarizing the results, we have

Temperature (degrees F)	40	65	100
Sag (in.)	6·63	9	13·9
Tension (lb)	1,578	1,160	752
Stress (lb per in.²)	12,550	9,230	6,000

The *sag to be given to a trolley wire* at any temperature should be such that, under the severest conditions of weather, the wire is not stretched beyond its elastic limit. In tramway practice these conditions are represented by a temperature of 22°F, and a horizontal wind pressure of 20 lb per ft². If D is the diameter of the trolley wire in inches, the wind pressure will produce a loading of $0{\cdot}6D \times 20/12 = D$ lb* per foot run of the wire, and this must be added vectorially to the weight per foot run in order to obtain the resultant

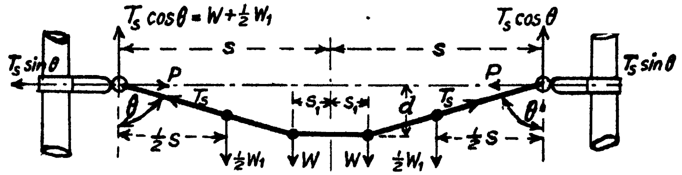


FIG. 281. DIAGRAM REPRESENTING GENERAL CONDITIONS OF LOADING OF SPAN WIRE

load. Thus, if w (lb) is the weight per foot of the wire, the resultant load per foot will be $w' = \sqrt{(w^2 + D^2)}$, and the tension T' is given by

$$T' = \sqrt{(l^2/2\delta')}\sqrt{(w^2 + D^2)} \quad . \quad . \quad . \quad . \quad (49)$$

where δ' is the sag corresponding to these conditions. This sag is, of course, to be measured in the plane of the wire, which will make an angle of $\tan^{-1}D/w$ to the vertical.

Applying these conditions to the above example, we obtain

$$w' = \sqrt{(0{\cdot}484^2 + 0{\cdot}4^2)} = 0{\cdot}628 \text{ lb}$$

and from equation (48),

$$T' = 1{,}000\sqrt{[535/(T' - 1{,}674)]} = 1{,}830 \text{ lb}$$

which is within the elastic limit.

Tension in Span Wires. The sag in a span wire depends on (i) weight supported; (ii) width of road; (iii) permissible tension.

In the general case of span-wire construction the conditions of loading are as represented in Fig. 281, in which

W = weight of each trolley wire and supporting device per span
W_1 = weight of the span wire
T_s = tension in the span wire
θ = inclination of the span wire to the vertical
$2s$ = horizontal distance between supports of span wire
$2s_1$ = distance apart of trolley wires
d = sag of span wire

By taking moments about the centre of the span we have

$$\begin{aligned} T_s \sin\theta \times d &= T_s \cos\theta \times s - \tfrac{1}{2}W_1 \times \tfrac{1}{2}s - Ws_1 \\ &= \{\tfrac{1}{2}(2W + W_1)s - \tfrac{1}{4}W_1 s - Ws_1\} \\ &= W(s - s_1) + \tfrac{1}{4}W_1 s \end{aligned}$$

* The total wind pressure on a cylindrical body = $0{\cdot}6 \times$ pressure on a flat surface of the same area as the projected area of the cylinder.

Writing $T_s \sin\theta = P$, we have

$$P = [W(s - s_1) + \tfrac{1}{4} W_1 s]/d \qquad (50)$$

Now $T_s \sin\theta$, or P, is the horizontal pull on the pole, hence

$$T_s = P/\sin\theta = P\sqrt{[d^2 + (s - s_1)^2]}/(s - s_1) = P\sqrt{[1 + d^2/(s - s_1)^2]}$$
$$= P[1 + d^2/2(s - s_1)^2]$$

The *weights of some typical overhead fittings* are given below—

Straight-line hanger with bolt	3 lb
Double pull-off with bolt	$4\frac{3}{4}$,,
Single pull-off with bolt	$4\frac{1}{4}$,,
12-in. ear	$1\frac{1}{2}$,,
6-in. ear	$\frac{3}{4}$,,
Globe strain insulator ($2\frac{1}{2}$ in. dia.)	1 ,,
Prescot loop type porcelain insulator ($4\frac{3}{8}$ in.)	1 ,,
Section insulator	18 ,,
Two-way switching frog	15 ,,
Two-way trailing frog	14 ,,
Crossing	18 ,,

Examples. 1. Determine the tension in a 7/0·125-in. span wire when supporting two 0·4-in. trolley wires 8 ft apart, the sag in the span wire being 1 ft. The poles of

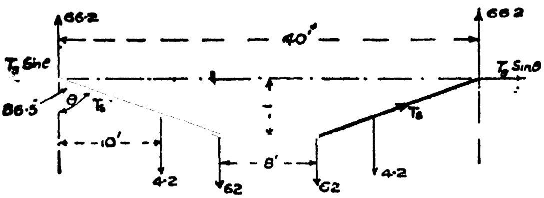


FIG. 282. LOADING OF SPAN WIRE IN EXAMPLE 1

each span are spaced at 43 ft apart, with a distance of 120 ft between spans, and the span wire is attached to bracket arms 18 in. long. We have

Weight of 120 ft 0·4-in. trolley wire = 0·484 × 120	= 58 lb
Weight of hanger, ear, etc.	= 4 ,,
	62 lb

Weight of 40 ft 7/0·125-in. span wire = 0·21 × 40 = 8·4 lb

The loading is shown in Fig. 282, and from equation (50) we obtain

$$P = 62(20 - 4) + \tfrac{1}{4} \times 8{\cdot}4 \times 20 = 1{,}031 \text{ lb}$$

and $$T_s = 1{,}034[1 + 1/(2 \times 16^2)] = 1{,}036 \text{ lb}$$

2. Similar conditions to above, but section insulators fitted to the trolley wires. We have

Weight of 120 ft 0·4-in. trolley wire	= 58 lb
Weight of section insulator with strain insulators	= 20 ,,
	78 lb

Assuming that the 7/0·125-in. span wire is used, we obtain,

$$P = 1{,}283 \text{ lb, and } T_s = 1{,}285{\cdot}5 \text{ lb.}$$

If the trolley wires are anchored, the span wire supporting the section insulators will be relieved of a portion of the load. A reference to p. 363 will show that the "medium" pole will be suitable for these cases.

Calculation of Tension in Pull-off Wires. Consider a curve on a tramway system where a single trolley wire is supported from span wires arranged, radially as in Fig. 283. The points of attachment of the pull-offs to the trolley wire are arranged on an arc of a circle of radius R ft. The trolley wire, in virtue of its tension, will form a series of chords in this circle. If T is the tension in the trolley wire, the horizontal component of this in the direction of the span wire will be $2T \cos \alpha$. As the triangles ABD and AOE are similar, we have

$$\cos \alpha = AD/AB = AE/AO = \tfrac{1}{2}l_1/R$$

In addition to this tension, the span wire is loaded with a tension due to the

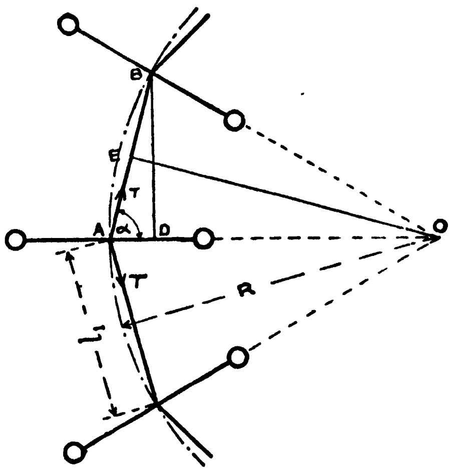


FIG. 283. DIAGRAM SHOWING POSITION OF TROLLEY WIRE ON A CURVE

weight of the pull-off, trolley wire, etc. This additional tension can be calculated by a method similar to that adopted with span wire construction. If P_1' represents the horizontal force due to the weight of the trolley wire and fittings, the pull on the poles to which the span wire is attached will be

$$P_1 = P_1' + Tl_1/R \quad . \quad . \quad . \quad . \quad . \quad (51)$$

on the outside of the curve, and

$$P_2 = P_1' - Tl_1/R \quad . \quad . \quad . \quad . \quad (51a)$$

on the inside of the curve.

The tensions in each portion of the span wire will be $P_1/\sin \theta_1$ and $P_2/\sin \theta_2$, where θ_1, θ_2 are the inclinations of the span wire to the vertical.

When Tl_1/R is greater than P_1', the portion of the span wire on the inside of the curve is not required. In this case it is usual to arrange the pull-off wires as in Fig. 277, and the problem does not permit of such an easy solution as above, due to an incomplete knowledge of the tension in the trolley wire, which in practice is adjusted to some arbitrary value when the line is erected. The method of calculation is better elucidated by working through an example.

Example. Determine the tensions in the pull-off wires and the sizes of poles for a single 0·4-in. trolley wire on a right-angle curve, of which a layout showing the positions available for the poles, is given in Fig. 284. There are seven pull-offs,

which are attached to the trolley wire at a radius of 50 ft. The tension in the trolley wire on the curve may be assumed at 700 lb. At each end of the curve there is a straight line which is suitably anchored, the tension in the anchor wire being assumed at 400 lb. The length of the first span of the straight sections is 60 ft.

The tension in the wires attached to pull-offs Nos. 3, 4, 5 can be obtained from a slight modification of equation (51). Thus, if ϕ is the angle between the pull-off

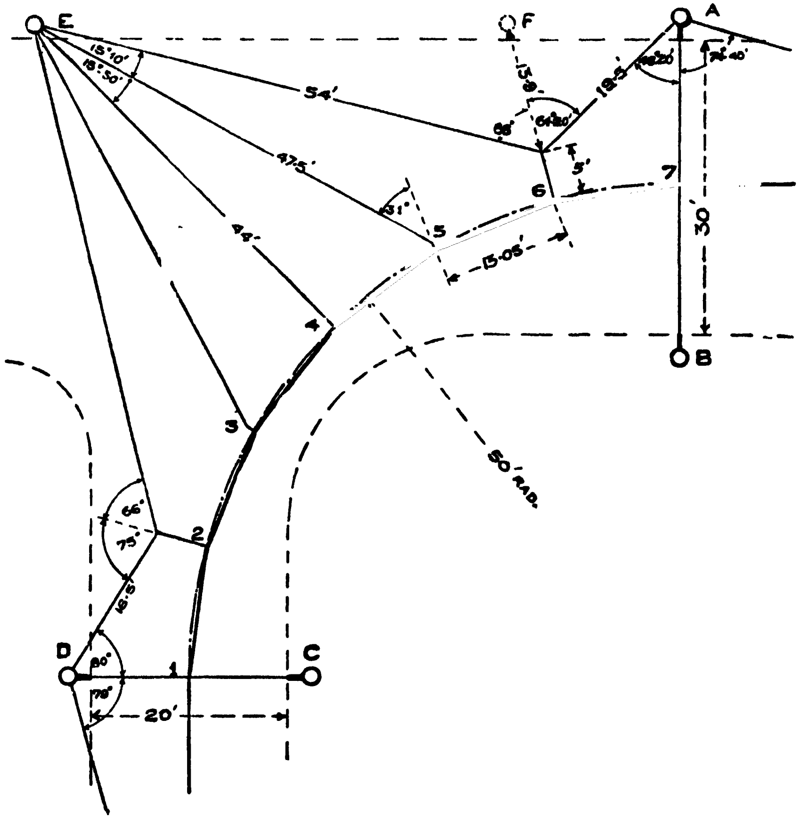


FIG. 284. LAYOUT OF OVERHEAD CONSTRUCTION FOR RIGHT-ANGLE CURVE

wire and a radius vector produced (Fig. 284), the horizontal pull on the pole is given by

$$P_1 = P_1' + Tl_1/R \cos \phi$$

where the first term is the pull due to the weight of trolley wire, fittings, etc., attached to the pull-off, and the second term is the component of the trolley-wire tension in the direction of the pull-off wire. To obtain P_1', let

W = weight of trolley wire, ear, and pull-off
W_2 = weight of pull-off wire
s = length of pull-off wire
d = sag in pull-off wire.

Then

$$P_1' = (Ws + \tfrac{1}{2}W_2 s)/d$$

If we assume that 7/0·125-in. steel wire is used for all the pull-off and span wires, then the weight of this wire for No. 4 pull-off will be $44 \times 0{\cdot}21 = 9{\cdot}2$ lb* and $47{\cdot}5 \times 0{\cdot}21 = 10$ lb for pull-offs Nos. 3 and 5. The distance between the pull-offs being 13·05 ft, we have: weight of trolley wire, ear, and pull-off $= 13{\cdot}05 \times 0{\cdot}484 + 1 + 3{\cdot}5 = 10{\cdot}8$ lb. If the sag in the pull-off wires is 4 ft, we obtain $P_1' = \frac{1}{4}(10{\cdot}8 \times 44 + 9{\cdot}2 \times 22) = 169$ lb for No. 4, and 188 lb for Nos. 3 and 5.

Hence, the tension in pull-off wire No. 4, which is radial, will be

$$169 + 700 \times 13{\cdot}05/50 = 169 + 183 = 352 \text{ lb},$$

and the tension in Nos. 3 and 5 will be

$$188 + 183/\cos 31^\circ = 188 + 213 = 401 \text{ lb}$$

We have now to deal with the bridles at Nos. 2 and 6, which can be done in the following manner. Consider the bridle at No. 6 replaced by a radial pull-off wire attached to a pole at F; calculate the tension in this wire, and resolve this tension into components along the directions of the bridle.† In this manner we obtain 266 lb for the tension in the radial pull-off wire, and $266/(\cos 66^\circ + \cos 61{\cdot}3^\circ) = 300$ lb in the bridle wire. Treating the bridle at No. 2 in a similar manner, we obtain the tension in the bridle wire as 360 lb.

The span wires Nos. 1 and 7 now demand attention. If we assume the sag in the span wire as 1 ft, we obtain, from equation (50), values of 128 lb and 201 lb for the tension in No. 1 and No. 7 respectively.

The resultant pull on pole E will be in the direction of pull-off wire No. 4, and its magnitude will be

$$352 + 2 \times 401 \cos 15{\cdot}83^\circ + (300 + 360) \cos 31^\circ = 1{,}690 \text{ lb}$$

The resultant pull on pole A is 540 lb, at an angle of 18° with the span wire (straight-line side), while the resultant pull on pole D is 393 lb at an angle of 12° with the span wire (straight-line side).

Summarizing, we have the following values for the pull on the poles: A, 540 lb; B, 201 lb; C, 128 lb; D, 393 lb; E, 1,690 lb; while the tensions in the respective span and pull-off wires are: No. 1, 128 lb; No. 2, (bridle), 360 lb; No. 3, 401 lb; No. 4, 352 lb; No. 5, 401 lb; No. 6 (bridle), 300 lb; No. 7, 201 lb; anchor wires, 400 lb.

Pole E must therefore be of the "heavy" class, while for the poles A, B, C, D the "light" class could be used.

The calculations for a double track are performed in a similar manner, but in this case the tension in the outer pull-off wires will be $P_1' + Tl_1/R \cos \phi$, and the tension in the wire between the trolley wires will be Tl/R.

If, with the assumed value for the tension in the trolley wire, the pull on the poles is excessive, then a lower value must be adopted and care taken that this value is not exceeded when the line is erected.

* The effect of the inclination of the pull-off wires is neglected in the calculations of weight and tension.

† As the bridle wire is threaded through a ring at the end of the pull-off wire, the tension in each portion of it will be the same.

CHAPTER XXIV

OVERHEAD CONSTRUCTION ON RAILWAYS

PART I. GENERAL CONSIDERATIONS

OVERHEAD construction is desirable for operating voltages above 750 volts. A sliding collector (bow or pantograph) is employed: its use, however, necessitates a level trolley wire in order that contact between the wire and collector may be maintained at high speeds.

The trolley wire must, therefore, be suspended with a very small sag, and to obtain this result without excessive tension in the wire, the span must be relatively short, i.e. of the order of 10 ft to 15 ft. For such short spans an

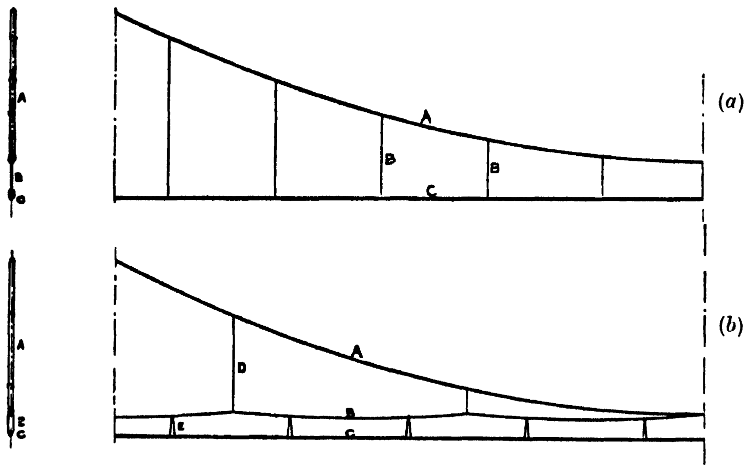


FIG. 285. TYPES OF CATENARY CONSTRUCTION
(a) Single catenary. (b) Compound catenary.

indirect method of suspension is desirable, the trolley wire being supported by another wire, which is suspended with considerable sag between supports fixed at moderate distances apart. The wire from which the trolley wire is supported is called the "catenary" or "messenger," and if this wire is insulated from the supports, no insulated hangers are necessary for the trolley wire. The object of the large sag in the catenary wire is to maintain the position of the trolley wire practically constant for the range of temperature occurring in practice.

Catenary Construction. Present-day catenary construction on railways is of either the single catenary or the compound catenary. The former consists of a suspended steel wire A, Fig. 285 (a), from which the trolley wire C is supported by means of droppers, B, clipped to A and C at equidistant horizontal intervals. On straight (or tangent) track the span of the catenary wire may be from 150 ft to 300 ft, with sags of from 3 ft to 6 ft respectively, and the distance apart of the droppers varies from 10 ft to 15 ft. On curves the span is reduced and the trolley wire is maintained in position by pull-off wires.

The compound catenary construction consists of three wires, all in the same vertical plane. The upper wire A, Fig. 285 (b), is the catenary wire, and is insulated from the supporting structures. From this wire the intermediate wire B is supported by droppers, D, clipped to both wires, and the trolley wire C is supported from the intermediate wire by the loops E. The trolley wire is maintained under a definite and constant tension by means of automatic tightening gear, and the loops E allow longitudinal movement of the trolley wire to take place without straining the suspension wires.

The *catenary wire* is usually of stranded steel, of seven or more strands, having the following average properties—

Ultimate tensile strength . . .	90 tons per in.^{2}
Elastic limit	40 tons per in.^{2}
Modulus of elasticity	30×10^6 lb per in.^{2}
Coefficient of linear expansion (per 1°F) .	0·0000064

Calculation of Tension in Catenary Wire. The sag of the catenary wire generally does not exceed three per cent of the span, and if the wire were

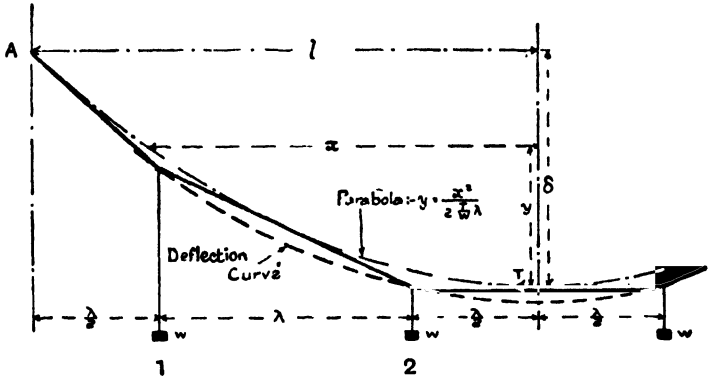


FIG. 286. LOADING OF CATENARY WIRE DUE TO TROLLEY WIRE

loaded only with its own weight, the tension could be calculated in the same manner as for a tramway trolley wire, as the curve of the sag could be considered to be a parabola.* The effect of the trolley wire is equivalent to a series of equal weights hung from the catenary wire at equi-distant intervals. If the weight of the catenary wire is neglected, the sections between the droppers are straight lines, and form the envelope to the parabola $y = x^2/2(T/W)\lambda$, where y is the ordinate corresponding to a distance x from the centre of the span, T the horizontal tension at the mid-span, and λ the distance apart of the weights W (Fig. 286).

The curve† joining the points of attachment of the droppers to the catenary wire differs slightly from this parabola,‡ as each section of the catenary wire is tangential to the latter.

* With a sag of 3 per cent of the span the error in calculating the sag from the parabola equation (instead of the catenary) is 0·2 per cent, while with a 2 per cent sag the error is 0·05 per cent.

† Called hereafter the "deflection" curve.

‡ It can be shown that at each dropper the deflection curve is (W/T) $(\lambda/8)$ vertically below the corresponding point on the above parabola.

In practice, the weight of the catenary wire is comparable with that of the trolley wire, so that the deflection curve will be intermediate between the above curve and the parabola $y = x^2/2T/w'$ (w' being the average weight of the trolley wire, catenary wire and droppers per foot of horizontal span).

The tension T is determined by considering one-half of the span and taking moments of all forces about the point of support (A, Fig. 286). Since the sag is small in comparison with the span, the weight of the catenary wire per horizontal foot of span may be considered to be identical with the weight per foot of wire. Hence w' may be considered as equivalent to the average weight of the whole suspension per foot run.

Considering an *even* number of droppers per span and taking moments about A (Fig. 286), we have—

$$\begin{aligned} T\delta &= \tfrac{1}{2}\lambda(w\lambda + w_1\lambda + w_2) + \tfrac{3}{2}\lambda(w\lambda + w_1\lambda + w_2) + \tfrac{5}{2}\lambda(w\lambda + w_1\lambda + w_2) + \ldots \\ &= \tfrac{1}{2}[\lambda(w + w_1) + w_2]\lambda(1 + 3 + 5 + \ldots + (2n - 1)) \\ &= \tfrac{1}{2}[\lambda(w + w_1) + w_2]\lambda n^{2*} \\ &= \tfrac{1}{2}l^2(w + w_1 + w_2/\lambda) \\ &= \tfrac{1}{2}w'l^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52) \end{aligned}$$

where w, w_1, denote the weight per foot of trolley wire and catenary wire respectively: w_2 denotes the average weight of a dropper and clips; $2l$ the distance (in feet) between supporting structures: δ the sag (in feet) of catenary wire at mid-span; λ the horizontal spacing (in feet) of the droppers; n the number of droppers per half-span, the centre one, if any, not being included.

For an *odd* number of droppers per span, we have

$$\begin{aligned} T\delta &= \tfrac{1}{2}(w + w_1)l^2 + w_2\lambda\tfrac{1}{2}(1 + 3 + 5 + \ldots + (2n + 1)) \\ &= \tfrac{1}{2}(w + w_1)l^2 + \tfrac{1}{2}w_2\lambda(n + 1)^2\dagger \\ &= \tfrac{1}{2}l^2(w + w_1) + \tfrac{1}{2}w_2\lambda(l^2/\lambda^2 + l/\lambda + \tfrac{1}{4}) \\ &= \tfrac{1}{2}l^2(w + w_1 + w_2/\lambda) + \tfrac{1}{2}w_2\lambda(l/\lambda + \tfrac{1}{4}) \\ &= \tfrac{1}{2}w'l^2 + \tfrac{1}{2}w_2\lambda(n + \tfrac{3}{4}) \quad . \quad . \quad . \quad . \quad . \quad (52a) \end{aligned}$$

Calculation of Lengths of Droppers. The deflection at any dropper can be obtained by taking moments, about the point of the catenary wire at which the dropper is attached, and dividing by the tension T. Thus for dropper No. 1 (Fig. 286) we have (assuming an *even* number of droppers per span)

$$Ty_1 = \tfrac{1}{2}(w + w_1)(l - \tfrac{1}{2}\lambda)^2 + w_2\lambda(1 + 2 + 3 + \ldots + (n - 1))$$

and for dropper No. 2,

$$Ty_2 = \tfrac{1}{2}(w + w_1)(l - \tfrac{3}{2}\lambda)^2 + w_2\lambda(1 + 2 + 3 + \ldots + (n - 2))$$

or generally,

$$Ty_m = \tfrac{1}{2}(w + w_1)[l - \tfrac{1}{2}\lambda(2m - 1)]^2 + w_2\lambda\tfrac{1}{2}[(n - (m - 1))(n - m)]\ddagger \quad . \quad (53)$$

where m is the number assigned to the dropper, the one nearest to the support (A, Fig. 286) being No. 1.

* The number of terms in the series $1 + 3 + 5 + \ldots + (2n - 1)$ is n; hence the sum $= \frac{1}{2}n(1 + (2n - 1)) = n^2$. [NOTE. $n = l/\lambda$.]

† The number of terms in the series $1 + 3 + 5 + \ldots + (2n + 1)$ is $n + 1$; hence the sum $= \frac{1}{2}(n + 1)[1 + (2n + 1)] = (n + 1)^2$. [NOTE $n = l/\lambda - \frac{1}{2}$.]

‡ The series for the mth dropper is $1 + 2 + 3 + \ldots + (n - m)$, the number of terms being $(n - m)$. The sum of this series is $\frac{1}{2}(n - m)(1 + (m - n))$, or $\frac{1}{2}(n - (m - 1))(n - m)$. For an odd number of droppers per span, the sum of the series becomes $\frac{1}{2}(n - (m - 1))(n - (m - 2))$, and equation (53) takes the form

$$Ty_m = \tfrac{1}{2}(w + w_1)\{l - \tfrac{1}{2}\lambda(2m - 1)\}^2 + \tfrac{1}{2}w\{\lambda(n - (m - 1)\}\{n - (m - 2)\} \quad . \quad (53a)$$

Calling these moments $M_1, M_2, \ldots M_m$, we have

$$Ty_1 = M_1,\ Ty_2 = M_2,\ Ty_m = M_m$$

whence $$y_1 = M_1/T,\ y_2 = M_2/T,\ \ldots\ y_m = M_m/T. \qquad (54)$$

The lengths of the droppers will be $(y_1 + z)$, $(y_2 + z)$, etc., where z is the distance between the catenary wire and the trolley wire at mid-span.

In cases where approximate results are sufficient, the deflection at each dropper can be obtained by assuming that the deflection curve is the parabola $y = x^2/2T/w'$, y being the deflection at a distance x from mid-span.

Example. Calculation of the deflections and lengths of droppers for an eleven-point single catenary suspension having the following constants—

Length of span ($2l$)	150 ft
Sag at mid-span (δ)	2·5 ft (30 in.)
Distance between catenary wire and trolley wire at mid-span (z)	0·66 ft (8 in.)
Numbers of droppers per span	11
Spacing of droppers (λ)	13·63 ft
Size of trolley wire	0·46 in. dia. (= 4/0 B. & S.G.)
Weight per foot of trolley wire (w)	0·641 lb
Size of catenary wire	7/16 in. dia.
Weight per foot of catenary wire (w_1)	0·384 lb
Average weight of dropper and clips (w_2)	0·5 lb

The average weight (w') of trolley wire, catenary wire, dropper, and clips per foot run

$$= 0{\cdot}641 + 0{\cdot}384 + 0{\cdot}5/13{\cdot}63 = 1{\cdot}062 \text{ lb}$$

Since there are 11 droppers per span, the value of n in equations (52, 53) will be 5. Hence the tension in the catenary wire at mid-span—obtained from equation (52a)—is

$$T = (\tfrac{1}{2} \times 1{\cdot}062 \times 75^2 + \tfrac{1}{2} \times 0{\cdot}5 \times 13{\cdot}63 \times 5{\cdot}75)/2{\cdot}5 = 1{,}214 \text{ lb}$$

From equation (53a) values for M_1, M_2, etc., are obtained as follows—

$$\begin{aligned} M_1 &= \tfrac{1}{2}(0{\cdot}641 + 0{\cdot}384)\{75 - \tfrac{1}{2} \times 13{\cdot}63(2 \times 1 - 1)\}^2 \\ &\quad + \tfrac{1}{2} \times 0{\cdot}5 \times 13{\cdot}63\,\{(5 - (1 - 1))(5 - (1 - 2))\} \\ &= 2{,}482 \end{aligned}$$

$$M_2 = 1{,}593 \qquad M_4 = 402{\cdot}5$$
$$M_3 = 897{\cdot}4 \qquad M_5 = 102$$

The deflections are obtained by dividing these values by the tension (1,214 lb), and the lengths of the droppers will be 8 in. greater than the deflections.

A summary of the results obtained by this method and by the approximate method is given in Table XIV.

TABLE XIV

LENGTHS OF DROPPERS FOR SINGLE CATENARY SUSPENSION

Dropper No.	1	2	3	4	5	6	7	8	9	10	11
Distance from mid-span (ft)	68·2	54·5	40·9	27·26	13·63	0	13·63	27 26	40·9	54 5	68·2
Deflection—by method of moments (ft)	2·043	1·312	0·74	0·332	0·084	0	0 084	0·332	0·74	1·312	2·048
Deflection—by approximate method (ft)	2·063	1·323	0·743	0·33	0·0862	0	0·0862	0·33	0·743	1·323	2·068
Length of dropper (in.)	32·5	23·8	16·9	12	9·01	8	9·01	12	16·9	23·8	32·5
Approximate length of dropper (in.)	32·8	23·9	16·9	11·96	9	8	9	11·96	16·9	23·9	32·8

In applying the approximate method—i.e. assuming the deflection curve to be a parabola—we have

$$T = w'l^2/2\delta = 1{,}195 \text{ lb, and } y = x^2/(2 \times 1{,}195/1{\cdot}062) = x^2/2{,}250$$

as the equation to the deflection curve.

Effect of Temperature on Level of Trolley Wire. The level of the trolley wire will be affected to some extent by changes of temperature, the variation of level depending upon the variation of the sag of the catenary wire with temperature. In considering the effects of temperature on the catenary wire, the latter will be treated as a simple catenary.

Thus, if l = length (in feet) of half the span

w' = equivalent weight per foot of catenary wire, trolley wire, and droppers

a = area of cross-section of catenary wire

α = coefficient of linear expansion

E = modulus of elasticity

T, T_1 = tension at lowest point of catenary wire corresponding to temperatures θ, θ_1 respectively

δ, δ_1 = sag at mid-span corresponding to temperatures θ, θ_1, respectively

then $\alpha(\theta_1 - \theta) = \frac{1}{6}(w'l)^2\,(1/T_1^2 - 1/T^2) + (T - T_1)/aE.$

Now $\delta = w'l^2/2T$, and $\delta_1 = w'l^2/2T_1$; therefore, on substituting for T and T_1, we obtain

$$\alpha(\theta_1 - \theta) = \tfrac{2}{3}(\delta_1^2 - \delta^2)/l^2 - (w'l^2/2aE)\,(1/\delta_1 - 1/\delta) \quad . \quad . \ (56)$$

which gives the relation between the sag and the temperature.

The value of α for steel is 0·0000064 (per 1°F), and E should be taken not higher than 25×10^6 lb per in.^{2} for stranded steel wires.*

Hence if the catenary wire in the above example is erected with the specified sag at 60°F, the sag at various temperatures, calculated from equation (56), will be—

Temperature (°F) .	24·3	42	60	78·9	98·1	118·2
Sag (in.) . . .	26	28	30	32	34	36

Thus the trolley wire, as a whole, will be level only at one temperature, and at other temperatures the sections between the supporting structures will be above or below their normal position, due to the variation of the sag in the catenary wire. For the case under consideration, if the trolley wire is level at 60°F, then, at the extreme temperatures of 22°F and 100°F, the portion at the centre of the span will be respectively 4·25 in. above and 4·2 in. below the normal position.

Layout of Trolley Wire. The position of the trolley wire relatively to the track should be such that the contact surface of the collector will be worn uniformly throughout its width. To obtain this result, it is necessary to "stagger" the trolley wire with respect to the centre line of the track, a stagger

* This value for E is considerably below that (30×10^6) for hard drawn steel, the low value being due to the tightening up of the strands with the load.

of 9 in. on each side of the centre line being the usual allowance,* although the value is influenced by the design of the collector, amount of side sway in the trolley wire, etc.

At curves, the super-elevation of the track rails and the swing or oscillation of the coaches must be carefully considered in locating the position of the trolley wire, since a slight tilt of the coach will correspond to a relatively large transverse movement of the collector on the wire. The T-square method

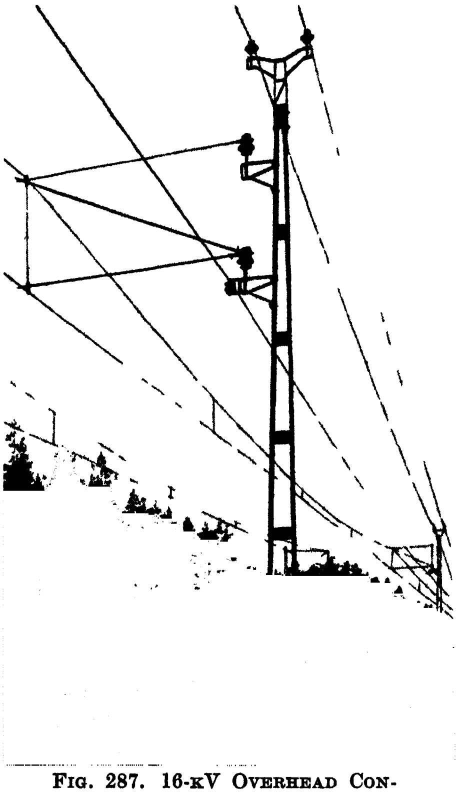


FIG. 287. 16-kV OVERHEAD CONSTRUCTION. STANDARD SPAN ON STRAIGHT TRACK (SWEDISH STATE RAILWAYS—A.S E A.)

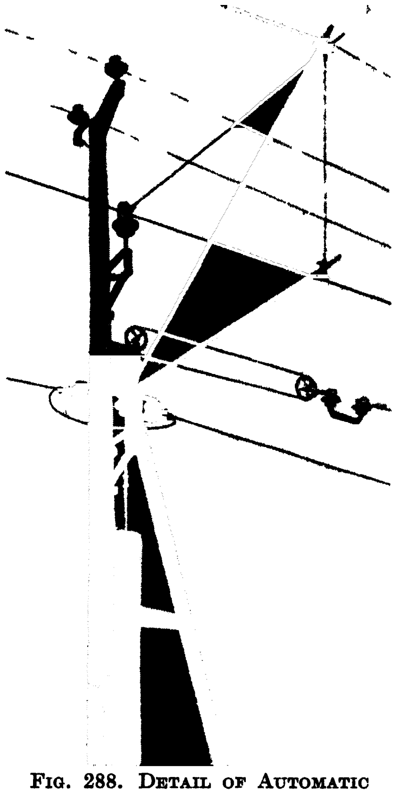


FIG. 288. DETAIL OF AUTOMATIC TIGHTENING DEVICE (SWEDISH STATE RAILWAYS—A.S E.A.)

is convenient for setting the trolley wire. The head of the T-square is arranged to fit the gauge of the track rails, and the centre line of the track and the limits of stagger are marked on the end of the blade which is adjustable vertically.

PART II. EXAMPLES OF OVERHEAD CONSTRUCTION

Single Catenary Construction. This form of construction is suitable for railways on which the traffic is not exceptionally heavy.

* The zigzag of the trolley wire should be arranged alternately on each side of the centre line of the track to equalize the wear on the collector.

Representative views of the construction on the Swedish and Norwegian State Railways are shown in Figs. 287, 288. The catenary wire is supported by cantilever trusses, each of which is supported by a pair of insulators mounted vertically one above the other on a pole at one side of the track. This design was adopted in order that the insulators should not be subjected to the hot blast from steam locomotives which may occasionally run over the track. The lower insulator supports also the pull-off or push-off; and both this member and the tubular compression member of the truss are hinged to the insulator in order not to restrict the vertical flexibility of the trolley wire. Moreover, the pins of both insulators fit loosely into the supporting brackets so as to allow small longitudinal movements of the wires, due to changes of temperature, to take place.

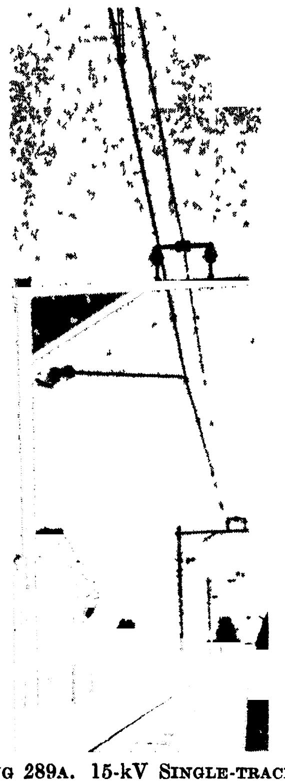


FIG 289A. 15-kV SINGLE-TRACK CONSTRUCTION, SWISS FEDERAL RAILWAYS

The trolley wire is of hard-drawn copper, having a cross-sectional area of 80 mm² (0·124 in.²). The catenary wire is of stranded steel and has a cross-sectional area of 50 mm² (0·0775 in.²). The normal span on straight track is 60 m (200 ft), and the droppers are spaced 20 m (66 ft) apart. The droppers are of steel wire about 5 mm (0·2 in.) diameter; the clips are of bronze and are light in weight so as not to produce "hard spots" in the trolley wire.

Both trolley and catenary wires are divided into sections, the normal length of which is 1,500 m (0·93 mile). Each section is anchored at its mid-point, and both ends are strained by 500 kg (1,100 lb) weights, Fig. 288, so as to maintain a constant tension in the wires, the tension being applied to the two (trolley and catenary) wires through an equalizing lever. By these means the variation in the sag of the trolley wire does not exceed 5 cm (2 in.) over a temperature range $-35°C$ to $+40°C$.

Typical construction on *Swiss main-line* railways is shown in Figs. 289A, 289B and 289C, the normal operating voltage being 15 kV.

Fig 289A shows the present normal construction on single track. The supporting structures are built up from channel and angle sections of structural steel. Double insulation is employed for both the catenary and the pull-offs or registration fittings. The catenary wire is attached to porcelain insulators of the "diabolo" or spool type, each of which is supported on a pair of pin-type insulators with double petticoats. Insulators of similar types (i e diabolo and pin) are employed for the registration fittings.

Fig. 289B shows examples of modern gantry construction for double and multiple tracks, and Fig. 289C shows an example of cross-catenary construction which is employed when the number of tracks to be spanned is such that gantry construction would be either impracticable or too costly.

The trolley wire is of hard-drawn copper, having a cross-sectional area of

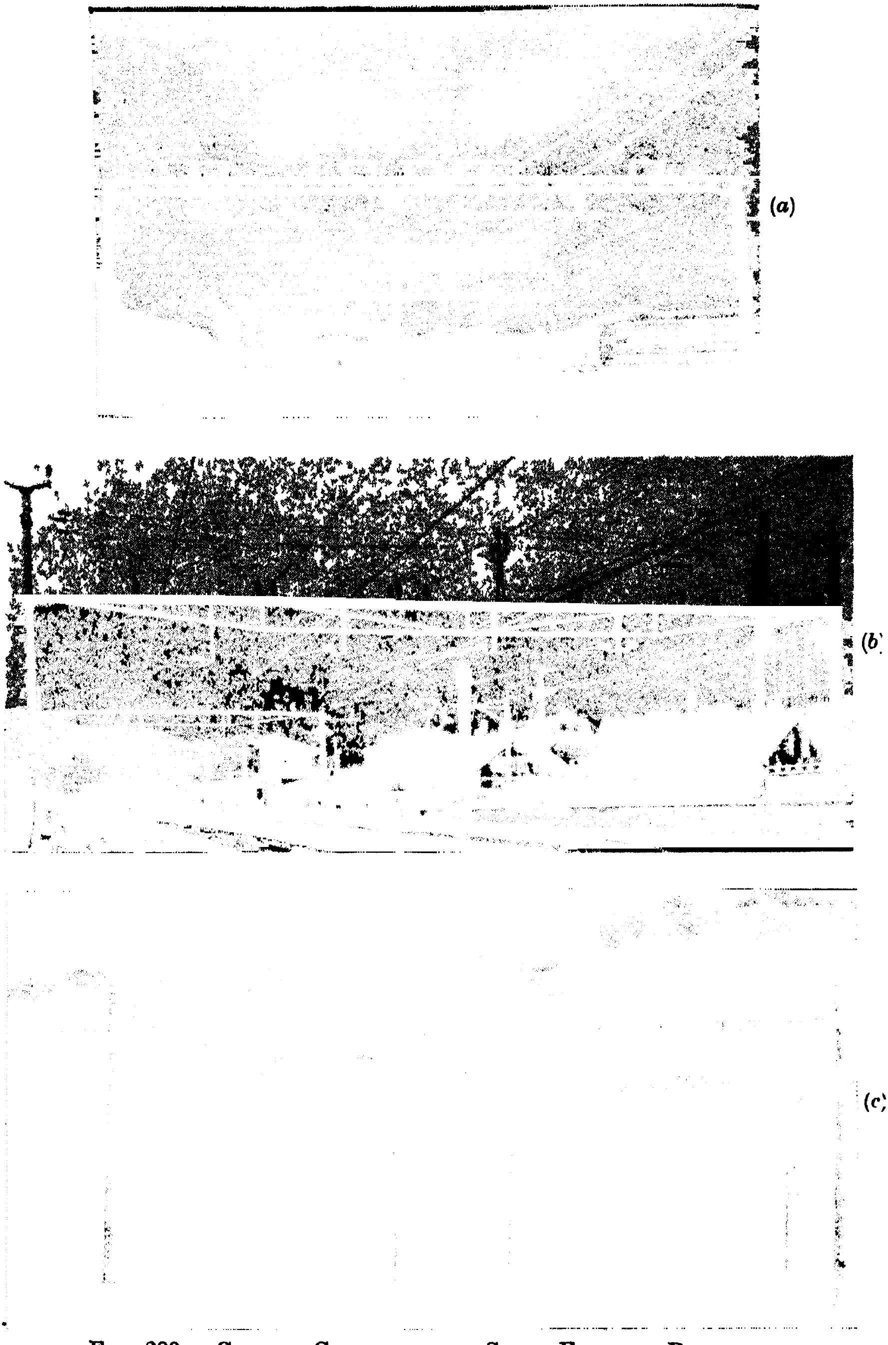
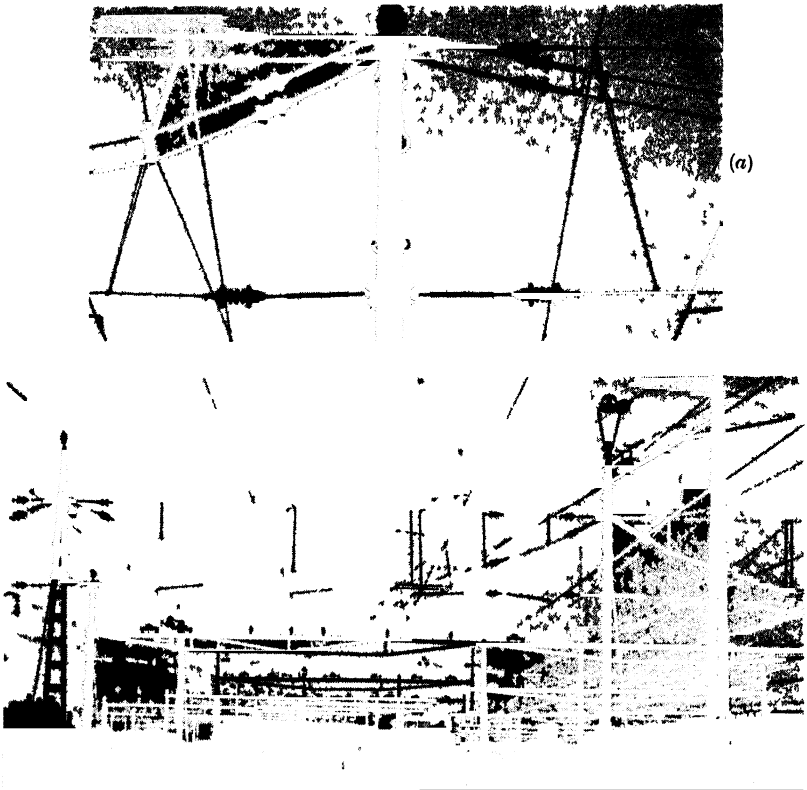


Fig. 289b. Gantry Construction, Swiss Federal Railways

(a) Welded-steel lattice gantry and supports, (b) 30-metre sectionalizing gantry and switching station, (c) gantry with concrete posts (war-time construction)

107 mm² (0·166 in.²), and the catenary wire is of stranded steel,* having a cross-sectional area of 50 mm² (0·0775 in.²). The normal span is 60 m (200 ft), but in some cases spans of 100 m (330 ft) are adopted. The droppers are spaced



(a)

(b)

FIG 289C CROSS CATENARY CONSTRUCTION NEAR ZURICH STATION, SWISS FEDERAL RAILWAYS

(a) Strain insulators, (b) cross-catenary span and commencement of gantry construction beyond station

10 m (33 ft) apart, and consist of 4 mm (0·158 in.) bronze wire. they are connected to the wires by light bronze clips

The sections of trolley and catenary wires are strained continuously by weights so as to maintain a constant tension of about 1,900 lb in the trolley wires.†

Modern single catenary construction on *American main-line* single-phase

* Copper-clad steel is employed in tunnels and on parts of the electrified system used by steam locomotives. Each of the seven individual wires of a strand is 9 mm overall diameter with a steel core 7·6 mm diameter

† This relatively high tension (850 kg) is employed to reduce radio interference due to sparking between the pantograph collectors and the trolley wire, the frequency being within the range of the Swiss National Broadcasting system

railways differs in a number of features from that on European railways. For example, (i) the spans are usually longer; (ii) the droppers and clips are much heavier; (iii) the contact wire is either of steel or a special hard-wearing, non-corrosive alloy (phono-electric) which is clipped to a copper wire suspended from the catenary wire, Fig. 290 (*a*); (iv) special inclined hangers, Fig. 290 (*b*), are usually employed at curved track to enable long spans to be retained without intermediate pull-off wires; (v) suspension-type insulators are employed in preference to pin-type insulators; (vi) the sections are anchored at both ends, and no devices are employed to maintain a constant tension in the wires.

The normal span is 300 ft and the sag at 60°F, with normal loading, is 6 ft 5 in. The catenary wire is of stranded steel $\frac{9}{16}$ in. in diameter; the contact wire is of alloyed bronze having a cross-sectional area of 0·166 in.^{2}, and the

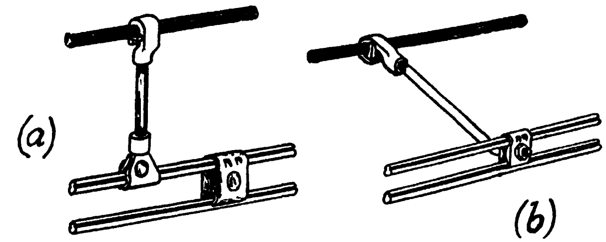


FIG. 290. HANGERS FOR STRAIGHT AND CURVED TRACK

auxiliary wire is of the same size but of hard-drawn copper. The hangers are spaced 10 ft apart and consist of $\frac{1}{2}$ in. galvanized steel rods with malleable-iron clips. The intermediate clips are of bronze. The normal tensions at 60°F are: catenary wire, 3,900 lb; contact wire, 1,815 lb; auxiliary wire, 1,600 lb.

An example of single catenary construction for 1,500-V d.c. railways is shown in Fig. 291 which refers to the present type of double-track construction on the *Nederlands Railways*. An equivalent copper cross-section of 500 mm^2 (0·775 in.^{2}) per track is provided by the twin trolley wires (each 100 mm^2), the catenary wire and a supplementary distributor (150 mm^2) which is arranged alongside the catenary wires as shown in Fig. 291. The normal span is 70 m (230 ft), the maximum sag is 2·15 m (7 ft) and the spacing of the droppers is 7 m (23 ft). A special feature of the construction is the provision of a supplementary catenary at each span, the ends of which are attached to the main catenary at points 10 m on each side of the supporting insulators. Three droppers are attached to each supplementary catenary as shown in Fig. 291. This arrangement was adopted to obtain more uniformity in the level of the trolley wires throughout the span, over a temperature range from $-$ 20°C to $+$ 40° C, than would be possible with a simple catenary.

The catenary and trolley wires are divided into sections of about 1·5 km (0·93 mile). A tension of 1,000 kg is applied to each trolley wire using 500 kg weights acting indirectly through differential pulleys, the diameters of which are in the ratio of 1 : 4.

Double insulation is employed for the catenary and the registration fittings. The diabolo type insulator, to which the catenary wire is clamped, is mounted

on a non-ferrous tube, the ends of which are fitted with porcelain caps which are clamped to U-shaped castings attached to the cross member of the supporting structure.

The feeding points are adjacent to the sub-stations (*see* Fig. 323) which are spaced at about 21 km (13 miles), and the overhead wires of the two tracks are paralleled, by means of high-speed circuit breakers, at two intermediate points 7 km apart.

Inclined Catenary Construction.* This is a modified form of single catenary construction in which the droppers occupy inclined or oblique positions at

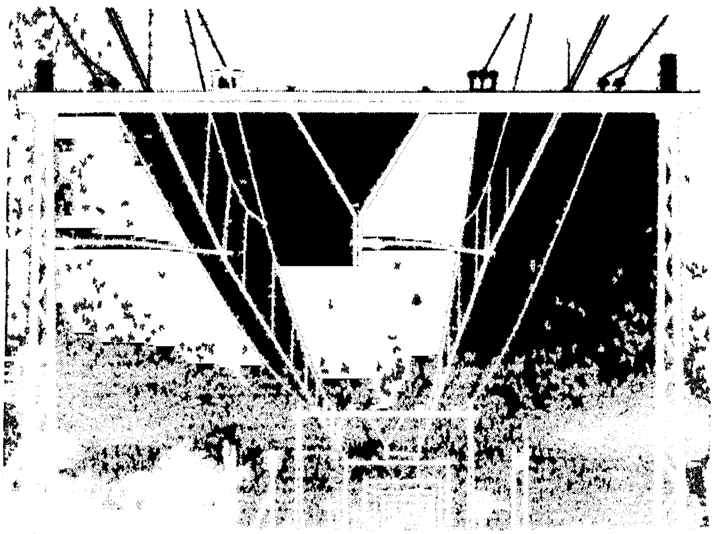
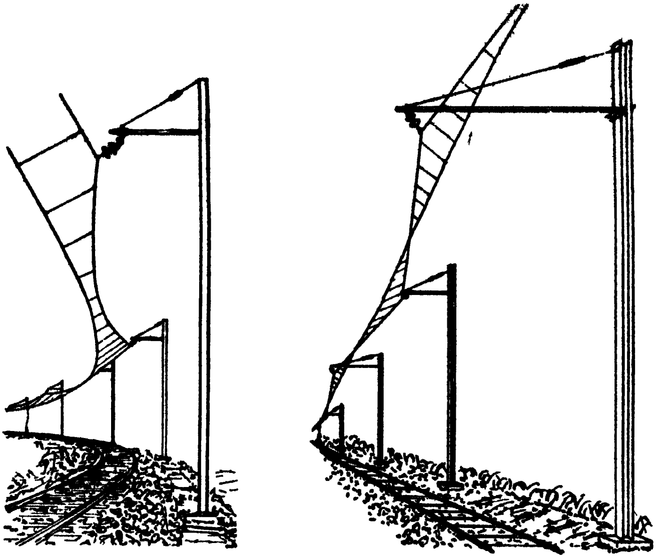


FIG. 291. 1,500-V SINGLE CATENARY CONSTRUCTION ON NEDERLANDS RAILWAYS

varying inclinations and maintain the trolley wire in its correct position without the use of pull-offs or registration fittings. It is employed extensively on curved track of American main-line single-phase railways and has recently been installed on both straight and curved track of the single-phase section of the French railways.

On straight track the catenary wire zigzags over the track and the droppers of adjacent spans are inclined alternately to the right and left as indicated at (*b*) Fig. 292. The trolley wire locates itself by its own weight and tension, the lengths of the droppers being chosen to give the required stagger.

* Catenary construction with inclined hangers was developed by the engineers of the New York, New Haven, and Hartford Railroad, with the idea both of eliminating to a large extent the necessity for pull-offs and of increasing the span on curves of moderate curvature. Details of methods of calculation of the lengths of the hangers and other features are given in a paper on "Catenary Design for Overhead Contact Systems," by Mr. H. F. Brown, *Trans. A.I.E.E.*, Vol. 46, p. 1,082.



(a) (b)

FIG 292 INCLINED CATENARY CONSTRUCTION ON CURVED AND STRAIGHT TRACK

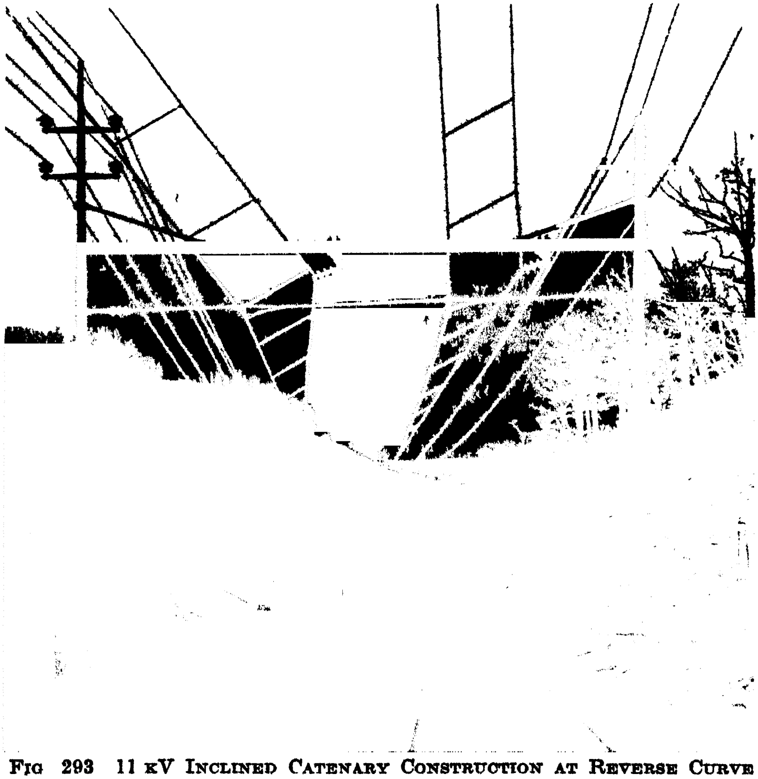


FIG 293 11 kV INCLINED CATENARY CONSTRUCTION AT REVERSE CURVE

On curved track the catenary wire is supported on the outside of the curve as shown in Figs. 292 (*a*), 293.

This construction gives the contact wire some lateral stability and provides a certain amount of automatic compensation for temperature. Moreover at

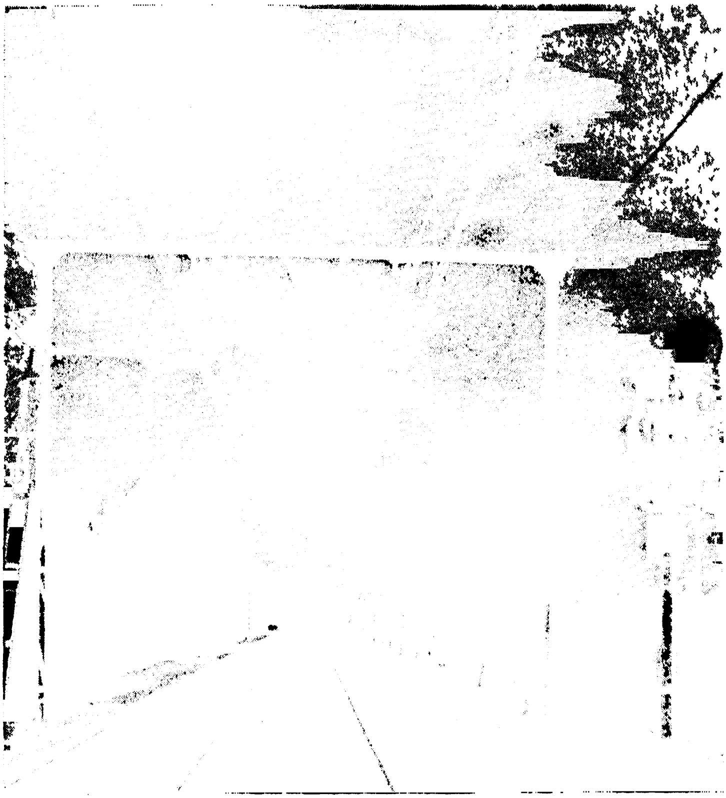


FIG. 294. 1,500-V COMPOUND CATENARY CONSTRUCTION ON LIVERPOOL STREET–SHENFIELD LINE (BRITISH INSULATED CALLENDERS CONSTRUCTION)

Mechanically independent registration on 2-track construction using broad-flange beam structures with diabolo supporting and cap-and-pin side-strain insulation at Shenfield

curves the alignment of contact wire and track is closer than that which is practicable with simple single-catenary construction. The spans are limited by the lateral displacement of the contact wire due to wind, and may be 200 ft on straight track and 100 ft on a curve of 1,000 ft radius.

Compound Catenary Construction. This form of construction was originally developed by Messrs. Siemens-Schuckert, to enable automatic tensioning devices to be applied to the trolley wire without causing displacements of the main hangers. The general features are described briefly on p. 373. This sytem

is installed on many 1,500-V, d.c. railways and an example is shown in Fig. 294.* An example of a single-phase installation is shown in Fig. 295.

Cross Catenary Construction. In cases where a large number of tracks have to be spanned—e.g. at goods yards, train-shed fanways, etc.—the catenary wires are suspended from transverse span wires, which are erected with considerable sag between towers on each side of the track. By this method, the heavy top girder, which would be required in gantry construction, is dispensed with, but higher towers are necessary on account of the sag in the span wire.

The main catenary insulators are suspended from the span wires by droppers

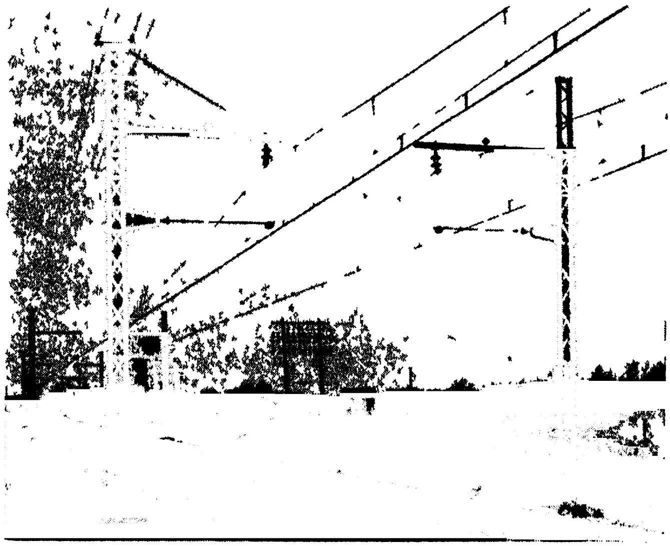


FIG 295. 20-kV COMPOUND CATENARY CONSTRUCTION ON THE 50-C/S ANNECY LINE, FRENCH NATIONAL RAILWAYS (OERLIKON)

Other types of single and inclined catenary construction are installed for trial purposes on single-track sections of the line between Aix-les-Bains and La Roche-sur-Furon The cross-section of the trolley wire is 107 mm² and that of the bronze catenary wire is 65 mm².

of various lengths, so that the insulators are in the same horizontal plane. Each span wire is provided with turn buckles, which are attached directly to the towers. Lateral stability is obtained by horizontal steady wires, which are insulated from the towers and from the catenary and trolley wires.

OVERHEAD CONSTRUCTION ON THREE-PHASE RAILWAYS

When the operating voltage does not exceed about 4,000 volts, insulated hangers, suspended from span wires in a manner similar to that adopted for side-pole construction on tramways, are practicable. But to avoid too large a sag in the trolley wire short spans must be adopted.

* Details are given in a paper, by H. H. Swift, on "The Electrification of the Liverpool St.–Shenfield Lines, British Railways." *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 42. See also p. 107, and Vol. 100, Pt. I, p. 133.

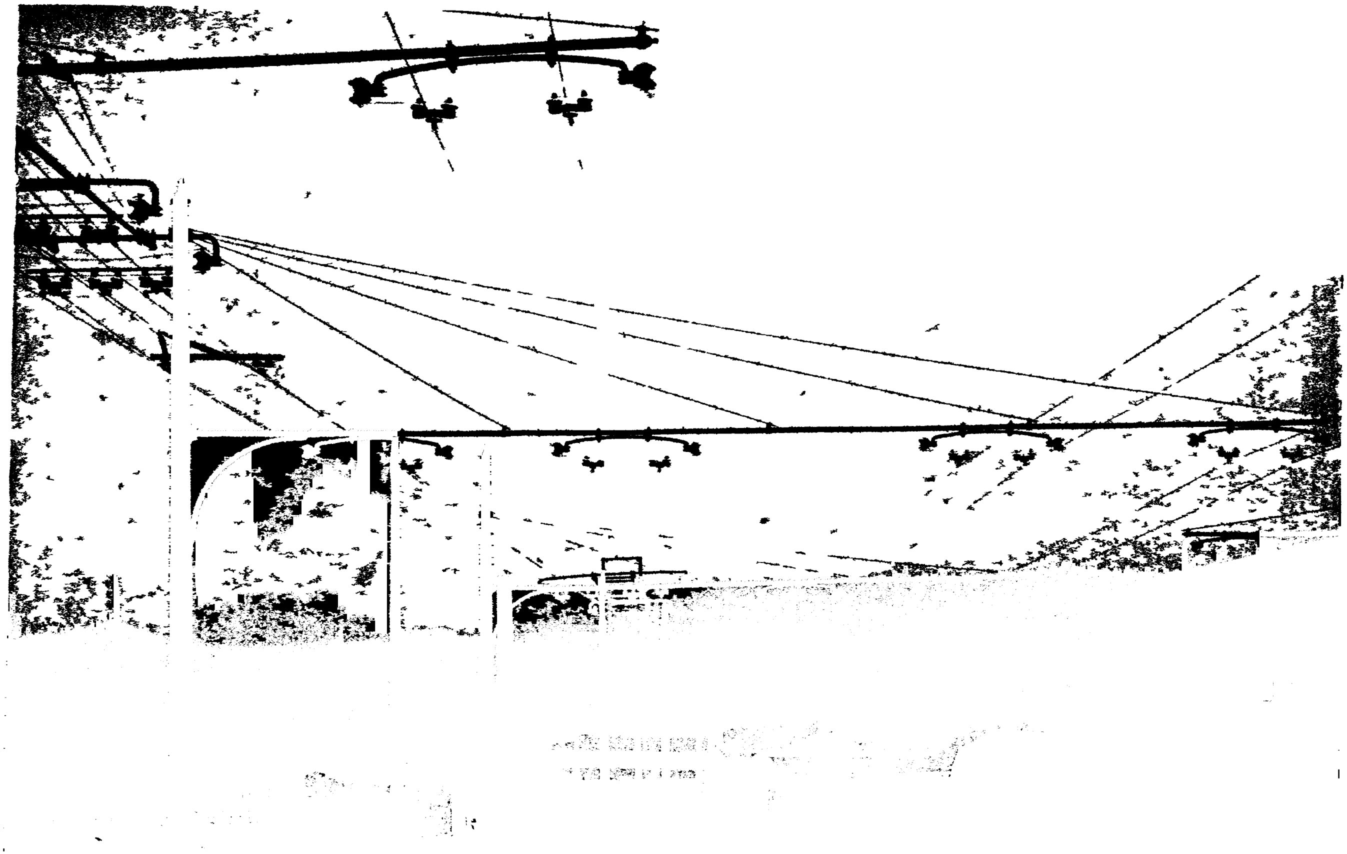


FIG. 296. A 16-METRE BRACKET ARM (FOR FOUR TRACKS) AT SAMPIERDARENA (3-PHASE, 3 3 KV)

Fig. 296 shows construction on the *Giovi-Genoa* lines of the Italian State Railways. Triple insulation is used throughout, and two types of hangers are used.*

On three-phase railways the portions of the overhead work requiring the most care in design and erection are *cross-overs* and *junctions*. At these places it is necessary to provide a continuous path for each bow collector, and at the same time prevent a bow from being in contact with wires of opposite polarity. It will be apparent, therefore, that neutral sections and section-insulators are required in addition to live wires.

Overhead Construction at Low Over-Bridges and Tunnels

The minimum height of the trolley wire above the track rails is determined by the minimum clearance allowed above the "loading gauge."

At very low bridges, which have been constructed with the minimum clearance, insufficient space will be available in which overhead wires and their supports can be erected. Therefore either the track must be lowered to obtain the necessary space, or a dead section (which consists of earthed guide wires attached directly to brackets fixed to the bridge) must be installed. In the latter case the trolley wire is dead-ended at each end of the bridge, and a neutral (or insulated) section is inserted between the dead wires and the trolley wires to provide a continuous path for the bow.

At bridges where the necessary space is available for the erection of overhead wires, the design of the supports for the insulators, etc., will be influenced largely by local conditions. In cases where the width of the bridge is not great, an insulated framework can be fitted to the underside of the bridge.

In all cases where live wires have to be taken under bridges, etc., the gradient between the two levels of the trolley wire must be such that the collector will maintain contact with the wire at all speeds, otherwise considerable flashing will occur. For high-speed running the gradient should be limited to 1 in 300.†

* The author is indebted to the *Tramway and Railway World* for Fig. 296.

† Much useful information is given in a paper, by Messrs. Kitchin and Holland, on "Design of Overhead Equipment," *Proc. I.E.E.*, Vol. 97, Pt. 1A, p. 107 and also in a paper, by Messrs. Crompton and Wallace, on "Economic Aspects of Overhead Equipment for D.C. Railway Electrification," *Proc. I.E.E.*, Vol. 100, Pt. I, p. 133.

CHAPTER XXV

FEEDING AND DISTRIBUTING SYSTEMS

WHEN electrical energy has to be supplied from a power station (or a sub-station) to a number of circuits at a constant voltage, the various circuits must be connected to distributing cables (called *distributors*), which are fed at suitable points (called *feeding points*) by other cables (called *feeders*). The latter cables connect the feeding points of the distributors to the station bus-bars, and their function is to maintain these points of the distributors at a definite voltage. The function of the distributors is to supply the circuits at practically constant voltage. This difference in the functions of distributors and feeders has an important effect upon the design of these cables.

Thus, in the design of a distributor, the principal consideration is the permissible variation of voltage along the distributor. On the other hand, a feeder should be designed for minimum annual operating expenses, which include the cost of the losses in the cable, together with the interest and depreciation charges.

Traction Distributing System. In electric traction systems the trolley wires and track rails—or, alternatively, the two sets of conductor rails—form the distributors. The loads are represented by the currents taken by the cars or trains, and are variable not only in magnitude but also in position.

In *tramway* and *trolleybus systems* operating in Great Britain the distributing system must conform to certain statutory regulations, chief among which are the following—

1. The voltage at the trolley wire shall not exceed 550 volts; and that at the generating station (or sub-station) shall not exceed 650 volts.
2. The trolley wire shall be divided into sections not exceeding one half-mile in length.
3. The potential difference between any two points of the (track) rail return system shall not exceed 7 volts.

Therefore, to comply with these regulations in the case of tramways separate feeding systems will be required for the trolley wire and the track rails.

In *d.c. railways* using the track rails as a return, the voltage drop in the track rails must be limited to values which will not affect the working of telegraphic apparatus by earth currents. The positive portion of the distributing system, however, has not to conform to statutory regulations with respect to voltage drop and sectionalization at every half-mile.

On account of the low voltage drop in the track rails, the variation of voltage at the cars in tramway systems depends almost entirely upon the voltage drop in the trolley wire. In any given case the permissible variation of voltage must obviously depend upon service conditions, and the minimum voltage must be such that the schedule speed can be maintained under all conditions of traffic. In some cases a maximum voltage variation of 10 per cent may be permissible, but in other cases—particularly in central districts with heavy traffic—a lower maximum variation will generally be desirable.

Length of Trolley Wire between Feeding Points. The determination of the feeding points for a traction distributor is not so straightforward as that for

a lighting distributor, on account of the loads being variable both in magnitude and position. Moreover, in practice, occasional blocks in the traffic will occur, with the result that the number of cars on one distributing section may be much greater than that under normal conditions of traffic. Hence a certain amount of judgment must be exercised in deciding upon the feeding points, and each case must be considered separately.

The permissible length of trolley wire between two feeding points may be determined when the voltage drop corresponding to normal conditions is fixed, and when the current and particulars of the trolley wire are known. For tramway traffic, the normal operating conditions may be considered to correspond to uniform spacing of the cars along the track. The current in a distributing section is obtained from a knowledge of the number of cars on the section and the current per car.

Thus, suppose the normal conditions correspond to a $2\frac{1}{2}$-minute service of cars operating at a schedule speed of 8 m.p.h., the average current per car being 30 amperes, and that a drop of 20 volts is permissible in the trolley wire, the resistance of which is 0·35 ohm per mile.

The average distance between consecutive cars $= 2{\cdot}5(60/8) = \frac{1}{3}$ mile, and the resistance of the trolley wire between these cars $= \frac{1}{3} \times 0{\cdot}35 = 0{\cdot}117$ ohm.

Hence the voltage drop in a 1-mile length of trolley wire

$$= 0{\cdot}117 \times 30(3 + 2 + 1) = 21 \text{ volts*}$$

Therefore the length of a distributing section is $20/21 = 0{\cdot}95$ mile; but, as the trolley wire has to be sectionalized every half-mile, the length of a distributing section would be made equal to 1 mile.

Feeders. If the feeding points of the distributors are maintained at constant voltage, the voltage drop in the feeders will have no effect upon the voltage variation in the distributors. Hence, under these conditions, the voltage drop in the feeders may be selected at a value which will result in the most economical operation.

Now the annual cost of a feeder comprises the cost of the energy dissipated, together with the interest and other charges on the capital expenditure. The latter can be divided into two portions, one being uninfluenced by the cross-sectional area of the cable and the other being directly dependent upon the cross section. For example, the costs of excavations, etc., laying of ducts and

* In a distributor fed from one end and loaded with a number of loads, the voltage drop is calculated as follows—

Let I_1, I_2, I_3, etc., denote the load currents in amperes; L_1, L_2, L_3 . . . denote the distances of the loads from the feeding point; l_2, l_3 . . . denote the distances between consecutive loads, l_2 representing the distance between the loads I_1 and I_2, etc. Then if r denotes the resistance of unit length of the distributor, the voltage drop (v) from the feeding point to the nth the load will be

$$v = r\{L_1(I_1 + I_2 + I_3 + \ldots I_n) + l_2(I_2 + I_3 + \ldots I_n) + l_3(I_3 + \ldots I_n) + \ldots l_nI_n\}$$
$$= r(L_1I_1 + L_2I_2 + L_3I_3 + \ldots L_nI_n),$$

since $L_2 = L_1 + l_2$; $L_3 = L_1 + l_2 + l_3$; etc.

In the special case when the loads are all equal and occur at equal distances along the distributor, we replace

$$I_1, I_2, I_3 \ldots \text{ by } I, \text{ and } L_1, l_2, l_3 \ldots \text{ by } l,$$

and obtain

$$v = rlI(n + (n-1) + (n-2) + \ldots 1) = rlI \times \tfrac{1}{2}n(n-1)$$

in which n denotes the number of loads.

drawing-in will depend only upon the length of cable and the number of cables being laid. Similarly, a portion of the cost of the insulation will be independent of the cross-section of the cable.

Therefore, let the cost per mile of the cable, laid and jointed be (£)$C = (A + Ba)$, where a denotes the cross-section of the cable, B the portion of the capital cost which is dependent on the cross-section, and A the remaining portion of the capital cost, which is independent of the cross section.

Then, if m is the percentage interest and depreciation charges on the capital cost of the completed cable, the annual charges per mile are

$$0{\cdot}01m(£)C = 0{\cdot}01m(A + Ba)$$

If I is the r.m.s. value, in amperes, of the current over a period of h hours per annum, the annual cost, £(E), of the I^2R losses in a cable 1 mile long and cross-section a sq. in. is given by

$$(£)E = I^2(0{\cdot}0425/a) \times ph/240{,}000$$

where p is the price, in pence per kWh, of energy delivered to the cable.

[NOTE—The numeral factor 0·0425 refers to the resistance of a copper cable 1 mile long and 1 in.^{2} in cross section.]

Therefore the annual cost of the cable is

$$0{\cdot}01m(£)C + (£)E = 0{\cdot}01m(A + Ba) + 0{\cdot}0425(I^2ph/a)/240{,}000$$

which will be a minimum when

$$0{\cdot}01mBa = 0{\cdot}0425(I^2ph/a)/240{,}000$$

Whence $$I^2/a^2 = (mB/ph) \times 5{\cdot}64 \times 10^4$$

or I/a (i.e. the current density $= 237{\cdot}5\sqrt{(mB/ph)}$ . . . . . (57)

Hence the *most economical current density* is that which makes the annual cost of the losses in the cable equal to the variable portion of the annual charges on the capital cost.* But if this current density exceeds that corresponding to the limiting operating temperature of the cable, the cross section must be chosen on the latter basis.

It is important to note that the voltage drop in the feeder does not appear in the above expression, although, when the current density is determined, the voltage drop is also indirectly determined.

Cases will arise with long feeders when the voltage drop, corresponding to the most economical current density, will be excessive, and in these circumstances either the cross section must be increased to give a lower voltage drop, or a booster must be used in conjunction with the cable.

POSITIVE FEEDING AND DISTRIBUTING SYSTEMS FOR TRAMWAYS

The method of feeding the distributing sections of the trolley wire will be influenced largely by traffic and economical considerations. In districts with very heavy traffic, the traffic considerations will usually preponderate, and the feeding system must be arranged so that the opening of a feeder circuit breaker at the generating station or sub-station will only affect a small portion of the traffic. Under these circumstances a separate feeder is desirable for each half-mile section of the trolley wire, and these feeders must be supplied from sub-stations.

* This relationship is usually known as Kelvin's law.

On the other hand, with light traffic, considerations of economy become of greater importance, and several half-mile sections of the trolley wire must be supplied from a single feeder. Two methods are shown diagrammatically in Fig. 297. In these diagrams the section insulators in the trolley wire are indicated, as well as the "section" and "feeder pillars," which contain the switches for isolating the sections of the trolley wire.

In one case the trolley wire is used as a distributor, and each distributing section comprises two or more half-mile lengths of trolley wire. In the other

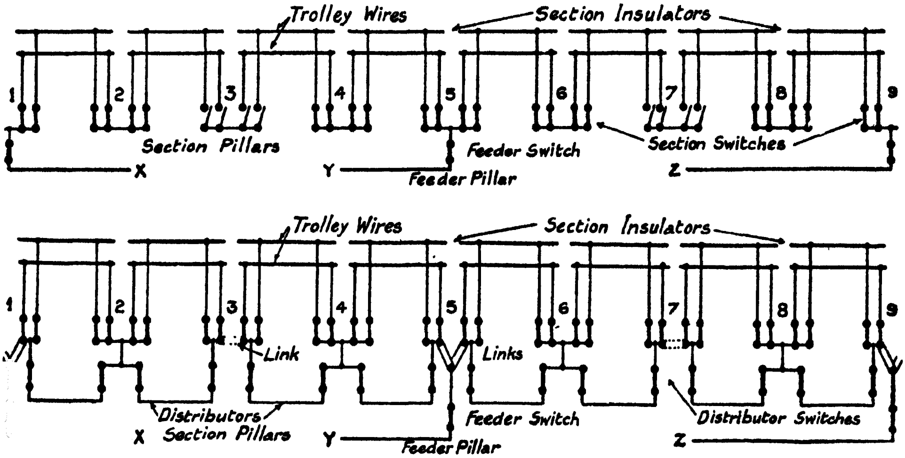


FIG. 297. METHODS OF FEEDING AND SECTIONALIZING TROLLEY WIRES FOR TRAMWAYS

case (which permits the use of a smaller number of feeders), feeders of large cross section are run to a few points in the system, from which the current is distributed to the various half-mile sections of the trolley wire by means of graded distributing cables. This method is more economical in copper than the first method, as longer distributing sections are obtained. Moreover, the variation of voltage between adjacent half-mile sections of the trolley wire is smaller than that for the first method.

NEGATIVE FEEDING AND DISTRIBUTING SYSTEMS FOR TRAMWAYS

Voltage Drop in Rails. The exact calculation of the voltage drop in the track rails is complicated on account of leakage currents due to the rails being uninsulated. For instance, a portion of the return current may reach the station via the earth, and the earth may also act as a diverter (or shunt) to the track rails, thereby relieving the rails of some of the return current. Although it is possible to calculate the voltage drop in the track rails under the latter conditions,* it is preferable to neglect the effect of the conductivity of the earth, and to design the distributing sections so that the voltage drop under the worst conditions does not exceed the statutory limit of 7 volts.

If the current in the rails is due to a number of cars, equally spaced along the track, the voltage drop in a length of the track rails can be calculated by a method similar to that adopted for the trolley wire.

* See *The Electrician*, Vol. 45, p. 595.

The average voltage drop in the rails under normal conditions should be assumed at a value not in excess of 5·0 volts, in order to allow for effects of blocks in the traffic, and "bunching" of the cars at certain parts of the system.

With the voltage drop given, and the resistance of the track known, the length of the rail distributing sections can be determined from a knowledge of the electric loading. In many cases it is convenient to consider that the loading is uniformly distributed throughout the distributing section (i.e. the current decreases uniformly from the feeding point), so that the voltage drop along the section follows a parabolic law.* The length of a distributing section corresponding to a definite voltage drop is then readily determined, and the position of the feeders naturally follows. As the rails are continuous throughout the system, the curve of voltage drop between adjacent feeding points will be a parabola with the zero points at the feeding points. Fig. 298 represents these

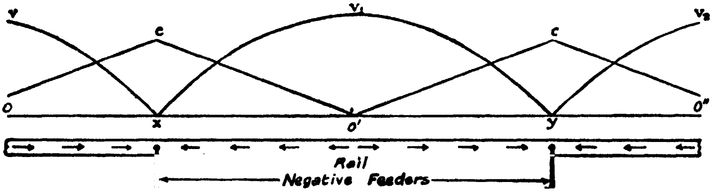


FIG. 298. VARIATION OF VOLTAGE ALONG RAIL WITH UNIFORM CURRENT LOADING

conditions, the straight lines $o\ c\ o'\ c\ o''$ representing the distribution of current along the rail, and the parabolas $v\ x\ v_1\ y\ v_2$ the variation of the voltage drop; the feeding points $x\ y$ being at the same potential.

Now the feeders connect the points x, y of the rails to the negative bus-bar, and, therefore, with *uniform electric loading* of the rail sections, the feeders must be *designed for the same voltage drop*. Hence, when long and short feeders are in use on the same system, equality of voltage drop in the several feeders must be obtained either by connecting resistances in series with the short feeders, or by neutralizing a portion of the voltage drop in the long feeders by means of boosters.

Ideal Conditions. The ideal conditions for a negative feeding system are represented diagrammatically in Fig. 298. The voltage drop along each distributing section of the rails is the same throughout the system, and, consequently, there is no tendency for leakage currents to pass between the sections. If the conductivity of the earth is uniform throughout the sections, the distribution of the leakage currents will be as indicated in Fig. 299 (A). On the other hand, if the voltage drops are unequal, there is an interchange of leakage current between adjacent sections as indicated in Fig. 299 (B). Hence, to prevent the interchange of leakage currents when adjacent sections are unequally loaded, the voltage drops must be equalized by adjusting the lengths of the

* If i is the rate of increase of the current along the rail from the dividing point (where the current is zero) and r is the resistance of unit length of the rail, then the current at any point, distant x from the dividing point, will be ix, and the voltage drop in an element of rail length dx will be $ix\ r\ dx$. Hence the voltage drop (v) in a length L from the dividing point will be given by

$$v = \int_o^L ix\ r\ dx = ir\left[\frac{x^2}{2}\right]_o^L = \tfrac{1}{2}irL^2$$

Thus v is proportional to L^2, and the curve connecting v and L is a parabola.

sections in the inverse ratio of the square roots of the loading. Thus, if I_1, I_2, denote the intensity of the loading (amperes per mile) in sections of lengths L_1, L_2 respectively, then $\frac{1}{2}I_1rL_1^2 = \frac{1}{2}I_2rL_2^2$, or $L_1/L_2 = \sqrt{(I_2/I_1)}$, where r denotes the resistance per mile of track.

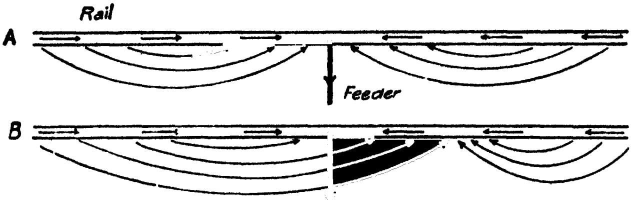


FIG. 299. DISTRIBUTION OF LEAKAGE CURRENTS
A—Equally loaded rail sections. *B*—Unequally loaded rail sections.

Connexions of Boosters. Although the boosters for the positive and negative feeders are series machines, with similar characteristics, the methods of operating them are different. Thus, the positive boosters are operated self-excited. The negative boosters, however, are separately excited; the armature being connected in series with the negative feeder, and the field winding being connected in series with the positive feeder supplying the corresponding

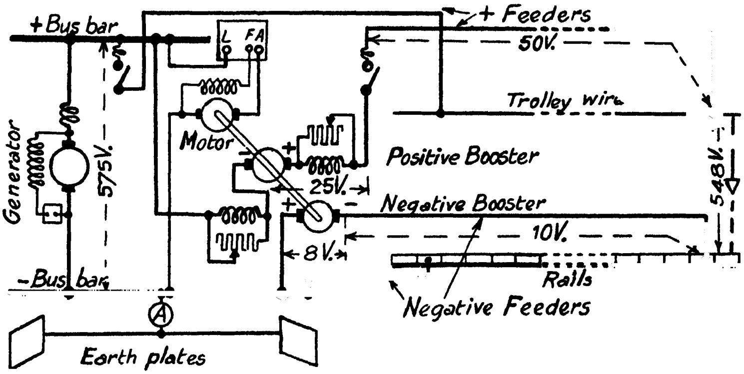


FIG. 300. CONNEXIONS OF FEEDERS AND BOOSTERS IN TRAMWAY SYSTEM
Typical circuit voltages and voltage drops are indicated.

sections of the trolley wire. The "boost" on the feeder is adjusted by means of a diverter rheostat connected in parallel with the field winding. Fig. 300 shows the connexions.

FEEDING AND DISTRIBUTING SYSTEMS FOR TROLLEYBUS ROUTES

In a double-track tramway system the rail return is equivalent electrically to a copper conductor having a cross section of about $3\frac{1}{2}$ in.^{2}, and in consequence practically the whole of the voltage drop in the distributing system occurs in the trolley wires. Hence in a trolleybus system the feeding points must be much closer together than in the corresponding tramway system, which condition usually necessitates, from economic and technical considerations, the placing of sub-stations, of relatively small capacity, at frequent intervals (two to three miles) along the route as indicated in Fig. 301. This is practicable

with mercury-arc rectifiers of the glass bulb, or air-cooled, pumpless, steel-tank type, which can be housed, singly if necessary, in kiosks or small buildings and remotely controlled from a central depot.

FEEDING AND DISTRIBUTING SYSTEMS—DIRECT-CURRENT RAILWAYS

Location of Sub-stations. The design of the feeding and distributing system for urban and suburban railways, operating at an average voltage of 600 volts, is closely connected with the location of the sub-stations. The distance between the sub-stations is generally determined from considerations of the permissible variation of voltage at the trains, but it is advisable to consider also the location which will result in the minimum annual cost. In considering the voltage drop

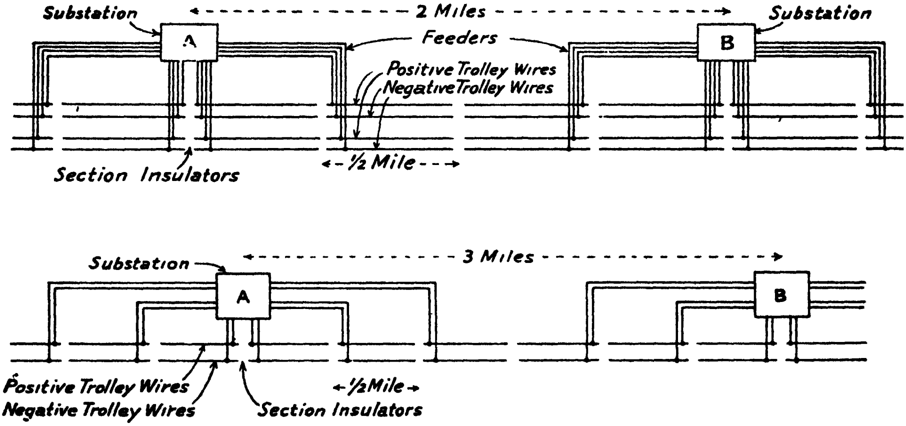


FIG. 301. FEEDING SYSTEMS FOR TROLLEYBUS ROUTES WITH TWO-WAY AND ONE-WAY TRAFFIC

permissible in the conductor rails, the possibilities of delays due to signal stops should be taken into account, as well as the maximum service which is likely to occur on the different sections. Moreover, when operating with the maximum service, the headway between consecutive trains will be smaller than that under normal service conditions, and, consequently, the checks at signals will probably be increased. For this reason, it is important to consider the position of the signals (especially the "stop" signals) and the "block" sections in arranging the positions of the feeding points.

The number of trains on a given section of the track is best determined from a *graphic time-table*, which is really a distance-time chart for each individual train. By means of this chart the position of trains at important junctions—where two trains may have to use the same crossover—can readily be seen. As a first approximation in the preparation of such a chart from a completed time-table, the distance-time curves of the trains may be assumed as straight lines. When the time-table is not available, the chart must be prepared from the running curves of the trains. If the positions of stations and the "stop" signals are indicated on the chart, an approximate idea of the maximum load on the section can be obtained.

Method of Determining Distance between Sub-stations. The method to be employed is best illustrated by considering a specific case. For example, a

service of 195-ton motor-coach trains has to be run over a double-track railway, on which the distance between the stations is 2,560 ft, the schedule speed being 16 m.p.h., and the duration of stop being 20 seconds. The track may be assumed to be straight and level. The train equipments are identical with those of the 195-ton six-coach train for which the speed-time curve and energy consumption were calculated in Chapter XIX.

The average voltage at the trains is to be 600 volts, and the converting machinery, to be used in the sub-stations, is designed to give 600 volts at no load and 630 volts at full load. The distributing system consists of two (positive and negative) conductor rails weighing 100 lb per yard.

The maximum service on the railway is 45 trains per hour in each direction,* which corresponds to a headway of (3,600/45 =) 80 sec, and the voltage drop in the distributing system under these conditions is not to exceed 40 volts.

The distance between trains, on a given track, taking maximum current at the same instant is determined by constructing a series of distance-time and current-time curves for consecutive trains over a fairly long length (3 miles) of track. Such a set of curves, constructed from data in Chapter XIX, is given in Fig. 302. Hence, if a group of consecutive stations be designated A, B, C, D, etc., then at a given instant trains are starting simultaneously from stations A, D, G. (There is, however, a short time interval of 7 seconds between the successive starts.) Also, when a train is starting from station B, other trains are starting from stations E and F. Thus, the maximum currents may be considered to occur simultaneously at stations 7,580 ft apart.

The positions of the sub-stations can now be fixed when the system of feeding has been decided. In order to simplify matters, we will assume that the conductor rails are continuous between the sub-stations and that the feeding points are opposite the sub-stations. A little consideration will show that if the sub-stations are located at the passenger stations, A, D, G, etc., the maximum voltage drop will occur at the intermediate stations B, C, E, F, etc., and will equal 39·6 volts,† which is equal to the specified value. Thus, with this arrangement of the feeders and conductor rails, the sub-stations are 7,680 ft (1·45 miles) apart.‡

Sectionalization of Distributing System. With low-voltage d.c. railways, the lengths of the distributing sections are arranged to suit the requirements of the traffic, due consideration being given to the sectionalization at cross-overs and junctions.

The methods of sectionalizing the conductor rails are influenced by the method of operating adjacent sub-stations (i.e. whether these sub-stations are operated in parallel or separately): they also depend upon whether or not the "up" and "down" conductor rails of the same polarity are paralleled.

When sub-stations operate in parallel a section insulator is inserted in the conductor rail opposite each sub-station, and the ends of the conductor rails

* This is equivalent to the maximum service run over a portion of the District Railway, London.

† When a train is starting from station B, two-thirds of the starting current will be supplied by sub-station A, and the remaining one-third by sub-station D, assuming equal voltages at the sub-stations. Hence, as the resistance of the bonded conductor rails is 0·068 ohm per mile of track, the maximum voltage drop between a feeding point and a train is

$$\tfrac{2}{3} \times 1{,}800 \times 0{\cdot}068 \times 2{,}560/5{,}280 = 39{\cdot}6 \text{ V}$$

‡ Further information on the spacing of sub-stations is given in a paper by W. J. Webb. *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 88.

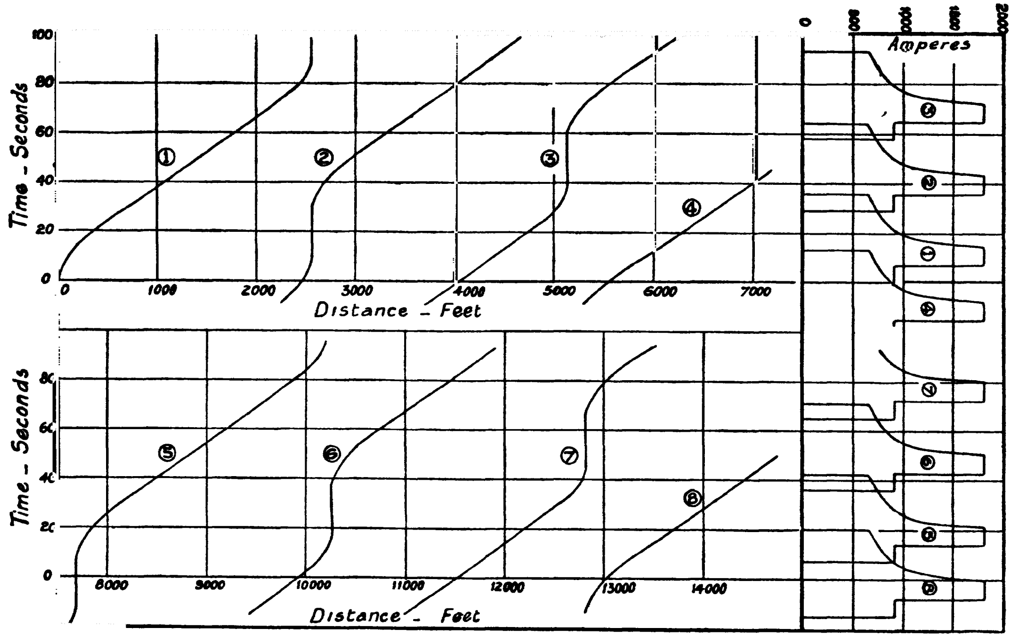


FIG. 302. METHOD OF DETERMINING DISTANCE BETWEEN TRAINS TAKING MAXIMUM CURRENT

are connected to double-throw (single-pole) switches, (as shown in Fig. 303.) When the switches in adjacent sub-stations are thrown into the upper contacts, the section of conductor rail between these sub-stations is fed from each end, and the load on this section is therefore supplied from both of the sub-stations. In the event of one sub-station being shut down, the switches are thrown into

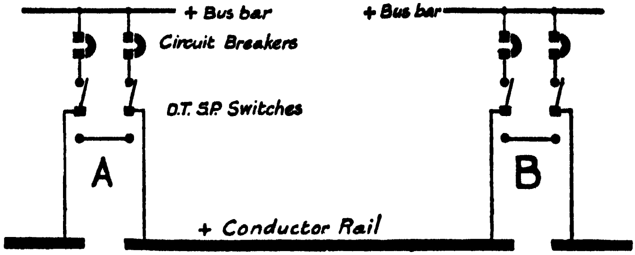


FIG. 303. METHOD OF FEEDING CONDUCTOR RAILS WITH SUB-STATIONS OPERATING IN PARALLEL

the lower contacts, thereby bridging the section insulators and transferring the load to the sub-stations on either side.

This method of sectionalization, however, is not suitable when the traffic is operated with motor-coach trains having a power cable between the motor coaches, as if a fault occurred when the train was passing over the section insulator the circuit breakers on both sections would be tripped, thereby cutting

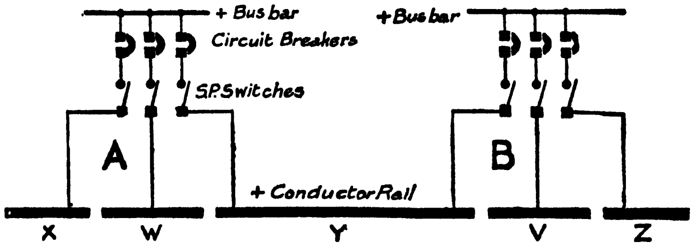


FIG. 304. METHOD OF FEEDING CONDUCTOR RAILS AND "TRAIN SECTIONS" WITH SUB-STATIONS OPERATING IN PARALLEL

off power from a considerable length of track. This objection can be removed by inserting a separate section (the length of which is slightly greater than the extreme distance between the front and rear collector shoes) between the main sections, and feeding this section separately. Thus, in Fig. 304, two sub-stations are represented at A and B. Sub-station A supplies the sections X, W, Y, while sub-station B supplies the sections Y, V, Z. Of these sections, X, Y, Z, represent the main sections of the conductor rails, while W, V represent the special short sections (called *train sections*) opposite each sub-station. It is apparent that an overload on a main section only, or a main section and a train section together, will only shut down one main section.

In those cases where the sub-stations do not operate in parallel provision must be made for bridging the sections supplied from neighbouring sub-stations, so that, in an emergency, these sections can be supplied from one sub-station. The switches for this purpose are generally located in section pillars adjoining the track.

Another feature in which a system supplied from separate sub-stations differs from one in which the sub-stations operate in parallel is the *paralleling of the conductor rails* of the same polarity. This paralleling is usually carried out by switches located in pillars adjacent to signal cabins, so that the switches

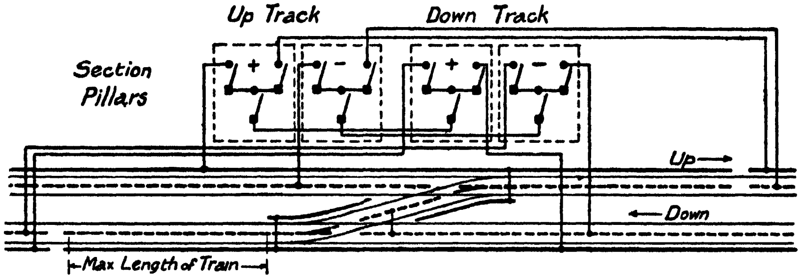


FIG. 305. ARRANGEMENT OF CONDUCTOR RAILS AND SECTIONALIZING SWITCHES AT A CROSS-OVER

may be operated by the signalman on instructions from the responsible department. Under normal conditions of operation the switches are closed, and the "up" and "down" conductor rails of the same polarity are connected together at a number of points, so that the full conductivity of the conductor rails is available.

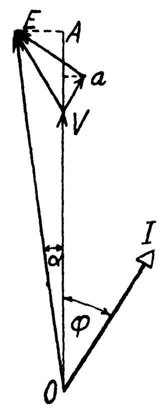


FIG. 306. VECTOR DIAGRAM FOR FEEDER

The method of sectionalizing and feeding the conductor rails at *station cross-overs* is important, as provision must be made, in the case of a breakdown, for single-line working or for shuttle working. A diagram showing the switches and conductor-rail sections to fulfil these requirements is shown in Fig. 305.

FEEDING AND DISTRIBUTING SYSTEMS—ALTERNATING-CURRENT RAILWAYS

Calculation of the Voltage Drop. This calculation, for a feeder or distributor, involves the consideration of the effects of resistance, self-inductance, mutual-inductance, and power factor.

The effect of power factor on voltage drop is shown, for the case of a feeder, in the vector diagram of Fig. 306, in which OI represents the current, OV the voltage at the load or receiving end of the feeder, and OE the voltage at the generating station. OE is compounded from OV and VE, which represents the voltage drop due to the impedance of the feeder. The impedance voltage, VE, may be resolved into a component, Va, in phase with the current, and a component, aE, in quadrature (leading) with the current. These components are numerically equal to IR and $2\pi fLI$ respectively, where I denotes the current, f the frequency, and R, L the resistance and inductance, respectively, of the feeder.

The actual voltage drop (v) in the feeder (i.e. the arithmetic difference between the voltages at the two ends) is given with sufficient exactness by VA, since, in practice, α is a very small angle. Hence

$$v = I(R \cos \phi + 2\pi fL \sin \phi) \quad . \quad . \quad . \quad . \quad (58)$$

Inductance of Trolley-wire and Rails. With the normal heights and spacings of the trolley-wire on a double-track railway the inductance of the loop of trolley-wires and track rails is of the order of 2·5 milli-henrys per mile of route.

Resistances of Trolley-wire and Rails. At the frequencies employed for traction the current density in the (non-magnetic) trolley-wire may be assumed to be constant over the cross-section, and therefore the resistance of the trolley-wires when carrying alternating current will be the same as that when carrying direct current.

With the rails, however, the internal flux due to the current is considerably greater than that in the trolley-wire, and, in consequence, the induced e.m.f. causes the current to be confined to a thin surface layer or "skin," the depth of which decreases as the frequency increases. The ratio of the resistance of this current-carrying layer (called the effective resistance) to the actual resistance of the rail is about 3·3 when the frequency is $16\frac{2}{3}$ c/s and 7·8 when the frequency is 50 c/s.

Voltage Drop in Trolley-wires and Rails. Assuming double track laid with 100-lb per yd rails and 0·25-in.^{2} copper trolley-wires, the actual resistance per route mile of cross-bonded track is about 0·018 ohm, and the resistance per route mile of the two trolley-wires connected in parallel is 0·08 ohm.

The reactance per route mile of the loop of trolley-wires and rails at $16\frac{2}{3}$ c/s $= 2\pi \times 16\frac{2}{3} \times 0{\cdot}0025 = 0{\cdot}26$ ohm, and at 50 c/s $= 2\pi \times 50 \times 0{\cdot}0025 = 0{\cdot}79$ ohm.

Hence the impedance per route mile

$$= \sqrt{[(3{\cdot}3 \times 0{\cdot}018 + 0{\cdot}08)^2 + 0{\cdot}26^2]} = 0{\cdot}3 \text{ ohm at } 16\tfrac{2}{3} \text{ c/s, and}$$
$$= \sqrt{[(7{\cdot}8 \times 0{\cdot}018 + 0{\cdot}08)^2 + 0{\cdot}79^2]} = 0{\cdot}82 \text{ ohm at 50 c/s.}$$

These values enable the voltage drops due to impedance to be calculated when the currents are known.

The voltage drop in the distribution system (i.e. the difference between the voltages at the feeding point and locomotive) is calculated by the application of equation (58). At very low power factors, such as occur at starting, the system voltage drop is practically equal to the voltage drop due to impedance.

Voltage Drop in Rails. This is of importance because leakage currents in the general mass of earth may cause interference with telecommunication systems.

If the inductance of the rail system is assumed to be one half of the total inductance of the trolley-wire-rail loop, the reactance per mile for the above double-track is 0·13 ohm at $16\frac{2}{3}$ c/s and 0·4 ohm at 50 c/s. Hence the impedance per mile

$$= \sqrt{[(3{\cdot}3 \times 0{\cdot}018)^2 + 0{\cdot}13^2]} = 0{\cdot}143 \text{ ohm at } 16\tfrac{2}{3} \text{ c/s, and}$$
$$= \sqrt{[(7{\cdot}8 \times 0{\cdot}018)^2 + 0{\cdot}4^2]} = 0{\cdot}42 \text{ ohm at 50 c/s.}$$

Therefore with 150 A in the track rails (which corresponds approximately to the rated current of a light passenger locomotive on a 15-kV system) the voltage drop per mile is 21·4 V on a $16\frac{2}{3}$-c/s system and 63 V on a 50-c/s system. Obviously, the latter value could not be tolerated and even the lower value may cause interference.

Methods of Relieving Rails of Return Current. If the potential difference between any two points of the rail return is to be maintained at a low value the rails must be relieved of a large portion of the return current, which may be effected by the use of a distributor cable in conjunction with booster transformers. Such a system is shown diagrammatically in Fig. 307, in which the sections of the trolley wire are supplied from the inner conductor of a concentric distributor, which is divided up into the same number of sections as the trolley wire, the sections being connected through the primary windings of the booster transformers (of 1 : 1 ratio). Thus the primary windings of the booster transformers are all connected in series. The secondary windings of these transformers are also connected in series by a single insulated cable (called the "booster"

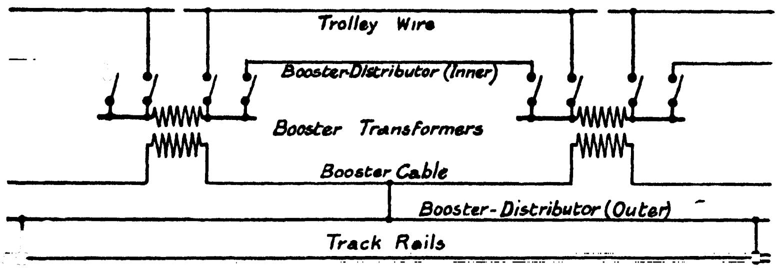


FIG. 307. METHOD OF CONNECTING BOOSTER TRANSFORMERS FOR NEUTRALIZING VOLTAGE DROP IN RAILS

cable), which is connected at certain points to the (earthed) outer conductor of the concentric distributor, this conductor being also connected to the rails at frequent intervals.

Hence, since the booster transformers have a ratio of unity and the secondary winding is connected to a closed circuit, the primary and secondary currents will be practically equal (the difference between these currents being equal to the magnetizing current of the transformers), and consequently the cable connecting the secondary windings of the transformers must carry a current approximately equal to that in the inner conductor of the concentric distributing cable. Therefore practically all the return current will be drawn from the rails into the booster cable.

The e.m.f. in the secondary winding of the booster transformers (which neutralizes the impedance voltage drop in the secondary circuit) is obtained from the primary winding, so that the voltage drop in the return conductor is transferred to the outgoing conductor (i.e. the inner conductor of the concentric distributing cable).

Fig. 308 shows a booster transformer and section switch installed on the 16,000-volt lines of the Swedish State Railways. The booster transformers are located at intervals of about 2·8 km (1·75 mile). In this case, however, the booster cable is a single bare conductor supported, by insulators, from the poles at approximately the same height as the trolley wires.

Interference Effects on Telegraph and Telephone Circuits. Single-phase currents in trolley wires produce electromagnetic and electrostatic disturbances in parallel telegraph and telephone circuits. In some cases the disturbances have been so severe that the telephone circuits have had to be run underground,

and in other cases various devices have had to be adopted to neutralize the inductive effects in the auxiliary circuits.

The electromagnetic inductive effects can be minimized by employing booster transformers and a return conductor arranged parallel to the trolley wires, as in Figs. 287, 308.

Alternatively, the scheme shown diagrammatically in Fig. 309 may be employed. In this case the feeders are supplied at double the voltage of the

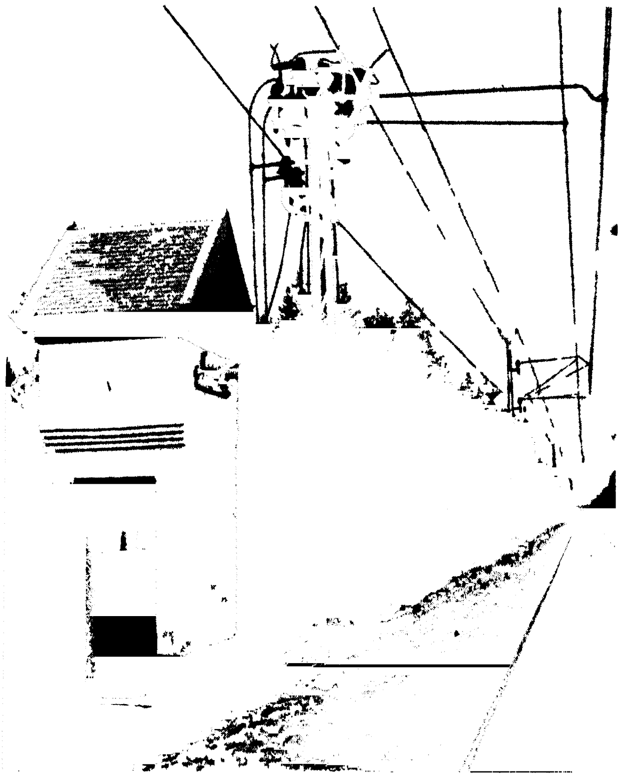


Fig. 308. Booster Transformer and Sectionalizing Switch on Swedish State Railways (A.S.E.A.)

trolley wire, the latter being fed from the former through auto-transformers. The centre point of the winding of each auto-transformer is connected to the rails. Hence the currents in the portions of the trolley wire between adjacent transformers are in opposite directions—as shown in Fig. 309—so that the resultant inductive effect on a neighbouring circuit is practically zero. This arrangement of the feeders and trolley wires also minimizes the electrostatic effects on neighbouring circuits.

On the New Haven electrified lines the auto-transformers are of the outdoor type, and are placed alongside the track at intervals of from 2 to 8 miles, according to the density of the traffic. The sectionalizing switches—which also form overload circuit-breakers—are of the electrically-operated oil-break

type, and are placed on the top of the sectionalizing gantries, the switches being controlled from an adjacent signal cabin.

Feeding Systems for 50-c/s Railways. These railways are supplied from transformer sub-stations receiving energy from a three-phase transmission

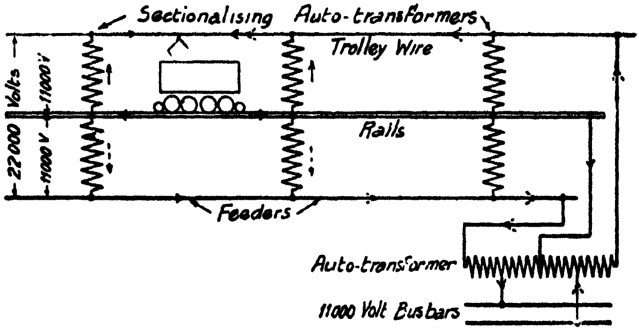


FIG. 309. METHOD OF CONNECTING OVERHEAD FEEDERS AND TROLLEY WIRE TO MINIMIZE INTERFERENCE EFFECTS ON NEIGHBOURING CIRCUITS

network or "grid." Although the traction load may be relatively small in comparison with the industrial load, steps should be taken to reduce the unbalancing effects on the three-phase system.

The Scott- or T-connected three-phase/two-phase transformer provides a simple solution to this problem if the phases on the output (two-phase) side supply adjacent tracks and the loads are approximately balanced.

The low power factor of 50 c/s motors, particularly at starting and low speeds, may cause large voltage drops in the distributing system and feeders, and banks of capacitors may be required to compensate a portion of this voltage drop in order to obtain more uniformity in the voltage at the trolley-wires.*

The percentage voltage drop, however, may be made comparable with that on a low-frequency ($16\frac{2}{3}$ c/s) system by adopting a higher voltage (e.g. 20 to 25 kV instead of 15 kV).

* A 10 MVAr capacitor is installed on the 25-c/s lines of the New York, New Haven and Hartford Railroad. See *Trans. A.I.E.E.*, Vol. 67, p. 258.

CHAPTER XXVI

SUB-STATIONS

THE supply of energy to the distribution systems of extensive railways is best effected from sub-stations, as, even with single-phase electrifications, the distribution voltage is too low for the transmission of large amounts of energy over long distances. The type of sub-station plant employed will depend upon the nature of the primary supply and the system of electrification. Thus if the railway purchases energy from a general extra-high-voltage three-phase network (or "grid"), rotating converting machinery will be necessary for low-frequency a.c. systems and mercury-arc rectifiers for d.c. systems.

I. SUB-STATIONS FOR SINGLE-PHASE RAILWAYS

In many European single-phase railways the primary supply system is designed solely for the railway load; the generators being single-phase low frequency ($16\frac{2}{3}$ c/s) machines, wound for the distribution voltage (15,000 V), in order that tracks near the generating station may be supplied direct. The sub-stations are supplied by high-voltage (66–132 kV) transmission lines, which are fed by transformers at the generating station. But with the continued extension of an electrified system—as on the Swiss Federal Railways—the situation may arise when extensions to the railway generating stations become impracticable, and the best economic solution would be the purchase of power from a three-phase national network. The sub-stations must then be equipped with converting plant (frequency changers).

With the object of saving the cost of expensive buildings for housing the transformers and switchgear, modern transformer sub-stations are of the outdoor type. The transformers and extra-high-voltage switchgear are installed out of doors, the switches being power operated and remote controlled from a central control room containing desk-type switchboards on which are mounted the control switches, measuring instruments, and remote indicating devices. The high-voltage lines entering the sub-station, and also the feeders for the traction distributing system are dead ended to steel structures; and the connexions to the switches, transformers, etc., are made by bare copper tubes or cables.

Fig. 310 shows a typical sub-station of the *Swiss Federal Railways*. The transformers and switches are mounted above ground level on concrete blocks; the layout being such as to facilitate the removal and replacement of any piece of apparatus. When sufficient ground area is available the apparatus is so arranged that the interconnexions are self-supporting, thus eliminating a large number of steel structures and insulators which would otherwise be necessary.

These outdoor sub-stations have given very satisfactory service in Switzerland under all conditions of weather, and the switchgear has been unaffected by heavy snowfalls (the sub-stations continuing in service with the 15-kV switchgear insulators covered in snow.)

The Stockholm-Gothenburg main lines of the *Swedish State Railways*, receive energy from a 50-c/s, three-phase, 132-kV national network, and accordingly the sub-stations have had to be equipped with converting plant. The sub-stations are situated at railway stations about 80–100 km (50–62 miles)

FIG. 310. OUTDOOR SUB-STATION (60 15 kV) OF SWISS FEDERAL RAILWAYS

Note.—The 60-kV switchgear and bus-bars are on the right, the 15-kV switchgear bus-bars and outgoing feeders are on the left (Oerlikon)

apart, and are equipped with transformers and motor-generators. The three-phase step-down transformers and the 132-kV switchgear are of the outdoor type; but the single-phase step-up transformers, the (16-kV) switchgear for the traction circuits, and the switchgear for controlling the motor generators, are of the indoor type. The motor-generators (of which there are twelve distributed between five sub-stations) each consist of a three-phase 12-pole synchronous motor (3,200 kVA, 6,300 V), a single-phase 4-pole alternator (2,400 kVA, 3,000 V), and exciters. The motor is arranged for self-starting from an auto-transformer, and the alternator of each set is directly connected to a step-up transformer (3,000/16,000 V) from which the traction circuits are supplied. This transformer protects the alternator windings from the effects of short-circuits and lightning discharges on the trolley wires.

The sub-station supplying the 50-c/s (Annecy—Aix-les-Bains) line of the *French National Railways* receives energy from a three-phase network. The equipment consists of a three-phase/two-phase transformer and switchgear of the out-door type, the latter being remote controlled from an adjacent signal cabin.

The sub-stations of the 50-c/s *Belgian Congo Railway* also receive energy from a three-phase industrial network, but in this case the transformers are of the single-phase type (each 6,000 kVA, 120/25 kV) as the traction load is small in comparison with the industrial load. Three transformers are in normal service, the secondary windings supplying different sections of the traction distribution system at 25 kV, and the primary windings being connected successively to the three phases of the supply system. Transformers and switchgear are of the outdoor type.

II. Sub-stations for Direct-current Tramways, Trolleybus Routes and Railways

These sub-stations are now equipped with mercury-arc rectifiers (also called mutators) which have superseded rotary (synchronous) converters on account of their much higher all-day efficiency, reliable operation at all voltages and adaptability to remote control with simplicity of operations. By locating the supply transformer out of doors only a simple building or kiosk is required for housing the rectifier.

The rectifier, however, is non-reversible. Hence when a rectifier sub-station feeds a system on which regenerative braking is employed provision must be made for the reception of returned energy. In cases where this energy may be considerable it may be returned to the a.c. supply system by means of an inverter (i.e. a grid-controlled rectifier arranged to operate with reversed voltage). In other cases the energy is dissipated in a loading resistor which is connected across the d.c. busbars by a quick-acting switch of the electronic type in conjunction with a voltage relay.

Mercury-arc Rectifiers

General. The mercury-arc rectifier consists essentially of a mercury cathode and a number of graphite or iron anodes enclosed in a highly exhausted bulb or cylinder. The rectifying action is due to electronic emission from a hot spot on the cathode, the conditions upon which this action depends being (i) the maintenance of a high vacuum (0·005–0·001 mm of mercury) in the electrode chamber; (ii) the maintenance of the high temperature (about 3,000°C) of the cathode hot spot; (iii) the prevention of emission from the anodes (i.e. these electrodes must be suitably cooled). The high temperature of the cathode hot spot causes evaporation of the mercury,

and therefore means must be provided for condensing the vapour and returning the mercury to the cathode. The presence of the mercury vapour in the electrode chamber gives a low resistance to the electron stream.

Types. The rectifiers originally developed for heavy traction service were of the steel tank, water-cooled, pumped type with either six or twelve main anodes.

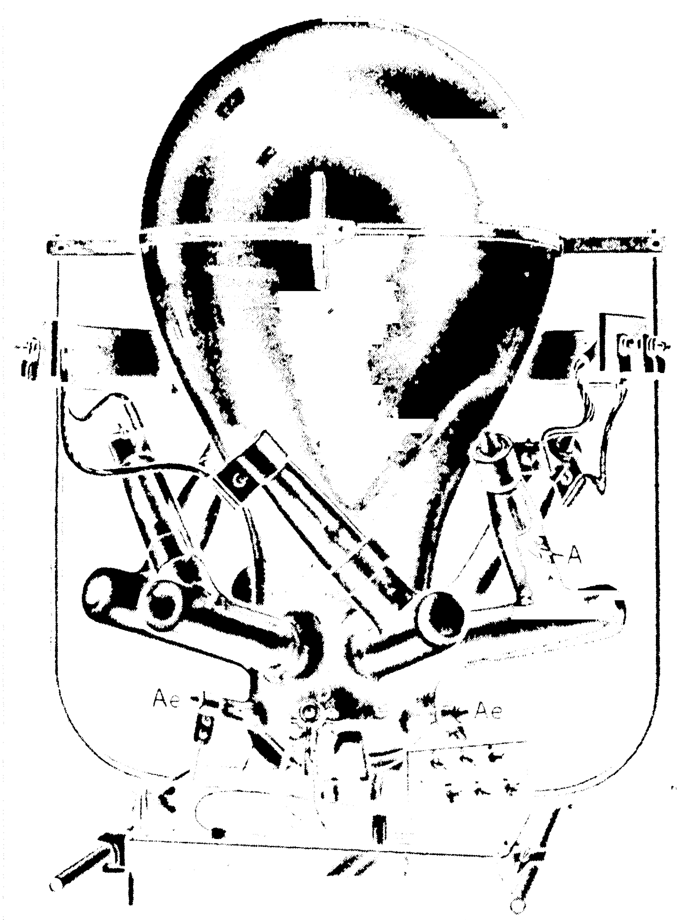


FIG. 311. GLASS BULB RECTIFIER (HEWITTIC)
A, main anode, *Ae*, excitation anode, *C*, mercury pool, *D*, ignition anode

Although rectifiers of this type, with ratings of 2,000 kW, are installed in many sub-stations, the tendency in this country and Europe is towards the use of multi-unit, pumpless, air-cooled rectifiers of either the glass bulb or the hermetically-sealed steel-tank type; a number of rectifiers of moderate output being operated in parallel and supplied from one transformer. In such cases the individual rectifiers may have either three or six main anodes, and the connexions may be arranged to give the equivalent of a 12- or 24-phase supply to the anodes (depending on the

number of rectifiers) thereby reducing considerably the harmonics in both the a.c. (primary) supply system and the d.c. output.

Single-tank rectifiers of the ignition type are installed in many American traction sub-stations as such rectifiers, although water-cooled and pumped in the larger sizes, are considered to be more reliable and convenient than those of the multi-anode type.

Construction. *Glass bulb rectifiers* are constructed of tough heat-resisting glass. The bulb, Fig. 311, is of inverted pear shape with elbow-shaped arms, containing

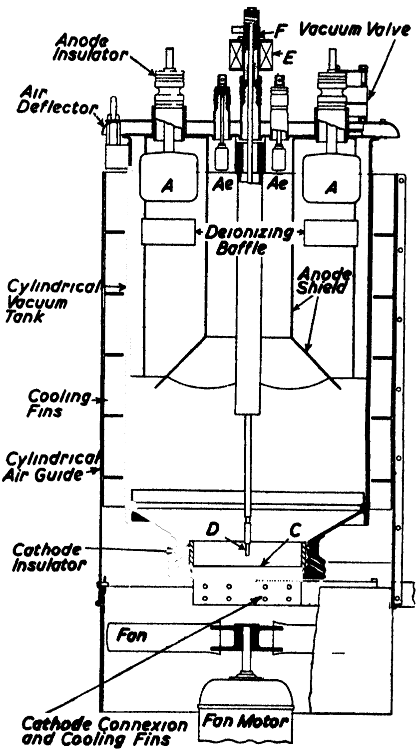


FIG. 312 ARRANGEMENT OF ELECTRODES, TANK AND AIR-COOLING SYSTEM OF PUMPLESS RECTIFIER (B.T.-H.)

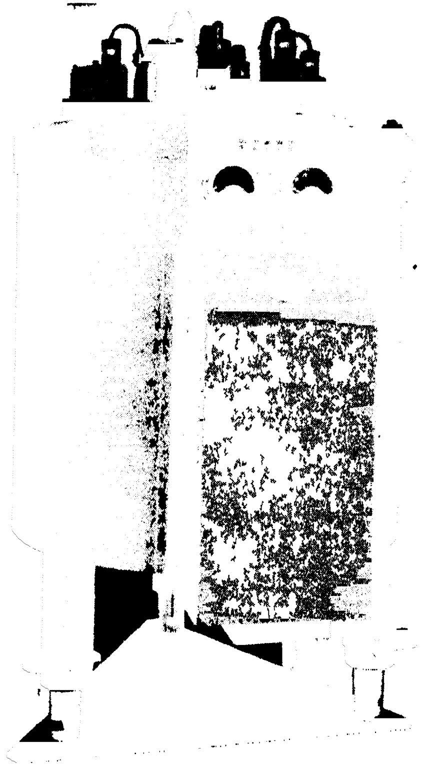


FIG. 313. AIR-COOLED, PUMPLESS, STEEL-TANK RECTIFIER (B.T.-H.)

the anodes, fused to the lower end, below which is the mercury pool. The portion of the bulb above the anodes forms the condensing chamber for the mercury vapour, and to augment the cooling of this chamber and the anodes air is blown upwards over their surfaces by a propellor-type fan located below the rectifier. Ignition is effected by an auxiliary anode, fixed to a horizontal spring-steel support with external electromagnets, the lower one, when excited, pulling the anode into the mercury pool and the upper one, when excited, lifting it clear of the pool.

Fig. 312 shows the arrangement of electrodes, tank and air-cooling system of a pumpless rectifier of the *cylindrical tank* type, and Fig. 313 shows the complete rectifier for an installation where the rectifiers are not enclosed in cubicles.

The six main anodes, together with the auxiliary anodes for ignition and

excitation, are mounted on the top plate, the joints being welded. The insulators, one of which is shown in Fig. 314, are of vitreous enamel. A sector-shaped shield of sheet steel is arranged around each anode to protect it from the direct blast of mercury vapour, and to prevent particles of mercury being projected on to its surface.

The tank is cooled by a stream of air blown upwards by a propellor-type fan mounted below the cathode. The air passes over the surfaces of numerous fins

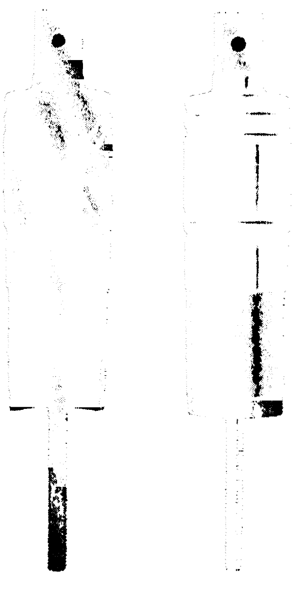


Fig. 314. Cross-section and Outside Views of Vitreous-enamel Insulator (B.T.-H.)

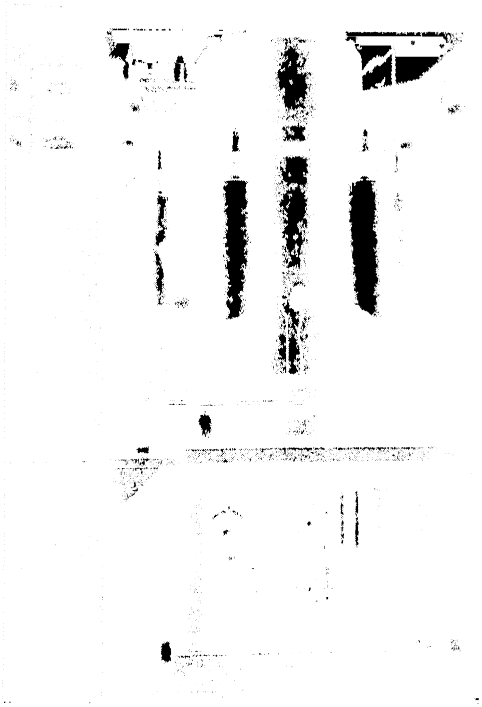


Fig. 315. Air-cooled, Pumpless, Steel-tank Rectifier with Accessories, Installed in Cubicle in Sub-station of Nederlands Railways (G.E.C.)

which are welded to the outside of the tank, the air channels being formed by a closely fitting outer cover.

The ignition anode, D (Fig. 312), is of the plunger type and is operated by a solenoid, E, above the top plate. The core or plunger, F, is supported on a spring and when the solenoid is excited it is pulled downwards and plunges the tip of the anode into the mercury pool, C. The circuits for ignition and excitation are shown in Fig. 316.

Fig. 315 shows an alternative form of pumpless air-cooled rectifier in which the main anodes are mounted in separate elbow-shaped arms projecting from a central tank which contains the cathode and auxiliary anodes, and which forms the condensing chamber for the mercury vapour.*

* Further details are given by J. C. Read in a paper, "Mercury-arc Rectifiers for Medium-voltage Applications." (A Review of Progress), *Proc. I.E.E.*, Vol. 99, Pt. I, p. 252.

Main Connexions. Fig. 316 shows these connexions for a six-anode rectifier. The transformer has double secondary windings which are connected to form two symmetrical three-phase star groups having a mutual phase difference of 180 degrees. The two neutral points are connected by a centre-tapped reactor, or inter-phase transformer (also called an "absorption coil"), the centre point of which forms the negative terminal of the direct-current circuit. This coil is wound in sections, which are so connected that (i) the complete winding possesses a high reactance, (ii) the direct currents passing in the two halves of the winding produce no resultant magnetization of the core. Hence the theoretical voltage ratio is the same as that for a three-phase rectifier. But the pulsations of the load voltage are the mean of

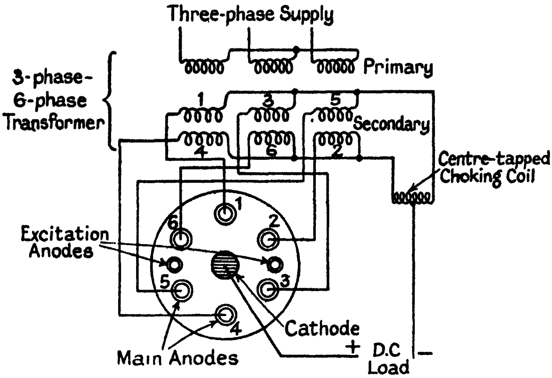


FIG. 316. MAIN CONNEXIONS OF SIX-ANODE RECTIFIER

those of two three-phase systems 180 degrees apart, and are therefore equal to those of a six-phase rectifier.

This three-phase double-star connexion possesses other features. Thus, (i) opposite anodes operate in parallel, and (ii) a closed path, internal to the rectifier and transformer, is formed for the circulation of triple-frequency currents (which are due to the triple-frequency e.m.f.'s. existing between the neutral points of the secondary windings). Both of these features result in the voltage drop, due to a given load current, being smaller than that in a six-anode rectifier with the six-phase star connexion (i.e. with the two neutral points of the transformer winding solidly connected). Moreover, the three-phase double-star connexion is more favourable than the six-phase star connexion for the design of the transformer, both with respect to power factor and the utilization of copper in the windings.

This connexion, without the reactor, is also advantageous for 3,000-V systems, as with a group of six ignitrons a *cascade arrangement* may be obtained in which each ignitron operates at 1,500 V, thereby ensuring great reliability. Thus the neutral point of one group of secondary windings (e.g. Nos. 1, 3, 5, Fig. 316) forms the negative terminal of the d.c. side; the cathodes of tanks Nos. 1, 3, 5, are connected together and to the neutral point of the other group (Nos. 2, 4, 6) of secondary windings; and the cathodes of tanks Nos. 2, 4, 6, are connected together to form the positive terminal. Such a scheme has been adopted in some of the more recent sub-stations of the Italian State Railways.

At light loads, with the connexions of Fig. 316, a 15 per cent rise occurs in the output voltage due to insufficient magnetizing current in the reactor. This voltage rise may be avoided by an auxiliary magnetizing winding on the reactor supplied with triple-frequency current obtained from the opened delta secondary winding of a small three-phase star/delta transformer.

Ignition and Excitation Connexions. Fig. 317 shows these connexions for a rectifier with a plunger-type ignition anode. When the supply is switched on to the auxiliary transformer, the solenoid, E, is excited and the anode, D, is plunged into the mercury pool, C, the current being limited by the resistors R_1, R_2. Relay B

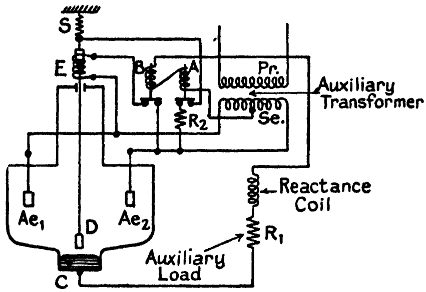


FIG. 317. CONNEXIONS OF EXCITATION AND IGNITION CIRCUITS

then operates and opens the circuit of E, causing D to be withdrawn from the mercury (by the action of the spring S) and an arc to be struck. If this arc is of the correct polarity the excitation anodes, Ae_1, Ae_2, come into action, and the increased current causes relay A to operate and isolate D.

Alternative connexions for the ignition circuits of glass-bulb rectifiers are shown

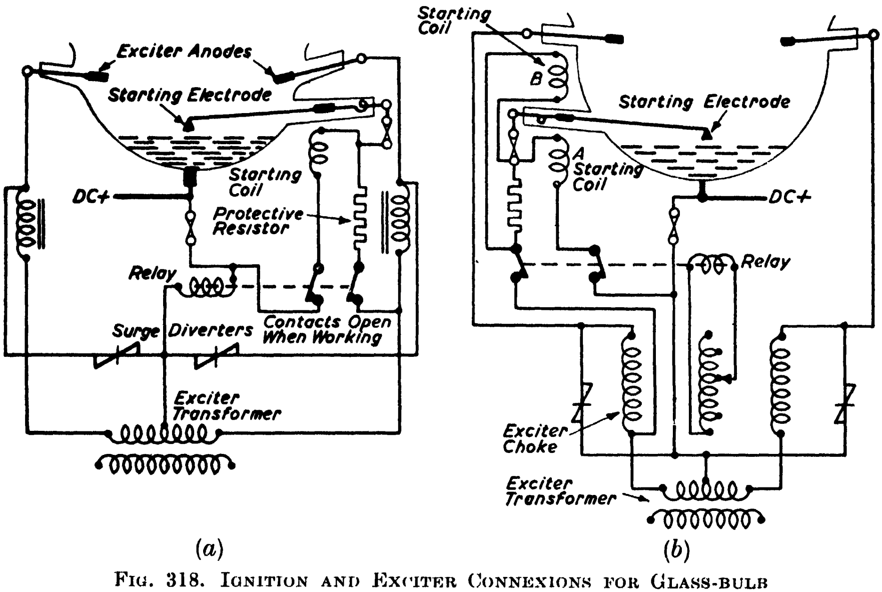


FIG. 318. IGNITION AND EXCITER CONNEXIONS FOR GLASS-BULB RECTIFIER (HEWITTIC)

in Fig. 318. In one case (a) a single electromagnet or starting coil is employed, which when energized pulls the starting electrode into the mercury pool. This action short circuits the coil and the electrode breaks contact with the mercury. If the arc is of correct polarity the exciter anodes come into action and the relay opens the starting coil circuit.

In the other case (b) two starting coils and a three-limbed exciter choke are

employed. When the lower coil, A, is energized the starting electrode is plunged into the mercury pool. This action short circuits the coil and applies sufficient voltage to the upper coil, B, to cause the electrode to be withdrawn from the mercury. With this arrangement the bulb does not require such precise setting in its cradle as in the previous case.

Voltage Ratio. The theoretical voltage ratio is easily calculated if the voltage drops in the arc and other parts of the circuit are ignored, and the alternating voltages follow sine laws. Thus, if m is the number of phases in the secondary system supplying the anodes, E_a the r.m.s. value of the e.m.f. of each phase, $2\pi/m$ the mutual phase difference between the several e.m.f's., the output voltage (E_d) is the mean value of the phase e.m.f. taken over an interval π/m on each side of the maximum value, i.e.—

$$E_d = \frac{m}{2\pi}\int_{-\pi/m}^{+\pi/m} \sqrt{2}\,.\,E_a \cos \omega t\,.\,d\omega t = \left(\sqrt{2}E_a \sin\frac{\pi}{m}\right)\Big/\frac{\pi}{m}$$

Whence

$$E_d/E_a = [\sqrt{2}\sin(\pi/m)]/(\pi/m) \quad . \quad . \quad . \quad . \quad (59)$$

The values of this quantity are—

1·17 for a three-phase rectifier (3 anodes)
1·17 for a double three-phase rectifier (6 anodes)
1·35 for a six-phase rectifier (6 anodes)
1·4 for a twelve-phase rectifier (12 anodes)

Operating Features. The voltage drop between the electrodes of the rectifier is about 25 volts and is practically constant at all loads. Hence, as this constitutes the only loss in the rectifier itself, the theoretical efficiency (neglecting the excitation circuit) is a function of the output voltage. For example, for output voltages of 600, 1,500, and 3,000 the theoretical efficiencies are (600/625 =) 0·96, (1,500/1,525 =) 0·984, and (3,000/3,025 =) 0·992. When the losses in the transformer, and the power supplied to the auxiliary circuits, are taken into account, the overall efficiencies may be—

1. 600-volt plant—2,000-kW rectifier supplied from 11,000-volt, 50-c/s, three-phase system—

Percentage of full load .	25	50	75	100	125	150
Percentage of efficiency .	93·9	94·8	95	94·8	94·7	94·6

2. 1,500-volt plant—2,000-kW rectifier supplied from 11,000 volt, 50-c/s, three-phase system—

Percentage of full load .	25	50	75	100	125	150
Percentage efficiency .	96	96·5	96·5	96·2	95·9	95·6

Owing to the relatively low light-load losses the all-day efficiency of a traction sub-station equipped with rectifiers is much higher than that of a similarly loaded sub-station equipped with rotating converting machinery. This higher all-day efficiency results in a very considerable annual saving of energy.

The instantaneous *overload capacity* of a rectifier is very large, and the permissible short-time and sustained overload periods for the combination of rectifier and transformer may be 50 per cent overload for 2 hours, 200 per cent overload for 5 minutes and 400 per cent overload for 15 seconds.

The *voltage characteristic*, or regulation curve, resembles that of a shunt generator. The percentage voltage drop from light load to full load depends upon the connexions employed and the impedance voltage of the transformer. A value between 3·5 and 5 per cent is representative for a double three-phase rectifier.

Suppression of Backfires. Rectifiers for 1,500-V and 3,000-V systems are fitted with a grid in front of each anode for the purpose of suppressing rapidly the internal current due to a backfire (caused by a hot-spot on an anode, and that anode becoming temporarily a cathode) or a short-circuit on the d.c. side. The grids are

normally excited in phase with the corresponding anode voltages, but in the event of a backfire (which is equivalent to a short-circuit across two anodes) or a d.c. short-circuit, an ultra-rapid relay, energized from a current transformer on the a.c. side, causes a negative potential to be applied to all the grids. Hence the arc at the conducting anode cannot transfer to an adjacent anode, as in normal operation, and therefore extinguishes at the end of the half cycle.

Inverter Operation. When a rectifier sub-station has to receive energy from regenerating vehicles and the conditions warrant the expense of equipment to return this energy, less losses, to the a.c. supply system, additional rectifiers, arranged for inverter operation, must be installed. Control grids are essential. One transformer may be employed to feed both a rectifier and an inverter if separate secondary windings are provided. The main connexions between the anodes and the appropriate secondary winding are the same in both cases, but the neutral point of the inverter winding is connected to the positive d.c. busbar and the cathode to the negative.

Inverter operation is obtained by delaying the firing of each anode (by the action of the control grid) until the voltage of the appropriate secondary winding becomes negative relatively to the voltage at the d.c. busbars (i.e. these voltages oppose each other). The current returning to the busbars is forced through the inverter—from anode to cathode—because the voltage at the d.c. busbars is higher than the opposing voltage of the secondary winding plus the voltage drop in the arc.

The current in the secondary winding (on the assumption of unity power factor) is in opposition to the e.m.f. induced in this winding, and therefore the corresponding current in the primary winding is in the same direction as the induced e.m.f. in that winding (i.e. in opposition to the supply voltage), which is the condition of a generator feeding into a supply system.

The control gear for inverter operation is considerably more complicated and costly than that for simple rectifier operation. The largest installations are on the South African Railways.*

Unattended (Remote-controlled and Automatic) Sub-stations

General. Since mercury-arc rectifiers require no "running" attention and the starting is automatic, their installation in unattended sub-stations presents no practical difficulties, and if transformers and a.c. switchgear are of the outdoor type, the buildings have to house only the rectifiers and d.c. switchgear. The operations of starting and shutting down an unattended sub-station are effected automatically, and may be initiated either automatically by relays responsive to a demand for power or by remote (supervisory) control from a central control room or the train dispatcher's office.

High-speed Circuit-breaker. The special features of this important accessory for heavy traction plant are—(i) extremely high speed of opening for any rate of rise of current; (ii) electromagnetic release not involving latched parts; (iii) powerful magnetic blow-out with narrow arc shute. Such a circuit breaker can clear a severe short circuit on a 600-volt system in about 0·01 sec, whereas about 0·15 sec would be required with an ordinary circuit breaker. Moreover, the current peak when a short circuit is cleared by a high-speed circuit breaker is only about 50 to 60 per cent of that when an ordinary circuit breaker is employed under similar circuit conditions.

* Some particulars of the equipment and operation are given in a paper, by A. J. G. Gosling on "The 3,000-V mercury-arc-rectifier traction sub-stations of the South African Railways." *Proc. I.E.E.*, Vol. 97, Pt. IA, p. 152.

Fig. 319 shows the scheme of connexions for remote manual control. The closing is effected by energizing both the holding and reset coils, the mechanism being designed so that the final closing of the contacts is effected by springs when the reset coil is de-energized. The contacts are held closed by the holding

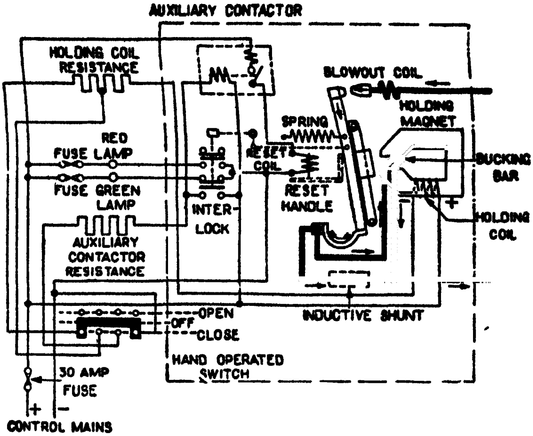


FIG. 319. CONNEXIONS OF HIGH-SPEED CIRCUIT-BREAKER (B.T.-H.)

magnet acting on the armature attached to the moving contact. Tripping is effected by the magnetic effect of the main current in a loop (called the "bucking bar") arranged in the air gap between the poles of the holding magnet. When the current in this loop exceeds a predetermined value sufficient flux is diverted from the armature to cause its release by the pull-off springs.

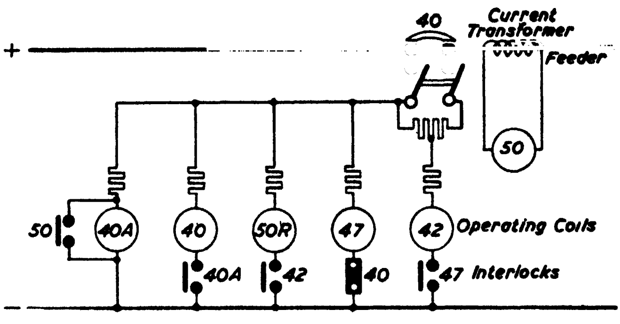


FIG. 320. SCHEMATIC DIAGRAM SHOWING CONNEXIONS FOR AUTOMATIC RE-CLOSING CIRCUIT BREAKER FOR FEEDER (METROPOLITAN-VICKERS)

40—circuit breaker.
40A—auxiliary contactor.
42—relay.
47—auxiliary contactor.
50—discriminating relay.
50R—re-set coil for 50.

Automatic Switchgear for d.c. Feeders. This switchgear comprises (i) an electrically-operated circuit breaker, (ii) relays for controlling the closing and tripping of the circuit breaker.

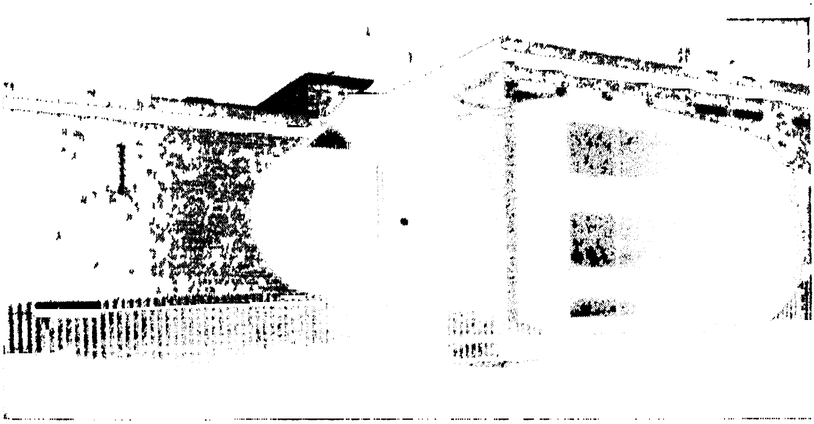
The simplest method of operation (which gives no discrimination between short circuits and legitimate sudden overloads) is to arrange that when the circuit

breaker opens, due to an overload or short circuit, it is re-closed again automatically after a short time delay. If the circuit breaker re-opens, it is re-closed again, and, if necessary, the cycle is repeated for a pre-determined number of times, when, if the fault still persists, the circuit breaker is locked out. This action is obtained by means of a time-delay repeat-action relay, which automatically re-sets if the circuit breaker remains closed for a pre-determined time interval, but requires to be re-set by hand when the circuit breaker is locked out.

FIG. 321. SUB-STATION WITH CUBICLE-MOUNTED RECTIFIERS (HEWITTIC)
Each cubicle contains four 3-anode bulbs

Discrimination between short circuits and sudden overloads is obtained by a current transformer and a quick-acting relay, the latter controlling the tripping circuit of the circuit breaker. The primary winding of the current transformer is connected in series with the feeder and the secondary winding is connected to the relay. The operation of the relay therefore depends upon the *rate* of increase of current in the feeder and the duration of the transient effect. Such a device gives satisfactory service in cases where the circuit conditions are suitable, but in some cases (e.g. with very heavy suburban railway service) difficulty may be experienced in obtaining sufficient discrimination between short circuits and heavy demands for power.

Automatic re-closing of a feeder circuit breaker upon the clearance of a fault is obtained by means of a load resistance relay and an auxiliary bridge network of fixed resistances. The operation of the relay is dependent upon the combined resistance of the load and fault. If this resistance exceeds a predetermined value the relay causes the circuit breaker (which is assumed to have opened



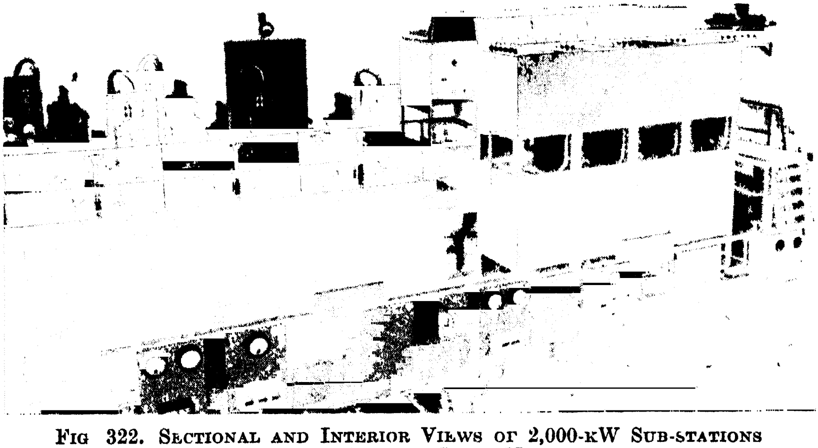


Fig 322. Sectional and Interior Views of 2,000-kW Sub-stations on Euston–Watford Line (Hewittic)

The lower view shows the two-tier rectifier equipment installed in a sub-station originally equipped with rotary converters The high-speed circuit-breakers for the traction circuits are shown on the gallery

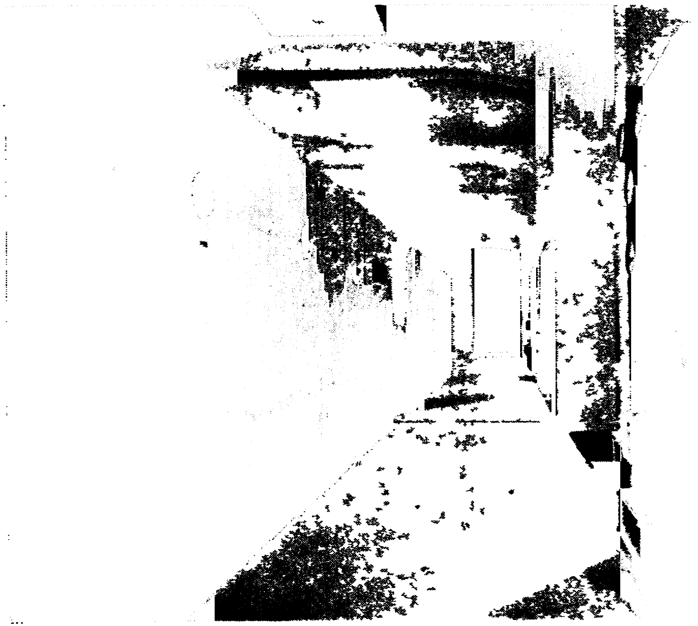


Fig 323 Tiete (Brazil) Sub station showing 2,000 kW, 3,000 V Air cooled, Pumpless, Steel Tank Rectifier Equipment and Control Panels (English Electric)

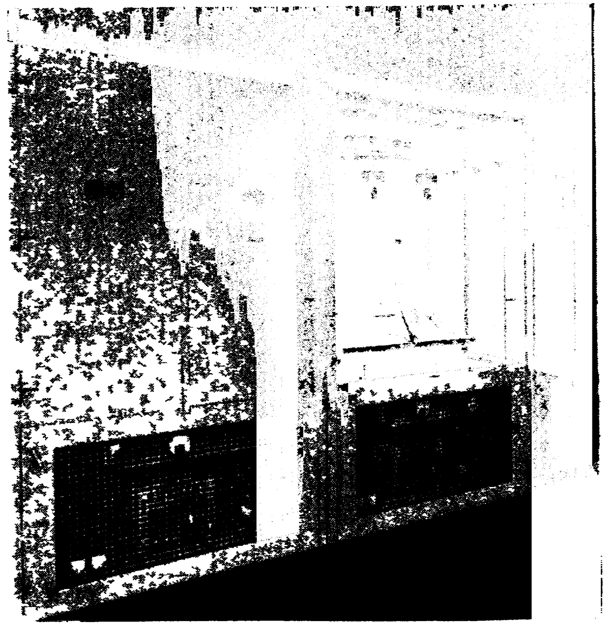


Fig. 324 Rectifier Cubicles with 500-kW, 3,000-V Rectifiers (English Electric)

due to a temporary short circuit) to re-close, but if the resistance is below this value the relay prevents the re-closing of the circuit breaker.

A schematic diagram of the circuits is shown in Fig. 320. The closing coil of the circuit breaker (40) is controlled by a small contactor (40 *A*), the operating coil of the latter being connected in parallel with the contacts of the short-circuit discriminating relay (50), the re-set coil (50 *R*) of which is controlled by the load-resistance relay (42).

When the circuit breaker opens, the auxiliary resistances are connected in series with the feeder, and an auxiliary switch on the circuit breaker closes

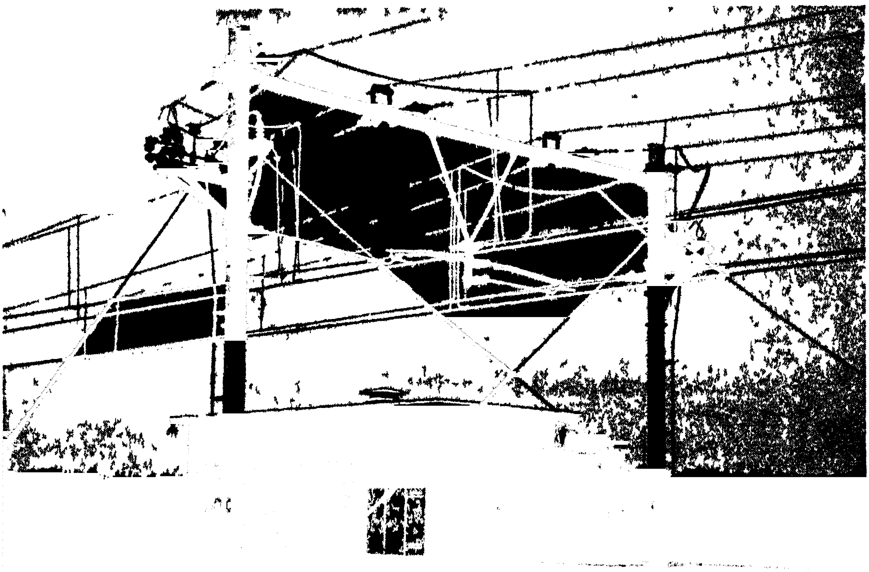


FIG. 325. SUB-STATION ON THE 1,500-V SYSTEM OF THE NEDERLANDS RAILWAYS (G E C EQUIPMENT)

the operating coil of contactor (47), the contacts of which close after a short time delay and connect the relay (42) to the bridge network. If the combined resistance of the load and fault is below a predetermined value the current passing through the relay (42) will be insufficient to operate its contacts, and in consequence the feeder circuit breaker will remain open, due to the contacts of relay (50) shunting the coil of contactor (40 *A*). But as soon as the fault clears, sufficient current passes through relay (42) to close its contact. The re-set coil of relay (50) is therefore actuated and the short circuit across the operating coil of contactor (40 *A*) is removed, thereby allowing the circuit breaker to re-close.

EXAMPLES OF TRACTION SUB-STATIONS

Modern rectifier sub-stations in this country and Europe are invariably equipped with pumpless, air-cooled rectifiers of either the glass bulb or steel-tank type.

Glass bulb rectifiers in multiple units have been in service for many years and are characterized by their reliability and extremely low maintenance. Steel-tank rectifiers of the type shown in Fig. 313 have been in service in railway sub-stations since 1936, groups of four rectifiers being connected in parallel and supplied from one transformer.

The modern tendency with British manufacturers is to adopt the cubicle type of installation in which the rectifier is mounted, together with its accessories, in a cubicle of sheet steel.

Fig. 321 shows a trolleybus sub-station equipped with two 500-kW cubicles each containing four 3-anode bulbs connected for 12-phase rectification.

Fig. 322 shows sectioned and interior views of 2,000-kW sub-stations on the Euston-Watford line of the London Midland Region. The rectifier equipment in each sub-station is arranged for 12-phase rectification, but provision is made so that, when desired, alternate sub-stations can be connected with staggered phasing to give the equivalent of 24-phase operation. No smoothing circuits on the d.c. side, however, have been found necessary with 12-phase rectification. All sub-stations (17 in all) are remotely controlled from a central control room.

Fig. 325 shows the standard form of sub-station building, and the method of supplying the trolley wires, adopted on the 1,500-V system of the Nederlands Railways. The building contains two British-built rectifiers of the pumpless, air-cooled type shown in Fig. 315. The two rectifiers, operating in parallel, have a normal rating of 1,224 kW at 1,530 V, but are capable of operating at 150 per cent overload for two hours, and of withstanding, during this period, two current peaks of 4,000 A each of one minute duration. The equipment had to be designed for a close voltage regulation of 2·7 per cent.

Fig. 323 is an interior view of a sub-station (with British-built equipment) for supplying a 3,000-V railway. The rectifiers are of the pumpless, air-cooled type and each group of four is rated at 2,000 kW. The panels and cubicles on the right-hand side contain indicating instruments, protective relays and control gear for the circuit breakers. Fig. 324 shows two of the rectifier cubicles.*

* Further particulars of the sub-stations and details of the electrification are given in a paper by Messrs. Chatterton and Rooney on "The first stage of the electrification of the Estrada de Ferro Santos a Jundiai (late São Paulo) Railway," *Proc. I.E.E.*, Vol. 100, Pt. II, p. 319.

APPENDIX

ABSTRACTS FROM THE STANDARDS OF THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

RAILWAY MOTORS

Rating

THREE ratings are standardized and are specified as follows—

(i) The *one-hour rating* of a railway motor shall be the output at the motor shaft measured in horse power (or kilowatts) which the motor can carry for one hour on a stand test, starting cold, at its rated voltage (and frequency in the case of an a.c. motor) with the ventilation system as in service, without exceeding the temperature limits given in the accompanying table. The initial temperature of the windings should be within 4°C of the ambient temperature, and the latter should preferably be not less than 15°C.

(ii) The *continuous rating* of a ventilated motor shall be the output at the motor shaft measured in horse power (or kilowatts) which the motor can carry for an unlimited period on a stand test, with the ventilating system as in service, without exceeding the temperature limits given in the table.

(iii) *Thermal capacity rating* (which represents the capability to absorb heat energy and is a basis of comparison between machines of the same class) is the time in seconds for the temperature to rise 1°C above the normal working temperature (110°C) when operating at 60 per cent above the continuously-rated current and normal voltage. It is determined by commencing the overload run with the windings at 110°C and continuing the run until approximately the limiting peak temperature is reached. The temperature rise during the test is then computed (from resistance measurements), and is divided into the duration of the test to obtain the thermal capacity rating (i.e. sec per 1°C).

ITEM	Type of Enclosure	Method of Temperature Determination	Limiting Temperature Rise °C			
			One-hour rating		Continuous rating	
			A*	B*	A*	B*
Armature winding	Ventilated	Resistance Thermometer	100 75	120 90	85 65	120 90
Field winding	Ventilated	Resistance Thermometer	100 75	130 95	85 65	130 95
Commutator	Ventilated	Thermometer	90	90	90	90

* A denotes "Class A" insulation, i.e cotton, silk, paper, and similar organic substances when impregnated, also enamel as applied to conductors.

B denotes "Class B" insulation, i.e. inorganic materials (mica, asbestos) in built up form, combined with binding substances. If Class A material is used in small quantities for structural purposes only the combined material may be considered as Class B, provided that the electrical and mechanical properties of the insulated winding are not impaired by the application of the temperature permitted for Class B.

Nominal line voltages for d.c. railways are—600, 750, 1,200, 1,500, 3,000 volts.

Standard (*rated*) *voltages* of d.c. motors are—300, 375, 500, 600, 750, 1,000, 1,200, 1,500 volts. Motors shall operate satisfactorily at voltages up to 10 per cent above the rated voltage when motoring and 20 per cent when regenerating.

Rated voltage of motors for Diesel-electric locomotives corresponds to 40 per cent of maximum safe armature speed with continuously-rated current.

Rated voltage of single-phase motors corresponds to 70 per cent of maximum speed with continuously-rated current.

Temperature Limitations and Measurement

The *limiting observable temperatures recommended for service* are—

	Normal Values	*Peak Values*
Class A insulation . .	90–110°C	100–125°C
Class B insulation (armature)	115–145°C	125–160°C
Class B insulation (field windings)	120–155°C	130–170°C

The "normal" values are those which will be obtained in normal service with cooling air at a temperature of 25°C. The "peak" values are those which will usually be found with maximum motor loads and highest cooling air temperatures. In each case the lower limit refers to the temperature as determined by thermometer, and the upper limit refers to the temperature as determined by resistance measurements.

The *temperature rise, when determined from resistance measurements,* is computed from the formula—

$$t_2 - t_a = [(R_2 - R_1)/R_1](234{\cdot}5 + t_1) + (t_1 - t_a)$$

where t_2 and t_1 denote the final (hot) and initial temperatures of the windings corresponding to the resistances R_2 and R_1, respectively, and t_a is the ambient air temperature at shut-down (mean value over last 15 min of test).

The hot resistance, R_2, is to be corrected to the value at the instant of shut-down by taking resistance measurements at 2-min intervals for approximately 20 min, plotting a (resistance/time) cooling curve and extrapolating to the instant of shut-down.

Characteristic Curves

350. The *characteristic curves* of railway motors shall be plotted with the current as abscissae, and the tractive effort, speed, and efficiency as ordinates. In the case of a.c. motors, the power factor shall also be plotted as ordinates.

351. *Characteristic curves of d.c. motors* shall be based on full rated voltage.

352. *In the case of field-control motors,* characteristic curves shall be given for all operating field connexions.

353. *Speed curves* shall be substantially of the same shape as, and shall not deviate more than $\pm$ 5 per cent from, the specified speed curve over the range 75 per cent to 150 per cent of the continuously-rated current. At the continuous rating the deviation of the speeds of individual motors from the accepted speed curve shall not exceed — $\pm$ 3 per cent for d.c. motors with full field; $\pm$ 4 per cent for single-phase motors and d.c. motors with weak field. These tolerances also apply to the speeds for clockwise and counter-clockwise rotation.

Efficiency and Losses

301. The following *method of determining efficiency* is recognized as standard for d.c. railway motors—

Conventional efficiency. The efficiency is obtained from the component losses, most of which are accurately determinable and the remainder of which are assigned conventional values.

302. *Normal conditions for conventional efficiency tests* and calculations—

(*a*) The efficiency shall be determined for the rated voltage.

(*b*) When the efficiency is stated without specific reference to load conditions, the continuously-rated load shall be understood.

(*c*) The efficiency of all apparatus at all loads shall be corrected to a reference temperature of 110°C*, but tests may be made at any convenient cooling air temperature, preferably not less than 10°C.

* Class B insulation. 75°C for Class A insulation.

303. *Conventional efficiencies of d.c. motors* shall be based upon the following losses*—

(*a*) I^2R losses in armature and field windings; (*b*) brush friction, armature bearing friction, and windage losses; (*c*) no load core loss; (*d*) brush contact loss; (*e*) stray load losses.

304. *Losses in d.c. motors* shall be determined as follows—

The I^2R *losses* shall be based upon the (current)^2 and the measured resistance, corrected to 110°C. (Class B insulation.)

The *no-load core loss, brush friction, armature-bearing friction and windage* shall be determined as a total under the following conditions—

In making the test, the motor shall be run without gears. The kind of brushes and the brush pressure shall be the same as in commercial service. With the field separately excited, such a voltage shall be applied to the armature terminals as will give the same speed for any given field current as is obtained with that field current when operating at normal voltage under load. The sum of the losses abovementioned is equal to the product of the counter e.m.f. and the armature current.

The no-load core loss is determined from the total losses thus obtained by deducting the power required to drive the motor light at the corresponding speeds. (For this test the machine is run as a series motor at low voltage. The product of the counter e.m.f. and the current at any speed shall be the sum of the brush friction, armature bearing friction, and windage losses.)

Brush Contact Loss. A total drop of three volts shall be assumed as the standard drop in determining brush contact loss for carbon and graphite brushes where no shunts (pigtails) are attached. Two volts drop shall be allowed where shunts are attached.

The *stray load losses* are given conventional values, as follow—

Input (% of 1-hour rating) . . .	200	150	100	75	50	25 (and under)
Stray load loss (% of no load core loss) .	65	45	30	25	23	22

305. *Conventional efficiencies of single-phase motors* shall be based upon the following losses—

(*a*) I^2R in armature, stator windings and interpole shunts; (*b*) brush friction, armature bearing friction and windage; (*c*) alternating core loss at line frequency; (*d*) rotational core loss; (*e*) brush contact loss; (*f*) stray load loss.

306. *Losses in single-phase motors* shall be determined as follows—

The I^2R *losses* shall be based upon the (current)^2 and the measured resistances, corrected to 110°C (Class B insulation), increased by 10 per cent to allow for eddy currents.

The *alternating core loss* is determined by exciting the stator (excitation) winding at line frequency and measuring the input over a range of currents with all brushes lifted from the commutator. The input (in watts), corrected for I^2R loss, gives the core loss.

The *friction, windage and rotational core losses* are determined by the auxiliary motor method as described in Chapter VII.

The *voltage drop at the brushes* shall be assumed to be—

$$2{\cdot}5 + \text{current density (A per in.}^2)/50$$

The *stray load loss* shall be calculated as 1·5 per cent of the rated kW input at rated voltage and is assumed to be constant at all motor currents.

600. The *losses in gearing and axle bearings* for single-reduction, single-geared motors varies with type, mechanical finish, age, and lubrication. The following values, based on accumulated tests, shall be used in the comparison of single-reduction single-geared motors—

Input (% of 1-hour rating) .	200	150	100	75	60	50	40	30	25
Losses (% of input) . .	2·5	2·5	2·5	2·5	2·7	3·2	4·4	6·7	8·5

* When the efficiency of the motor, including the losses in the gearing, is required, the conventional values given in §600 should be included.

Dielectric Test

The test voltage, of commercial frequency, shall be applied for one minute with the machine hot. The r.m.s. voltages are—$2E + 1{,}500$, with a minimum of 2,500 V, for machines operating on a grounded circuit, and $2E' + 1{,}000$, with a minimum of 2,000 V for machines operating on an ungrounded circuit, where E denotes the highest voltage to ground, and E' the rated voltage of the generator or the highest tapping voltage on the transformer.

(NOTE—Commutation tests are similar to those given in British Standard No. 173.)

NOTES ON THE BRITISH STANDARD No. 173 FOR THE ELECTRICAL PERFORMANCE OF TRACTION MOTORS

Rating and Temperature Limitations

The *one-hour* and *continuous ratings* are the same as those given, on page 419, in the Standards of the A.I.E.E.

The *permissible temperature rise* and the limiting normal and peak temperatures in service are also the same as those given in the Standards of the A.I.E.E.

Characteristic Curves and Efficiency

The *rated speed* and the *characteristic speed curve* of the motor shall be corrected to a motor temperature of 110°C (class B insulation)* as measured by resistance. The rated speeds (at the continuous and one-hour loads) shall not differ from the declared speeds by more than ± 3 per cent. Any statement of *efficiency* shall be based upon direct measurement and shall be corrected to correspond to a motor temperature of 110°C (class B insulation).* Unless otherwise stated, the efficiency shall be understood as referring to the motor only, and shall exclude losses due to gearing.

High-voltage Test

The test (r.m.s.) voltage shall be $(2E_n + 1{,}500)$ volts, applied for one minute with the motor hot (at the conclusion of the one-hour test).

Tests on New Machines

These comprise—heat runs at the 1-hour and continuous ratings; speed characteristic; overspeed, commutation and interruption tests, all carried out with the machine hot.

In the *overspeed* test the motor is run for 2 min at a speed 10 per cent above that corresponding to the declared maximum vehicle speed.

Commutation tests for all d.c. motors include runs of 30-sec duration in each direction of rotation (i) at rated voltage with twice rated current† and minimum field, (ii) at a voltage 20 per cent above the nominal maximum voltage of the system (E_m)‡ with rated current and minimum field, (iii) at the same voltage $(1{\cdot}2E_m)$ and field but with the speed corresponding to the maximum service speed.

Commutation tests for motors connected permanently in series include runs as above—items (i), (ii), (iii)—and additional runs (30 sec in each direction of rotation) at a motor voltage of $(1{\cdot}5E_m/$number of motors in series) with full field and 60 per cent of rated current (which may be increased if necessary to prevent overspeed).

Commutation tests for motors when employed for regenerative braking include 30-sec runs (as generator) in each direction of rotation at a voltage $1{\cdot}2E_m$ with an armature current 25 per cent in excess of the rated current, and a speed 150 per

* 75°C for class A insulation.

† Continuous rating for ventilated motors; 1-hour rating for non-ventilated motors.

‡ Defined as the voltage at the sub-station when the plant is loaded to its full capacity. The rated motor voltage is assumed to be $0{\cdot}9E_m$.

cent of that (as a motor) corresponding to rated current, full field and maximum system voltage (E_m).

Commutation tests for motors when employed for rheostatic braking include 30-sec runs, as above, at a voltage $1{\cdot}5E_m$ with twice rated current in the armature and maximum service speed.

Commutation tests for single-phase motors include runs of 30-sec duration in each direction of rotation (i) at a voltage of $1{\cdot}2E_m$* with (*a*) rated current, (*b*) the current corresponding to maximum service speed, (ii) at the maximum system voltage (E_m) with 170 per cent of rated current, (iii) at 170† per cent of rated current and a voltage adjusted to give 10 to 12 per cent of maximum service speed.

Interruption tests for motors of ratings (1 hour) up to 300 kW are carried out by running the motor on load (at a specified field strength) up to a speed above that specified for the re-application of power, interrupting the power supply (including the excitation of the load-generator) for not less than one second, and re-applying full voltage when the speed reaches the specified value.

* Defined as the no-load secondary voltage of the transformer (when the line voltage is equal to the nominal maximum voltage of the system) divided by the number of motors permanently connected in series.

† 200 per cent for motors starting with weakened field.

EXAMPLES

THIS collection of examples includes a large number of numerical questions set at the following examinations—

University of London, B.Sc. (Eng.)* Reference *L.U.*
City and Guilds of London Institute, Electrical Engineering (Final Grade). Reference *C.G.*

I—TRAIN MOVEMENT (CHAPTERS II AND III)

1. On an electric railway the stations are 2,560 ft apart and the trains operate to a schedule speed of 16 m.p.h. The duration of the station stops is 20 sec, the maximum speed is 22 per cent higher than the average running speed and the braking retardation is 2 m.p.h.p.s. Calculate the acceleration required to operate this service. State the necessary assumptions made. (*C.G.*)

[*Ans.* $\alpha = 1{\cdot}19$ m.p.h.p.s.]

2. Draw a typical speed-time curve for an electric train for suburban service. State usual values for acceleration and retardation.

On a certain line, with an average of $1\frac{1}{4}$ miles between stops, the schedule speed is 24 m.p.h. If the maximum speed attained is 36 m.p.h., the stops 20 sec., and the retardation 2 m.p.h.p.s., find the acceleration required. (*C.G.*)

[*Ans.* $\alpha = 0{\cdot}538$ m.p.h.p.s.]

3. An electric train has an average speed of 24 m.p.h. It is accelerated at 1 m.p.h.p.s., and is braked at 2·0 m.p.h.p.s. The distance between stations is 4,000 ft, and the track is level. Draw the speed-time curve for the run. Estimate the energy consumption, at the axles of the train, per ton-mile. Assume tractive resistance constant at 10 lb per ton, and allow 10 per cent for the effect of rotational inertia. (*L.U.*)

[*Ans.*

Time (sec)	0	29·9	98·6	113·6
Speed (m.p.h.)	0	29·9	29·9	0

Specific energy consumption = 55 Wh per ton-mile.]

4. A train weighing 200 tons is accelerated uniformly from rest to a speed of 28 m.p.h. up a uniform gradient of 1 in 500, the time taken being 30 sec. The power is then cut off and the train coasts down a uniform gradient of 1 in 1,000 for a period of 40 sec, when the brakes are applied for a period of 15 sec so as to bring the train uniformly to rest on this gradient.

Calculate (*a*) the maximum power output from the driving axles, (*b*) the energy taken from the conductor rails in kWh.

Assume the tractive resistance to be 10 lb per ton at all speeds, and allow 10 per cent for rotational inertia. (*L.U.*)

[*Ans.* (*a*) 1,330 kW; (*b*) 9·2 kWh, assuming energy efficiency during acceleration = 60 per cent.]

5. A train driven by 8 motors and weighing 320 tons is accelerated uniformly from rest to a speed of 30 m.p.h. in 20 sec up a gradient of 1 in 100. The driving wheels are 36 in. diameter, the gear ratio is 4 : 1 and the average resistance to traction is 12 lb per ton. Allow 10 per cent for the effect of rotational inertia and assume efficiency of gearing is 93 per cent. Calculate the torque per motor. (*L.U.*)

[*Ans.* 3,270 lb-ft.]

* The Engineering Examination Papers of the University of London are published regularly by the University of London Press.

6. A motor-coach train weighing 200 tons accelerates uniformly on the level to a speed of 27 m.p.h. in 20 sec. Power is then cut off and the train coasts until the speed is 21 m.p.h., when the brakes are applied and the train is brought to rest in 10 sec. Allowing a train resistance of 10 lb per ton during acceleration and 13 lb per ton during coasting, and 12 per cent for rotational inertia, calculate (*a*) the maximum h.p. developed at the driving axles and (*b*) the specific energy output. (*C.G.*)

[*Ans.* (*a*) 2,230 h.p.; (*b*) 50·9 Wh per ton mile.]

7. An electric train weighing 200 tons (accelerating weight 220 tons) runs between two stations, situated on a uniform gradient of 1 in 80. For the run *down* the gradient the acceleration is 1·5 m.p.h.p.s., which is maintained until the speed reaches 32·5 m.p.h. Power is then cut off and the train coasts for 90 seconds, when the brakes are applied to bring the train to rest with uniform retardation.

Draw the speed-time diagram and calculate the energy (kWh) output from the driving axles. Assume the train resistance to be 12 lb per ton and the braking effort due to the brakes alone to be 47,000 lb. (*L.U.*)

[*Ans.*

Time (sec)	0	21·7	111·7	134·9
Speed (m.p.h.)	0	32·5	45·3	0

Energy output = 5·935 kWh.]

8. The following data refer to the speed-time curve of an electric train for the run on level track between two stations on a suburban railway—

Time (sec)	0	18	20	23	28	33	38	62·6	76
Speed (m.p.h.)	0	18·7	20·6	23	26	28·2	30	27·1	0

Determine the energy output, in kWh, from the driving axles for the run. Determine also the energy output per ton-mile.

The dead weight of the train is 150 tons, and the accelerating weight is 166 tons. Assume the train resistance as 10 lb per ton.

[*Ans.* 4·65 kWh; 71·3 Wh per ton mile.]

9. Calculate the accelerating weight of the six-coach train of which data follow—

Composition of train—2 motor coaches, 4 trailer coaches.

Motor coaches—Weight, 27·5 tons. Equipment: one 4-wheel motor truck with 36-in. wheels and two 200-h.p. motors, gear ratio 3·2 : 1; one 4-wheel trailer truck with 30-in wheels. Diameter of armature, 18·5 in. Weight of armature, 1,800 lb.

Trailer coaches—Weight, 16·2 tons. Equipment: two 4-wheel trucks with 30-in. wheels.

Weight of wheels: 36-in., 900 lb each; 30-in., 650 lb each.

[*Ans.* 133 tons.]

II—Motors (Chapters IV–VII)

1. The current/tractive-effort characteristic of a motor, based on a wheel diameter of 36 in., is as follows—

Current (amp)	80	160	240	320	400
Tractive effort (lb)	400	1,350	2,470	3,700	4,950

A motor bogie is fitted with two of these motors, one pair of wheels being 36 in. in diameter and the other pair 35 in. The motors are operated on the series-parallel system. If the tractive-effort at the 36-in wheels is 3,000 lb, what will be the current and tractive-effort of the other motor (*a*) in full series, and (*b*) in full parallel?

[*Ans.* (*a*) 276 A, 3,085 lb; (*b*) 257 A, 2,790 lb.]

2. Explain why d.c. series motors are preferred for electric traction on suburban railways.

The characteristics of a d.c. motor when operating at 600 V and driving 42-in. wheels are as follow—

Amperes	500	400	300	200
Train speed (m.p.h.)	22·1	23·9	26·4	31·4
Tractive effort (lb)	5,700	4,330	2,950	1,600

Two of these motors are mounted on a motor-coach. The wheels driven by one motor (A) are 42 in. in diameter, and those driven by the other motor (B) are 40 in. in diameter. Calculate (a) the power input to, and (b) the tractive-effort of, each motor when (i) the motors are in parallel and the train speed is 25 m.p.h., (ii) the motors are in series and the current is 400 A. The resistance of each motor is 0·08 ohm. (*L.U.*)

[*Ans.* (i) (a) 210 kW, 184 kW; (b) 3,600 lb, 3,170 lb.
(ii) (a) 122·6 kW, 117·4 kW; (b) 4,330 lb, 4,540 lb.]

3. Discuss the reasons why series motors are employed for electric traction on suburban railways. Why are trolleybuses usually equipped with compound motors, and why is a single motor employed in preference to two motors? Explain how low rheostatic losses at starting may be obtained with such equipment.

A motor-coach is equipped with two d.c. series motors, the characteristics of each at 675 V with 42-in. wheels are as follows—

Current (amp)	160	240	320	400
Speed (m.p.h.)	34·5	28·8	25·5	23·2
Tractive effort (lb)	1,350	2,470	3,700	4,950

If the wheels driven by one motor (A) are 42 in. in diameter and those driven by the other motor (B) are 40 in. in diameter, determine the total tractive-effort at a train speed of 27 m.p.h. when the motors are operating in parallel at normal voltage. (*L.U.*)

[*Ans.* 5,780 lb (A, 3,030 lb; B, 2,750 lb).]

4. The bogie of a motor-coach is equipped with two motors each having characteristics as given in Example No. 3, which apply to 42-in. wheels. If the wheels driven by one motor (A) are 41 in. diameter and those driven by the other motor (B) are 40 in. diameter, determine, when the motors are operating in parallel and the train speed is 30 m.p.h., (i) the current input to each motor, (ii) the total tractive-effort, (iii) the output at each driving-axle. (*L.U.*)

[*Ans.* (i) 210 A (A), 198 A (B); (ii) 4,020 lb; (iii) 123·5 kW (A), 116·5 kW (B).]

5. Discuss the advantages of motor control by field weakening.

A d.c. traction motor has the following characteristics—

Current (amp)	45	67·5	90	112·5	135	180	225
Train speed (m.p.h.)	41	33·3	28·4	25·2	23·8	21·1	20
Tractive effort (lb)	415	990	1,585	2,190	2,790	4,050	5,300

Calculate and plot the corresponding values when the motor is working with weakened field by cutting out one third of the turns of the field winding. (*C.G.*)

[*Ans.*

Current (amp)	45	67·5	90	112·5	135	180	225
Speed (m.p.h.)	68·8	48·9	36·2	30·6	27·9	23·7	22·1
Tractive effort (lb)	270	675	1,240	1,800	2,380	3,600	4,800]

6. Discuss the relative advantages of shunting and tapping the field of a series traction motor.

The characteristics of a motor are given in Example 3. Draw the new characteristics with the field shunted 30 per cent.

[*Ans.*

Amperes	160	240	320	400
Tractive effort (lb)	1,040	2,080	3,200	4,370
Speed (m.p.h.)	44·8	34·2	29·5	26·3]

7. The characteristics of a compound-wound trolleybus motor at 525 V with the shunt field fully excited are as follows—

Armature current (amp)	25	100	150	200
Speed (m.p.h.)	12·8	11·3	10·6	10·1
Tractive effort (lb)	440	1,950	3,100	4,240

When operating with the series-field winding alone the current/speed characteristic is—

Armature current (amp)	100	150	200
Speed (m.p.h.)	26	19·2	15·8

Calculate the characteristic (speed and tractive effort) when operating as a compound-wound motor at 525 V with an external resistance of 350 ohms inserted in the shunt field circuit. Resistance of armature and series-field windings, 0·15 ohm; resistance of shunt-field winding, 175 ohms; number of turns in each shunt field coil, 1,200; number of turns in each series field coil, 12. (*L.U.*)

[*Ans.*

Armature current (amp)	25	100	150	200
Speed (m.p.h.)	21·9	16·75	14·5	12·8
Tractive effort (lb)	257	1,315	2,270	3,350]

8. A compound trolleybus motor when run as a generator on open circuit at 1,000 r.p.m. gave the following results—

Excitation (amp-turns per pole)	1,800	2,400	3,600	4,800	5,400
Open-circuit e.m.f. (volts)	285	349	450	517	546

Calculate, for the range 50 to 200 A, the speed/current characteristics when operating at a line voltage of 525 V as (i) a compound motor with full shunt field, (ii) a series motor. The resistances are—(armature + series-field winding), 0·23 ohm; shunt winding, 160 ohms. The number of turns per pole are—series, 15; shunt, 1,100. Limit your calculations to 50, 100 and 200 amperes. (*L.U.*)

[*Ans.*

(i)	Armature current (amp)	50	100	150	200
	Speed (r.p.m.)	1,035	948	873	823
(ii)	Current (amp)	50	100	150	200
	Speed (r.p.m.)	4,100	2,075	1,475	1,180]

9. The speed/current characteristic of a compound trolleybus motor at 500 V and with full shunt field is as follows—

Armature current (A)	− 200*	− 150*	− 100*	0*	50	100	150
Speed (m.p.h.)	25·5	20	17	14	13·1	12·4	11·8

Determine the characteristic, over the range 75–200 A, when operating with the series-field winding alone. Resistance of armature and series-field windings, 0·2 ohm; resistance of shunt-field winding 100 ohms; number of turns per pole in series-field winding 15; number of turns per pole in shunt-field winding 900. (*L.U.*)

[*Ans.*

Armature current (amp)	75	100	150	200
Speed (m.p.h.)	32·3	22·6	17·7	15]

* Operating as a differentially-compounded generator at 500 V.

10. The cold resistance of the field winding of a motor is 0·043 ohm, measured at an air temperature of 12°C, and the hot resistance is 0·067 ohm measured at an air temperature of 15°C. Calculate the temperature rise. The resistance coefficient is 1/234·5 at 0°C.

[*Ans.* 134·5 C. HINT—$R_\theta = R_0(1 + \theta/234{\cdot}5)$]

III—CONTROL (CHAPTERS VIII–XII)

1. Discuss the advantages of series-parallel starting over ordinary rheostatic starting for a pair of d.c. traction motors forming part of the equipment of a *suburban* train. Show by diagrams how transition from series to parallel is usually effected in such equipments.

A motor-coach train, equipped with two motors and a series-parallel controller, is accelerated uniformly to a speed of 22 m.p.h. in 20 sec, the tractive-effort per motor being 4,000 lb. Determine approximately the energy (in kWh) lost in the starting rheostats. (*L.U.*)

[*Ans.* 0·486 kWh.]

2. Two 600-V motors, each having a resistance of 0·1 ohm, are started on the series-parallel system, the mean current per motor throughout the starting period being 300 A. The starting period is 15 sec and the train speed at the end of this period is 18 m.p.h. Calculate (i) the rheostatic losses (in kWh) during (*a*) the series and (*b*) the parallel combinations of the motors, (ii) the train speed at which transition from series to parallel must be made. (*L.U.*)

[*Ans.* (i) (*a*) 0·16 kWh; (*b*) 0·198 kWh; (ii) 8·52 m.p.h.]

3. An electric train (dead weight, 130 tons; accelerating weight, 143 tons) is equipped with four 600-V motors, which are arranged in two pairs for series-parallel control. If, during series-parallel starting the current per motor is maintained at 400 A, calculate (i) the duration of the starting period, (ii) the speed of the train at transition, (iii) the rheostatic losses during (*a*) the series, and (*b*) the parallel, steps of the starting period.

At this current (400 A) and normal voltage (600 V) the tractive effort per motor is 4,330 lb and the train speed is 23·9 m.p.h. Assume the specific train resistance to be 10 lb per ton, and the resistance of one motor to be 0·1 ohm. (*L.U.*)

[*Ans.* (i) 21·7 sec; (ii) 11·1 m.p.h.; (iii) (*a*) 0·292 kWh, (*b*) 0·387 kWh.]

4. The characteristics of a compound trolleybus motor when operating with full field at 525 V are as follows—

Line current (amp)	150	200	250
Speed (m.p.h.)	13·4	12·9	12·5
Tractive effort (lb)	2,400	3,300	4,400

Calculate the rheostatic losses when starting a bus weighing 11 tons, assuming a level surface, a mean starting current of 230 A, a line voltage of 525 V and a tractive resistance of 40 lb per ton. The total resistance of the armature and series field windings is 0·25 ohm. Allow 12 per cent for rotational inertia.

Calculate also the energy taken from the supply system from the start until a speed of 13·4 m.p.h. is reached. (*L.U.*)

[*Ans.* 67 Wh; 160·5 Wh]

5. Distinguish between rheostatic and regenerative braking as applied to electric traction. Explain, with the aid of diagrams of connexions, how the series motors of a train or locomotive can be used for (i) rheostatic braking, (ii) regenerative braking. Explain also how the braking torque is controlled in each case.

The characteristics of a series traction motor at 525 V are as follows—

Current (amp)	50	70	80	90
Speed (m.p.h.)	21	16·7	15·6	14·75
Gross torque (lb-ft)	160	260	312	370

Determine the gross braking torque at a speed of 16 m.p.h. when operating as a self-excited series generator and loaded with an external resistance of 6 ohms. Resistance of motor, 0·5 ohm. (*L.U.*)

[*Ans.* 282 lb-ft]

6. Show by diagrams (i) how the compound motor of a trolleybus is connected for rheostatic braking, (ii) how the series motors on a d.c. locomotive are connected for regenerative braking. Describe how the braking torque is controlled in each case.

The characteristics of a series motor at 525 V are as follows—

Current (amp)	75	125	175	225
Speed (r.p.m.)	1,200	950	840	745

Calculate the current when operating as a generator at 1,000 r.p.m., and loaded on a rheostat having a resistance of 3·25 ohms. The resistance of the motor is 0·35 ohm. (*L.U.*)

[*Ans.* 150 A]

7. The data of the open-circuit characteristic of a compound-wound trolleybus motor, determined by running the machine as a generator at 1,000 r.p.m., are as follows—

Ampere-turns per pole	1,800	2,400	4,800	5,400
Open-circuit e.m.f. (V)	285	349	517	546

Calculate the vehicle speed (m.p.h.) and the approximate braking tractive-effort when operating in service as a differentially-compounded generator with a rheostatic load of 3·5 ohms and an armature current of 150 A, the *separately excited* shunt-field current being 3 A.

The resistance of the armature and series-field windings is 0·15 ohm, and the number of turns per pole in the shunt and series windings is 1,200 and 12 respectively. When operating in service as a compound motor at 525 V with 150 A in the armature and a total excitation of 5,400 ampere-turns per pole, the speed is 11·2 m.p.h. and the tractive-effort is 3,100 lb. The wheels are 38 in. in diameter and the gear ratio is 9·33 : 1. (*L.U.*)

[*Ans.* 23·3 m.p.h.; 1,615 lb]

8. The characteristics of a compound trolleybus motor at 525 V with the shunt field fully excited are as follows—

Armature current (amp)	25	100	150	200
Speed (m.p.h.)	12·8	11·3	10·6	10·1
Tractive effort (lb)	440	1,950	3,100	4,240

When operating with the series-field winding alone the characteristics are—

Armature current (amp)	100	150	200
Speed (m.p.h.)	26	19·2	15·8
Tractive effort (lb)	850	1,680	2,640

Calculate the speed and approximate tractive-effort when braking regeneratively (differentially compounded) at armature currents of 100, 150, 200 A, the line voltage being constant at 525 V. Resistance of armature and series windings, 0·15 ohm; resistance of shunt winding, 175 ohms; number of turns in each shunt field coil, 1,200; number of turns in each series field-coil, 12.

[*Ans.*

Armature current (amp)	100	150	200
Speed (m.p.h.)	16·4	20·3	27·7
Tractive effort (lb)	1,320	1,680	1,700]

9. A motor-coach is equipped with two motors which are controlled on the series-parallel system. The resistance of each motor is 0·15 ohm; the maximum permissible starting current per motor is 400 A, and the average current is 350 A. Assuming the voltage is constant at 600 V during starting, and that for the range considered the flux is proportional to the fourth root of the current, determine the values of the resistances required for the first and second notches in series.

[*Ans.* Resistance of rheostats for first notch = 1·2 ohms
,, ,, second ,, = 0·8 ohm]

10. What should be the special characteristics of rheostats for starting d.c. electric goods locomotives?

It is desired to arrange for starting a certain locomotive and train so that the tractive effort is within given limits. At the upper limit the speed of the locomotive with motors in full series is 6·8 m.p.h., and the resistance drop in the motor is 12 per cent of its terminal voltage. At the lower limit the speed is 7·1 m.p.h., and the resistance drop 10 per cent of the terminal voltage. How many series notches are necessary in the control?

[*Ans.* 10. HINT—Use equation (vii), p. 169. NOTE—$\lambda = 0{\cdot}85$, $\gamma = 0{\cdot}833$, $e/V = 0{\cdot}12$, $\zeta = 1{\cdot}0$.]

11. Calculate the resistances of the series sections of the starting rheostat for the series-parallel control of two railway motors having characteristics as given in Example V, 7, the number of steps in the series portion of the controller being five. The initial starting current is 300 A, and the upper and lower limits of current during starting are 350 A and 253 A respectively. The resistance of each motor is 0·1 ohm.

[*Ans.* $R_1 - R_2 = 0{\cdot}538$; $R_2 - R_3 = 0{\cdot}435$; $R_3 - R_4 = 0{\cdot}353$; $R_4 - R_5 = 0{\cdot}286$ ohm]

12. Find the resistance of each section of the starting rheostat for the series-parallel control of two 500-V 250-h.p. series motors. There are 4 series and 3 parallel sections, and the lower current limit per motor is 350 A in each case. Over the required range the relation between the flux (Φ) and the current (I) is given by $\Phi = 5{\cdot}34 + 0{\cdot}0076I$. The resistance of each motor is 0·127 ohm. The upper limit of current per motor is about 450 A in each case. (*C.G.*)

[*Ans.* $R_1 - R_2 = 0{\cdot}27$ ohm; $R_2 - R_3 = 0{\cdot}232$ ohm; $R_3 - R_4 = 0{\cdot}194$ ohm; $R_4 - R_5 = 0{\cdot}167$ ohm; $R_{1p} - R_{2p} = 0{\cdot}19$ ohm; $R_{2p} - R_{3p} = 0{\cdot}162$ ohm; $R_{3p} - R_{4p} = 0{\cdot}14$ ohm (per motor). HINT—Use equation (32), p. 170, to calculate R_{1p}, R_{2p}, etc. and obtain $I_{1p} = 445$ A by trial.]

13. Calculate the resistances of the sections of the starting rheostats for the series-parallel control of two 225 h.p., 600-V, d.c. traction motors. The motor controller is arranged for automatic notching; there are five series and four parallel steps, and bridge transition is employed. The initial starting current is 300 A. The upper limit of current is 348·7 A for the series steps and 326 A for the parallel steps. The lower limit of current on all steps is 251·3 A. Each motor has a resistance of 0·12 ohm, and the speeds (at 600 V) corresponding to the currents 348·7, 326 and 251·3 A are 17·9, 18·4 and 20·5 m.p.h.

[*Ans.* Calculated resistances of sections (per motor)—*Series*: $R_1 - R_2 = 0{\cdot}2955$, $R_2 - R_3 = 0{\cdot}238$, $R_3 - R_4 = 0{\cdot}912$, $R_4 - R_5 = 0{\cdot}1545$ ohm. *Parallel*: $R_{1p} - R_{2p} = 0{\cdot}32$, $R_{2p} - R_{3p} = 0{\cdot}269$, $R_{3p} - R_{4p} = 0{\cdot}226$ ohm. NOTE—A complete solution is given in *Journ. I.E.E.*, Vol. 60, p. 881.]

14. Explain how regenerative braking can be obtained with single-phase series traction motors.

The e.m.f. generated between brushes of such a machine is 500 V, and the brushes are connected through a 0·5 ohm reactor to a 400-V tapping on the supply transformer. Neglecting losses, calculate the power regenerated. (*L.U.*)

[*Ans.* 400 kW. HINT—Use vector diagram, Fig. 151.]

IV—Rolling Stock and Locomotives (Chapters XIII–XVII)

1. A locomotive weighs 120 tons of which 80 tons is adhesive weight. Determine the maximum trailing load which can be started with an acceleration of 0·25 m.p.h.p.s. up a gradient of 1 per cent. Assume the coefficient of adhesion as 0·25, the train resistance as 10 lb per ton and allow 10 per cent for rotary inertia.

If the acceleration is maintained until the speed is 20 m.p.h. determine the output from the driving axles and the pull at the drawgear.

[*Ans.* 625 tons; 1,782 kW; 40,910 lb.]

2. A locomotive is required to start a 500-ton train on a gradient of 1 in 200 and accelerate it at 0·35 m.p.h.p.s. Assuming the coefficient of adhesion to be 0·25, the train resistance to be 10 lb per ton and allowing 10 per cent for rotary inertia, determine the minimum adhesive weight of the locomotive.

[*Ans.* 58·5 tons.]

V—Service Characteristics, Speed-Time Curves and Energy Consumption (Chapters XVIII and XIX)

1. A 15-ton car, driven by two series motors in parallel, takes 120 A from a 500-V line in ascending a gradient of 1 in 15 at a speed of 10 m.p.h. The gear ratio is 4·73 : 1. If this ratio be changed to 5·25 : 1, find the speed of the car and the current taken when ascending the same gradient. The resistance to traction is 20 lb per ton, and it can be assumed that, over the required range, the speed of the motor is inversely proportional to the current taken. (*L.U.*)

[*Ans.* 9·46 m.p.h.; 114·5 A.]

2. Two d.c. motors, each rated at 40 h.p., are driving a car weighing 16 tons at 12 m.p.h. up a gradient of 1 in 40. The tractive resistance is 15 lb per ton. The resistance of each armature is 0·3 ohm, and that of the field coils of each motor is 0·15 ohm. The motors being in full parallel, and the line voltage 550 V, find the current per motor. The overall efficiency is 75 per cent.

The controller is moved to a tap-field point cutting out 25 per cent of the field turns. Find the alteration in the steady current and speed on the same gradient. Assume that the flux per pole is proportional to the current in the field windings. (*L.U.*)

[*Ans.* Current increases from 66 A to 76·2 A; speed increases to 13·85 m.p.h.]

3. The specific tractive resistance of a train and electric locomotive on level and straight track is as follows—

Speed (m.p.h.)	30	40	50	60	70	80
Specific tractive resistance (lb per ton)	7	9·5	13	17	21	27

The locomotive is equipped with four d.c. motors, each of which has the following characteristics at 1,500 V—

Current (amp)	600	500	400	300
Speed (m.p.h.)	53·5	58·9	67·2	83·2
Tractive effort (lb)	8,070	6,220	4,400	2,730

The total dead weight of the train and locomotive is 500 tons and the effective mass is 550 tons.

Determine (i) the free running speed of the train on level track and also up a uniform gradient of 1 in 200; (ii) the current input to the locomotive for both of these cases. (*L.U.*)

[*Ans.* Speeds, 76 m.p.h., 69·5 m.p h.; currents, 1,300 A, 1,520 A.]

4. A 130-ton train is equipped with four motors, the characteristics of each at normal voltage are as follows—

Current (amp)	100	200	300	400
Train speed (m.p.h.)	65	36·5	29·8	26·5
Tractive effort (lb)	330	1,450	2,740	4,100

Calculate and draw the speed-time curve from the start to a speed of 36 m.p.h. for a run on level track, and determine the r.m.s. current per motor during this period. Assume constant normal supply voltage, a constant motor current of 400 A during rheostatic acceleration, a constant tractive resistance of 10 lb per ton, and the accelerating weight of the train to be 10 per cent greater than the dead weight. (*L.U.*)

[*Ans.*

Speed (m.p.h.)	0	26·5	28·5	30·5	32·5	36
Time (sec)	0	25·6	27·8	30·7	34·4	43

R.m.s. current = 333 A.]

5. A two-coach train weighing 65 tons is equipped with two motors, each having the following characteristics at 675 V—

Current (amp)	80	160	240	320
Tractive effort (lb)	400	1,350	2,470	3,700
Speed (m.p.h.)	53	34·5	28·8	25·5

Calculate the speed-time curve and the energy input from the conductor rails for the portion of a run on level track from start to 36 m.p.h. Series-parallel control is employed at starting, and the mean current per motor during rheostatic acceleration is 320 A. The tractive resistance is to be assumed at 10 lb per ton throughout, and the accelerating weight is to be taken at 10 per cent greater than the dead weight of the train. (*L.U.*)

[*Ans.*

Speed (m.p.h.)	0	25·5	27·5	30	32·5	36
Time (sec)	0	27·6	30	34·1	39·8	51·5

Energy = 4·28 kWh.]

6. The following data relate to the speed-time curve of a 127-ton train working off a 1,375 V d.c. system—

Speed (m.p.h.)	0	13·25	26·5	30	32·5	35	37·5	40
Time (sec)	0	9·3	18·6	21·6	24·8	29	34·5	41·2
Current (amp)		450	900	650	530	460	405	370

The motor current may be assumed constant until 26·5 m.p.h., all resistance being then cut out. At 40 m.p.h. power is cut off, and the train allowed to coast until 30 m.p.h. is reached, when the brakes are applied. Determine the watt-hours consumed per ton-mile.

Take retardation during coasting as 0·12 m.p.h.p.s., and retardation during braking as 2 m.p.h.p.s. (*L.U.*)

[*Ans.* 74·4. NOTE—Distance of run = 1·16 miles.]

7. A six-coach electric train, consisting of two motor coaches and four trailers, operates on an underground tube railway at a schedule speed of 15 m.p.h. with stops of 15 sec duration. Calculate the speed-time curve and energy consumption of this train for a run between two stations 0·38 mile apart, assuming the train to be supplied at a constant voltage of 550 V and the track to be level and straight.

The dead weight of the train is 138·5 tons and the accelerating weight is 151·7 tons. Each motor coach is equipped with two 200-h.p., 550-V motors, which are geared to 36-in. wheels, the gear ratio being 3·37 : 1. The characteristics of the motors at 550 V are—

Amperes	350	300	250	200	150	100
Speed (m.p.h.)	17·5	18·7	20·1	22·9	27·5	37·4
Tractive effort (lb)	4,850	3,940	3,030	2,140	1,320	590

The average accelerating current per motor is 300 A, the braking retardation is 2 m.p.h.p.s., the train resistance may be assumed at 8 lb per ton for all speeds, and the apparent train resistance during coasting may be assumed at 11·3 lb per ton.

[*Ans.* Speed-time curve—

Time (sec) . .	0	19·6	23·2	26·2	32·5*	65·1†	76·2
Speed (m.p.h.) .	0	18·7	21·3	22·9	25·5	22·2	0

Energy consumption—82·6 watt-hours per ton mile.]

8. Under actual service conditions the train of the preceding example operates on a graded straight track with stations at the same level. The profile of the track between centres of station platforms, in the direction of running, is as follows—

210 ft level; 240 ft 1 in 30 down; 870 ft level; 480 ft 1 in 60 up; 210 ft level.

Calculate the speed-time curve and energy consumption of the train for the actual running conditions, assuming data as above.

[*Ans.* Speed-time curve—

Time (sec) .	0	18	20	23·1	28·5*	43·2	53·2	58·9	67·2†	76·2
Speed (m.p.h.) .	0	18·7	21·1	24	27·1	25·6	22·9	20·4	18	0

Energy consumption—71·8 watt-hours per ton mile.]

VI—Overhead Construction (Chapters XXIII and XXIV)

1. What considerations determine the permissible sag to be given to a trolley wire for an electric tramway? Calculate the sag to be given to a trolley wire 0·4 in. in diameter and weighing 0·484 lb per ft if, when erected with a span of 120 ft, the stress in the wire when erected is 9,000 lb per in.^{2}

[*Ans.* 9·25 in.]

2. In a catenary system the catenary wire is steel with a maximum permissible tension of 1,000 kg. The weight of the catenary wire is 1 kg per metre run, the weight of the trolley wire is 1·5 kg per metre run, and the allowance for droppers and fittings is 20 per cent of the trolley wire weight. If the span is 80 metres, what is the minimum sag of the catenary?

[*Ans.* 2·24 metres.]

3. On an overhead catenary construction with a span of 300 ft the tension in the catenary wire is 2,000 lb and the sag is 10 ft. With the same weight per foot run, but the span reduced to 280 ft, what would be (*a*) the sag with the same tension, (*b*) the tension with the same sag?

[*Ans.* (*a*) 8·72 ft, (*b*) 1,745 lb.]

VII—Feeding and Distributing Systems (Chapters XXV and XXVI)

1. The resistance of the track rails on a railway is 0·1 ohm per mile, and the normal return current is 500 A. What size of copper feeder would be used in parallel with the rails if the voltage drop is limited to 10 V per mile, the resistivity of copper being 0·75 microhm-in.

[*Ans.* 1·9 in.^{2}]

2. Describe the action of a negative booster and state the advantages gained by using it.

A section *ABC* of a tramway rail system is 3 miles long. *A* is earthed, and *B* is 2 miles from *A*. The total resistance of the rails is 0·03 ohm per mile; the loading is 400 amp per mile and may be assumed uniformly distributed.

A negative feeder (connected to a booster) is tapped to the rail at *B* and maintains

* Power off. † Brakes applied.

the potential of B at -2 volts. Calculate and plot a graph showing how the potential of the rail varies along its length, neglecting leakage currents. (*L.U.*)

[*Ans.*

Distance from A (miles)	. 0	0·25	0·5	0·75	0·917	1·0			
Voltage (from A) .	. 0	2·36	4	4·85	5·04	4·97			
1·25	1·5	1·75	1·835	2·0	2·185	2·25	2·5	2·75	3·0
4·37	3·0	0·88	0	− 2·0	0	0·62	2·5	3·62	4·0]

3. *An overhead trolley wire in a tramway system has a resistance of* 0·46 ohm per mile, and the maximum permissible voltage drop in a distributing section is 55 V. The cars on the section have a schedule speed of 10 m.p.h. and there is an interval of 2 min between the cars. The average current per car is 25 A. Find the length of distributing section corresponding to the maximum voltage drop, assuming the average voltage drop to be 50 per cent of the maximum. (*C.G.*)

[*Ans.* 1 mile.]

4. In a single-catenary system the cross-sections of the wires are: copper catenary, 0·2 in.^{2}; trolley wire, 0·1 in.^{2} The line is single track and the sub-stations are 8 miles apart. Determine the voltage drop in the overhead line when a locomotive taking a current of 500 A is 2 miles from a sub-station. Resistance of copper is 0·75 microhm-in.

[*Ans.* 120 V.]

5. On a 1,500-V d.c. railway there are two sub-stations A and B 10 miles apart. The first 4 miles of track from A is single and the remainder is double. The resistance per mile of the single track is 0·05 ohm and that of the double track is 0·025 ohm. The overhead line is of constant cross-section and the lines over the double track are cross bonded. Determine the minimum cross-section of the overhead lines in order that the maximum voltage drop shall be 100 V when a locomotive taking a constant current of 200 A runs from A to B. Resistivity of copper 0·75 microhm-in.

[*Ans.* 0·2015 in.^{2}]

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